An application of optimal control theory to agricultural policy analysis

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ABSTRACT


Since the early 1970s there has been interest in the application of optimal control theory to the management of economic systems. Specifically, optimal control theory prescribes policy strategies which optimise a quantifiable policy preference function subject to market equilibrium conditions. Problems of this kind have been identified among agricultural markets and this paper aims to illustrate the application of optimal control theory to the British potato market. The paper takes evidence from policy makers to derive target values for the producer price, imports, and the changes in the quota area from year to year. The constraints on optimisation are specified in terms of a partial equilibrium econometric model which specifies, demand, supply and trade relationships. The policy preference function is specified as a quadratic and a ‘revealed preference approach’ is employed to estimate the parameters which penalise market equilibria which over or under-shoot policy targets. The resulting optimal control problem is minimised by a dynamic programming routine. The results suggest that policy makers may benefit from taking dynamic effects directly into account when formulating policy strategies.

1. Introduction

Agricultural policy instruments are formulated for market systems which are stochastic and dynamic; that is, the effects of policies are uncertain and policy decisions in the current period affect the market both in the current period and in subsequent periods. Policy makers are thus confronted with planning problems which require an appreciation of the possible range of
market outcomes through time if policy is to be directed purposefully towards achieving specified objectives. Optimal control theory, a mathematical theory of dynamic systems, provides a coherent framework for representing such market management problems: in control-theoretic terms market outcomes represent the state of the system, while the policy instruments are the decision variables which drive the state variables towards target levels.

The suitability of control theory for the analysis of such policy problems has been reviewed in the economics literature in general (see Pindyck, 1973) and the agricultural economics literature in particular (see Burt, 1969; and Rausser, 1978). So far, the number of applications are limited; Freebairn and Rausser (1974) consider the applications of adaptive control in relation to U.S. beef import quotas, and Richardson and Ray (1982) apply control techniques to the analysis of U.S. commodity programmes. In part, this limited use may be attributable to the complexities of the techniques, particularly when allowing for uncertainty and learning processes. However, from both a management and an analytical perspective, there are many insights to be gained from considering policy formation as a process of maximising a specific policy preference function. In this article, we evaluate the application of control theory to the British potato market.

The paper tackles three issues related to control problems organised in the following sections. Section 2 considers the specification of a partial equilibrium market model of the potato market. Section 3 formulates policy decisions as a control problem based upon the market model and discusses the specification of policy targets and proposes a revealed preference approach to their derivation. Results and conclusions are presented in Sections 4 and 5.

2. Simulation model of policy

The objectives of the British potato market policy, implemented by the Potato Marketing Board (PMB), are (see Marsh, 1985): 'fair and reasonable' prices for producers and consumers; approximate self-sufficiency in main crop potatoes, interpreted as a small but positive import volume onto the British market to meet processing constraints; and market stability.

The major instrument in relation to the objectives is a quota area regime with individual producers being subject to financial penalties for planting in excess of their quota allocations. The quota is set via annual negotiations between the government, represented by the Ministry of Agriculture, Fisheries and Food and the producers, represented by the National Farmers' Union. A consumers' committee performs a monitoring role. In addition to
the quota, the PMB also has powers to implement a limited support buying programme to provide a price floor to the market in years of unusually high yields.

Since 1979 the British market has been open to trade in potatoes, and there are regular but small imports of maincrop\textsuperscript{1} potatoes from elsewhere in Europe, with the quantity responding to the British market price. Trade in effect imposes a constraint on the policy outcomes: if the government acts to increase the price of potatoes by reducing the quota, then imports are attracted to the British market, it becomes increasingly difficult to increase prices further and the level of self-sufficiency is reduced.

It is therefore essential to include trade linkages within a policy model. A convenient framework for modelling and analysis is in terms of excess supply and demand in the trading countries. The location and slope of the relevant supply and demand schedules are influenced by productive capacity, input costs, income levels and tastes. The model identifies as the principal trading partners The Netherlands and Great Britain, with Great Britain being an importer from The Netherlands. West Germany is included as a regular net importer from the Dutch market. Since transport costs for Dutch exports into the north of Germany are typically lower than those associated with exports to the British market and there is a long standing tradition of trade in potatoes between the two countries, it was considered reasonable to combine the two countries into a single trading bloc which would be, on average, an exporter to the British market. Thus the relevant trade functions are taken to be an import (excess) demand schedule for Britain and a net export (excess) supply function for the bloc combining Holland and Germany.

This study focuses specifically on the role of an area control policy which aims to stabilise the market over time by matching expected supply to trends in demand, taking into account developments in area planted and trends in yield. Accordingly, the model used is both dynamic and deterministic. The behavioural equations were estimated econometrically\textsuperscript{2} and are presented in summary form in the appendix. For each country we assume that demand is a non-stochastic function of price in that country; the econometric estimates of demand relationships suggested that a linear form was appropriate. For any given season, supply is taken to be perfectly price-inelastic. Over time, yield develops on trend with the linear specification being selected on the grounds of statistical performance. Area in the British market is determined recursively as a function of price expectations and an

\textsuperscript{1} There are two crops of potatoes grown in Britain: 'earlies' and 'maincrop'. The latter constitute about 75\% of the total supply, and the analysis in this paper applies to that crop.

\textsuperscript{2} Full details of the econometric estimation of the various functional relationships can be found in Ennew and White (1988a, b).
adjustment coefficient, while for Holland and West Germany, a simple autoregressive process is used to model plantings. Throughout the model exchange rates are exogenously determined and per unit transport costs are constant. For simplicity prices are expressed in terms of Britain's currency using appropriate exchange rates.

In order to apply this model of trade to maincrop potatoes in the context of British policy, the two instruments of quota area and a desired floor price coupled with limited support buying must be introduced into the model. The area quota is set to induce a specified level of plantings, but plantings themselves depend upon the quota in addition to price expectations. The basic model therefore treats quota levels and plantings as being simultaneously determined. The impact of intervention purchases is evaluated by prespecifying a desired minimum support price, and an upper limit on support buying. Support buying will be implemented when market clearing prices in Britain fall below the minimum support price. This can occur as a result of high domestic yields and a high level of imports (a consequence of high yields in The Netherlands/West Germany bloc). The policy maker's attitude to support buying in the potato market is that it should only be necessary in years of unusually good domestic harvests. Accordingly the price which triggers support buying is set at a low level compared with the mean price. Under autarky a large domestic supply shift is required to induce Britain to engage in support buying. However with trade the likelihood of support buying at a given support price increases since a relatively low support price in Britain may be relatively high for The Netherlands/West Germany bloc. Should the presence of imported potatoes force the PMB to engage in support buying then the result will be a higher price and a higher quantity traded than would have occurred in the policy-free situation (see Ennew et al., 1988).

The model was calibrated for the period 1981-85 and used to simulate the behaviour of the market in the period 1986-90 using the assumed trend yields. The structure of the model and details of the estimation procedures are given in Ennew and White (1988, b); the final forms of the estimated equations are given in the Appendix. The simulation model mimics the operation of policy and the responses of the market in accordance with some predetermined policy rules, but these rules are implicit rather than explicit. In the next section we specify the objective function explicitly, and consider the setting of policy within a dynamic context.

3. Optimal control

Dynamic optimisation of policy requires the specification of a formal objective function and the derivation of optimal policy instruments over a
specified time period subject to the constraints set out in a model of the relevant market. The objective function consists of a set of targets for policy variables and a set of relative weights attached to the targets. The choice of policy targets is the outcome of a bargaining process between government policy makers and the interest groups affected by policy. The policy preference function incorporates trade-offs between targets as reflected in the weights. In principle, the weights are 'revealed' by past levels of policy instruments, assuming that policy outcomes represent the consistent optimisation of the policy objective function. Hence, it may be possible to estimate the weights from time series data, given the knowledge of the underlying system generating values of the policy variables.

Here we estimate the policy preference function for British potato policy assuming annual decision making in line with the actual implementation of policy. More specifically, we assume that actual policy making implicitly embodies the optimisation of the appropriate policy preference function one year at a time. Then we apply control theory to determine the optimal setting of the appropriate policy instrument — area control — in a future multi-period setting through optimising the revealed policy preference function subject to the constraints imposed by a dynamic econometric model of the potato market. Thus we identify an optimal time path for policy taking account of dynamics and indicate that this approach has the potential to improve upon the existing rules employed by policy makers.

We specify the following implicit annual objective function within the institutional framework as:

\[
\text{Minimise } Z_t = 0.5[c_1(P_t - \bar{P}_t)^2 + c_2(M_t - \bar{M}_t)^2 + c_3(\Delta QA_t - \Delta Q\bar{A}_t)^2]
\]

where \(\bar{P}_t\) is the price target for year \(t\), \(P_t\) is the actual price in year \(t\), \(\bar{M}_t\) is the import target for year \(t\), \(M_t\) is the actual volume of imports in year \(t\), \(\Delta QA_t\) is the target for quota area change in year \(t\), \(\Delta Q\bar{A}_t\) is the actual quota area change in year \(t\), and \(c_1, c_2\) and \(c_3\) are relative weights.

The objective function includes the policy instrument, in the form of the desired change in quota area which will be negative. This can be viewed as a 'rule of thumb' which indicates to producers the general downward trend in quota allocation in response to rising yields in Britain as embodied in the actual policy process. Here it is proposed that deviations from this level of quota change impose adjustment costs upon producers and induce market instability. The control problem specifies that the objective function is optimized over a planning horizon subject to a set of constraints which are determined by the market structure. Formally the optimal control problem is defined as:
Minimise

\[ Z = 0.5 \sum_{t=1}^{T} \left[ c_1(P_t - \tilde{P}_t)^2 + c_2(M_t - \bar{M}_t)^2 + c_3 (\Delta QA_t - \Delta \bar{QA}_p)^2 \right] \]  \hspace{1cm} (2)

subject to

\[ AP_t = f(AP_{t-1}, \Delta QA_t, \Delta P_{b,t}, \Delta EL_t) \]
\[ MA_t = AP_t - EA_t \]
\[ QS_t = Y_t MA_t + CO_t - \text{WASTE}_t - \text{SEED}_t \]
\[ \hat{P}_{b,t} = f(QS_t) \]
\[ P_{b,t} = f(\hat{P}_{b,t}, \hat{P}_{d,t}, TC_t, QSB_t) \]

where \( \hat{P}_b \) is the autarky price in Britain, \( P_b \) the market clearing price in Britain under trade, \( \hat{P}_d \) the autarky price in The Netherlands/West Germany, \( M \) British imports from The Netherlands/West Germany, \( QA \) quota area, \( AP \) all plantings, \( MA \) maincrop area, \( EA \) early plantings, \( EL \) excess levy, \( QS \) maincrop supply, \( Y \) maincrop yield, \( CO \) carry over from the early crop, \( TC \) transport costs per metric tonne (t), \( QSB \) support buying, and \( t \) is the time subscript.

The policy objective function is presented as a quadratic, which implies that policy makers will be indifferent between undershooting or overshooting desired values of a target or control variable by a given amount. This is clearly a simplification of the true objective function but is necessary to give a function which is tractable for optimisation purposes.

A graphical interpretation of policy implementation under this preference function is given in Fig. 1 with reference to the price and import targets on-

![Fig. 1. Policy preference function.](image-url)
The cost of divergence of the actual price \( P \) from the target price \( \bar{P} \) is 
\[ 0.5c_1(P - \bar{P})^2, \]
so that the marginal cost of divergence from that target is 
\[ c_1(P - \bar{P}). \]
If, as is usually the case here, the market price is below the target price then we may represent this marginal cost in the left hand panel of the diagram by a straight line drawn from \( \bar{P} \) having (positive) slope \( c_1 \). The cost of the divergence from the import target is 
\[ 0.5c_2(M - \bar{M})^2, \]
so that the marginal cost of divergence from that target is 
\[ c_2(M - \bar{M}). \] We could represent this marginal cost by another straight line drawn from a point on the import axis, but this does not give a convenient comparison with the marginal cost of missing the price target. However, the excess supply curve gives us a means of converting this marginal cost into its ‘price equivalent’, since any level of imports may be converted uniquely to a price. This then allows us to represent the marginal cost of missing the import target as a straight line of (negative) slope \( c_2/\eta^2 \) drawn from point \( \bar{P} \) on the price axis, \( \eta \) being the slope of the excess supply curve. The market price \( P \) and import level \( M \) will then be the optimal outcome if the two marginal cost lines intersect at that price, and the total costs of missing the targets will be shown by the two shaded areas. The market outcomes for price and imports are determined by the equations:

\[
\begin{align*}
P_t &= (\eta + \beta_0)^{-1}(P_{b,t}\eta + \hat{\pi}_t \beta_0) \\
&= \hat{P}_{b,t} \\
M_t &= (\eta + \beta_0)^{-1}(\hat{P}_{b,t} - \hat{\pi}_t) \\
&= 0
\end{align*}
\]

for \( \hat{P}_{b,t} > \hat{\pi}_t \)

for \( \hat{P}_{b,t} \leq \hat{\pi}_t \) \hspace{1cm} (3)

for \( \hat{P}_{b,t} > \hat{\pi}_t \)

for \( \hat{P}_{b,t} \leq \hat{\pi}_t \) \hspace{1cm} (4)

with

\[
\hat{P}_{b,t} - \hat{P}_{b,t-1} = -\beta_0\Delta Q_t \equiv -\beta_0(Y_{t-1}\psi \Delta QA_t + A_t \Delta Y_t) \hspace{1cm} (5)
\]

\[ ^3 \text{The full policy preference function is not amenable to a graphical treatment since the target for change in quota area introduces a dynamic element into the analysis.} \]
Equations (3) to (5) represent equilibrium price and quantity outcomes derived from the model’s structural equations as presented in the Appendix. The market clearing price and quantity are given by $P_t$ and $M_t$; $\bar{P}_{b,t}$ is the British self-sufficiency price (price intercept of the $ED_b$ curve). $\pi_t$ is the minimum import price inclusive of transport costs, $Q_t$ is British production, $A_t$ is British planted area, and $\Delta Y_t$ is the constant increment in British yield. The response of area planted to a change in quota area is given by $\psi$. The parameter $\beta_0$ is the slope of the British excess demand curve, $ED_b$, and $\eta$ is the slope of the excess supply curve.

If the policy preference function (1) is minimised subject to (3), (4) and (5) then the equilibrium condition is:  

$$c_1(P_t - \bar{P}_t) = -c_2\eta^{-1}(M_t - \bar{M}_t)$$

$$+ c_3(\eta + \beta_0)\psi Y_{t-1}^{-1} (\Delta QA_t - \Delta QA_{t-1}) \tag{6}$$

Defining $(\beta_0\psi Y_{t-1})^{-1}$ as $\theta_{t-1}$ and $\eta + \beta_0$ as $\gamma$, assuming $M_t > 0$, and using (3) and (4) allows us to rewrite (6) as:

$$\bar{P}_{b,t} = (c_1\eta^2 + c_2)^{-1}\gamma(c_1\bar{P}_t + c_2\eta^{-1}\bar{M}_t) + (c_2 - c_1\eta\beta_0)\hat{\pi}_t$$

$$+ c_3\gamma^2\theta_{t-1}(\Delta QA_t - \Delta QA_{t-1}) \tag{7}$$

Actual policy decision making with respect to the level of quota area—the policy instrument—is assumed to produce the optimal outcome described in (6). However, we conceptualise the policy making process in terms of (7). Specifically, the quota area fixes the position of the excess demand curve in terms of $\bar{P}_{b,t}$, which given the position of the excess supply curve ($\hat{\pi}_t$) determines $P_t$ and $M_t$. This decision is constrained by the adjustment cost term; that is it takes into account costs associated with deviations of the change in quota area from its policy target value. However, the quota area and therefore $\bar{P}_{b,t}$ is determined prior to growers’ planting decisions when $P_t$ is unknown since the latter relates to the post harvest period. Consequently, $\bar{P}_{b,t}$ may be regarded as being determined in relation to a forecast of $P_t$ made at the time the quota area is determined. Let $\pi_t^f$ be this forecast, and let it replace $\hat{\pi}_t$ in equation (7).

In principle equation (7), with $\hat{\pi}_t$ replaced by $\pi_t^f$, provides a basis for

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4. (6) is obtained by forming a Lagrangean from (1), (3), (4) and (5) and setting the derivatives with respect to $P_t, M_t, \bar{P}_{b,t}$ and $\Delta QA_t$ equal to zero and substituting to eliminate the Lagrangean multipliers.

5. The forecast is assumed to be the one-step-ahead forecast from the econometric model. Note that this forecast is implicit in actual decision making since the $ES_d$ curve constrains the options open to policy makers.
estimating the policy weights from time series data if $\hat{P}_{b,t}$ is replaced by equation (5) and the resulting equation solved for $QA_t$, the policy decision variable. However, given a limited data set (7 years, 1981–1987) and the nonlinearity of the equation in terms of the policy weights, this was not feasible. Rather the following procedure was used. First, set $c_3 = 0$ and normalise $c_2$ on unity to give, from (7), an equation for $\hat{P}_{b,t}$:

$$\hat{P}_{b,t} = (c_1\eta^2 + c_2)^{-1}[\gamma(c_1\hat{P}_t + c_2\eta^{-1}\hat{M}_t) + (c_2 - c_1\eta\beta_0)\pi^f_t]$$

(8)

Second, estimate $c_1$ using nonlinear least squares from (8). Third, specify a relationship between $c_1$ and $c_3$, holding the ratio of $c_1$ to $c_2$ constant, that could be utilized for alternative runs of the optimal control model.

Time series data for the period 1981–1987 were used to estimate the policy weights. The policy targets were assumed to be constant over this period and were specified as follows. First, discussions with policy makers suggested that a real price of £70–75/t was a suitable figure for this period. Second, an import target of 25 000 t was selected after consideration of the import requirements of processors. Third, the rule of thumb for desired quota area adjustment was set at:

$$\Delta QA = -2.4 \text{ million ha}$$

(9)

a figure which just offsets the impact on production of the upward trend in yield in Britain. The econometric model reported in the Appendix provides information on the parameters $\eta$, $\beta_0$, $\gamma$ and $\theta_{t-1}$, and was used to calculate one step ahead forecasts $\pi^f_t$ and a series for $\hat{P}_{b,t}$.

Equation (8) was fitted with $\hat{M}_t$ set at 25 000 t each year and a search was made over values of $\hat{P}_t$, assumed constant for each year, between £70/t and £80/t in steps of £1/t. The minimum residual sum of squares was found with a value of $\hat{P}_t = £74/t$ giving an estimate of $c_1$ of 1916 with a standard error of 1092.

The relationship between $c_1$ and $c_3$ was specified to be negative whilst preserving the estimated relationship $c_1 = 1916c_2$. Thus if $c_3$ is zero, there is a static optimal trade-off between $\hat{P}_t$ and $\hat{M}_t$ which can be obtained by fixing the quota to a desired level. However if $c_3 > 0$, so that adjustment costs constrain the change in quota around the policy rule, then this implies that less weight is given to achieving the optimal static trade-off. The following pair of equations were used to fix the relationship between the weights:

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6 An import ban was in force in years prior to 1979 such that imports were only allowed in years of low British yields/high British prices. The period 1979 to 1980 was regarded as a period of adjustment in policy. This left 1981–1987 as the period of estimation.

7 The $R^2$ of the equation was 0.75 and the D-W statistic was 1.75.
\[ c_1 = 1916 - \delta c_3 \]  
\[ c_2 = 1 - c_3 \delta /1916 \]

The value of \( \delta \) fixes the weights \( c_1 \) and \( c_2 \) relative to \( c_3 \). Thus if \( \delta \) is 'low', \( c_1 \) and \( c_2 \) are invariant to the choice of the unknown \( c_3 \); conversely, if \( \delta \) is 'high', then \( c_1 \) and \( c_2 \) are affected quite markedly by the value of \( c_3 \). A figure for \( \delta \) was set in the following ad hoc manner. First, \( c_3 \) was arbitrarily constrained to lie in the range of zero to 4000. Second, \( \delta \) was constrained such that, for these values of \( c_3 \), the value of \( c_1 \) was always greater than 500. Third, \( \delta \) was chosen as 0.35, a figure which gave the maximum response of \( c_1 \) to \( c_3 \) within these constraints. This figure for \( \delta \) was used in conjunction with equations (10) and (11) to generate alternative weights for \( c_1 \) and \( c_2 \) for \( 0 < c_3 \leq 4000 \) for use in the optimal control model.

4. Results

The optimisation problem can be solved using dynamic programming to search over a grid of states for a specified time period in order to identify an optimal time path for policy. This method is rigorous in the sense that it becomes equivalent to the algebraic solution of an optimal control problem as the differences between the states tend towards zero (Chow, 1975, chapter 8).

The time period for solution was 1986–1990. Four optimisation runs were undertaken with high and low adjustment costs and with fixed and variable targets. In all cases, the end state is free which allows the model to select its own cost-minimising end state. The optimisation results are compared with the results of the simulation model outlined in Section 2.

The fixed targets were set at the values specified previously for the policy preference function; \( \bar{P}_t = \£74/t \), \( M_t = 25000 \) t and \( \Delta Q\bar{A}_t = -2.4 \) million ha. The specification of variable targets took into account trends in the import supply curve. Specifically, over the control period, the econometric model predicts that the import supply curve is shifting systematically downwards. The targets, \( \bar{P}_t \) and \( \bar{M}_t \), were adjusted through time such that for every £1 fall in the expected minimum import price, \( \bar{P}_t \) falls by £0.74/t and \( \bar{M}_t \) rises by 6700 t. These shifts hold the target price/target import trade-off for the policy weights \( c_1 = 1916 c_2 \). On the other hand the quota rule, \( \Delta Q\bar{A}_t = -2.4 \) million ha, was maintained. Thus policy makers under dynamic optimisation are assumed to adjust price and import targets in response to falling import prices but maintain the policy rule for quota area solely in response to the trend rise in British potato yield.

The alternative values for low and high adjustment costs were chosen as
TABLE 1

(a) Policy weights

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adjustment costs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>(c_1)</td>
<td>516</td>
<td>1741</td>
</tr>
<tr>
<td>(c_2)</td>
<td>0.27</td>
<td>0.91</td>
</tr>
<tr>
<td>(c_3)</td>
<td>4000</td>
<td>500</td>
</tr>
</tbody>
</table>

(b) Policy targets

<table>
<thead>
<tr>
<th>Year</th>
<th>Fixed targets</th>
<th>Variable target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Imports</td>
</tr>
<tr>
<td>1986</td>
<td>74.00</td>
<td>25.00</td>
</tr>
<tr>
<td>1987</td>
<td>74.00</td>
<td>25.00</td>
</tr>
<tr>
<td>1988</td>
<td>74.00</td>
<td>25.00</td>
</tr>
<tr>
<td>1989</td>
<td>74.00</td>
<td>25.00</td>
</tr>
<tr>
<td>1990</td>
<td>74.00</td>
<td>25.00</td>
</tr>
</tbody>
</table>

\(c_3 = 500\) and \(c_3 = 4000\) to give considerable sensitivity in the response of outcomes to different weights. Tables 1a and 1b outline the various targets and weightings used in generating the results which are presented in Tables 2 – 4. Included in the tables is a comparison of the costs generated by the ‘optimal’ policies and the costs generated by the simulation outcomes when the same policy cost function is applied.

Results from the forecast period allow us to examine the effects of switching from an existing set of rules for policy determination to a new set derived from optimal control. On the strength of these results a number of general points can be made. First, the policy costs of continuing to follow existing sets of rules as indicated by the simulation outcomes are high compared to what could be achieved from optimisation. This would indicate that in a deterministic world the current policy instruments are in some sense ‘inferior’, although it is not clear whether such results would ‘carry over’ if we allow for the presence of uncertainty. Second, the optimal control rules suggest a much greater overall reduction in quota and consequent higher price, with the result that we observe a markedly higher level of imports, but with the current objective function these are more than offset by the gains from higher prices. Following the optimal control rules would therefore allow
policy makers to obtain a better price (for producers) in the domestic market, although the penalty of so doing is a larger costs on imports and a slightly larger adjustment cost.
TABLE 4

Changes in the quota area

<table>
<thead>
<tr>
<th></th>
<th>Optimisation</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed targets</td>
<td>Variable targets</td>
</tr>
<tr>
<td></td>
<td>Adjustment costs</td>
<td>Adjustment costs</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>1986</td>
<td>-6.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1987</td>
<td>-3.4</td>
<td>-6.0</td>
</tr>
<tr>
<td>1988</td>
<td>-3.4</td>
<td>-1.4</td>
</tr>
<tr>
<td>1989</td>
<td>-9.5</td>
<td>-9.3</td>
</tr>
<tr>
<td>1990</td>
<td>-8.5</td>
<td>-14.7</td>
</tr>
</tbody>
</table>

5. Conclusions

The dynamic optimisation of a policy model in a deterministic framework is presented here as an analytical approach which may be regarded as an alternative to simulation. By treating the control problem as deterministic we avoid many of the difficulties of stochastic and adaptive control while still being able to focus explicitly on policy decision making in pursuit of specific objectives, and the specific adjustment costs and trade offs which this entails. In addition, it may usefully be regarded as complementary to simulation analysis in the sense that it facilitates comparisons between what might be regarded as 'optimal policies' and those generated by existing policy rules. While it seems unlikely that dynamic optimisation will become a management tool for the policy decision process itself, it may prove valuable for periodic evaluations of existing policy rules in terms of their deviation from some optimal rule.

References

Appendix: the linear model

Great Britain

Area planted

\[
\begin{align*}
\text{AP}_t &= \text{AP}_{t-1} + (\beta_7 + \beta_9) \Delta P_{t-1} + (\beta_8 + \beta_{10}) \Delta QA_t + \beta_9 \Delta EL_t \\
&= \text{AP}_{t-1} + 0.0561 \Delta P_{t-1} + 0.610 \Delta QA_t + 0.0108 \Delta EL_t
\end{align*}
\]

Early plantings

\[
\begin{align*}
\text{EA}_t &= \alpha_1 + \beta_1 \text{EA}_{t-1} \\
&= 10.77 + 0.77 \text{EA}_{t-1}
\end{align*}
\]

Maincrop area

\[
\begin{align*}
\text{MA}_t &= \text{AP}_t - \text{EA}_t
\end{align*}
\]

Maincrop yield

\[
\begin{align*}
Y_t &= \alpha_2 + \beta_2 t \\
&= 17.78 + 0.68t
\end{align*}
\]

Seed production

\[
\begin{align*}
S_t &= (\beta_3 + \beta_4) \text{AP}_{t-1} \\
&= 3.0 \text{AP}_{t-1}
\end{align*}
\]

Seed exports

\[
\begin{align*}
\text{SX}_t &= \alpha_5 + \beta_5 t \\
&= -0.903 + 1.88t
\end{align*}
\]
Demand
\[ P_{b,t} = \alpha_0 - \beta_0 Q_t \]
\[ = 530 - 0.111Q_t \]

*The Netherlands*

Area planted
\[ AH_t = \phi_2 + \varphi_2 AH_{t-1} \]
\[ = 21.24 + 0.78 AH_{t-1} \]

Yield
\[ YH_t = \phi_1 + \varphi_1 t \]
\[ = 25.11 + 0.510t \]

Seed exports
\[ SH_t = \phi_4 \]
\[ = 500 \]

Demand
\[ P_{h,t} = \phi_3 - \varphi_3 QH_t \]
\[ = 383.3 - 0.139QH_t \]

*West Germany*

Area planted
\[ AG_t = \lambda_2 + \omega_2 AG_{t-1} \]
\[ = 42.0 + 0.80 AG_{t-1} \]

Yield
\[ YG_t = \lambda_1 + \omega_1 t \]
\[ = 19.40 + 0.397t \]

Demand
\[ P_{g,t} = \lambda_3 - \omega_3 QG_t \]
\[ = 383 - 0.0528QG_t \]