Pricing under Uncertainty in Agricultural Grain Markets and the Objectives of Cooperatives: A Mixed Oligopoly Analysis

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Abstract

The lack of consensus on the objective of the agricultural cooperative (co-op) has been an obstacle for the advancement of co-op theory and empirical implications. This paper develops a theoretical model to compare pricing behavior of co-op and for-profit investor owned firm in the context of mixed oligopoly with uncertain future profit. We focus specifically on the change in market equilibrium as the objective of cooperative firm moves along the objective spectrum from profit-maximization towards maximization of member benefit.

Key words: Agricultural cooperative, Mixed oligopoly, Decision under uncertainty

JEL classification: D4, D43, L1, L2, Q1

Introduction

The objective, operating behaviors and subsequent performances of the agricultural cooperative (co-op) are purported to be different from investor owned firms (IOF) because of its unique ownership structure. The co-op’s ownership structure that is characterized by user-owner and user-benefit principles have given rise to diverse assumptions regarding the goal of co-op in the existing literature, which yet can be categorized into two main approaches: 1) the neoclassical microeconomic approach that regards the cooperative as a single objective-optimizing firm and 2) the game theoretical approach that focuses on the collective decision-making of a co-op’s heterogeneous members. A co-op could be an independent firm seeking for profit maximization(Sexton 1990) or be a vertical integration to producers’ on the farm business to increase the price for producers’ raw product(Emelianoff 1942). Alternatively, a co-op could only seek to maximize the profit of members(Choi and Feinerman 1993) or more generally to maximize the weighted sum of the member profit and the co-op’s profit(Phillips 1953). A recent reviews of cooperatives objectives and performances is provided by Soboh et al. (2009). Although each co-op’s objective is able to capture an aspect of co-op’s fundamental characteristics, the lack of consensus on the ob-
jective of cooperatives has been perhaps the greatest obstacle to advance the cooperative theory (Cotterill 1987) and to provide testable hypotheses for empirical studies Soboh et al. (2009). However, understanding the connections among various co-op objectives and their implications is crucial in the absence of clear cut agreement. LeVay (1983) discusses the implications of different objectives on co-op and its member behaviors while to our knowledge, there has been no published paper presenting a generalized model that accommodates several co-op objectives and compares the co-op’s decision to its IOF counterpart.

In this paper, we fill in this gap by constructing a mixed oligopoly model that features the pricing competition in the grain input market between an agricultural cooperative and investor owned firm under different objectives of co-ops. Acknowledging that cooperatives with different business orientations could also differ widely regarding their business objectives, we focus our attention in the agricultural marketing business to facilitate the analysis. We also take into account roles of the uncertain nature of the co-op’s business and its risk attitude played in the decision of member-producers, which has received little attention in existing co-op theories. However, this decision making under uncertainty of producer’s has important implication on the co-op’s operating behaviors as producer’s decision to sell to the co-op depends on how much patronage refund she will receive in the year end which is tied to the co-op’s future profit but unknown today.

Our results show that a risk-neutral co-op that only maximizes profit pays a higher mill price but has a smaller market share than the IOF. Such competitive disadvantage of co-op is worsen, measured by reduced market share, as co-op becomes risk averse. However, if a risk-averse co-op considers both cash payments to producers and its own profit, it’s feasible for the co-op to capture a bigger market share than the IOF by paying a higher mill price at equilibrium.

In the following subsection, we provide a brief literature review concerning theories of the co-op’s pricing strategies and competition with the IOF. We describe the model environment and introduce different players in Section 2. In section 3 we analyze different
market equilibriums as co-op changes its objective. The final section discusses empirical implications of our model and provides testable hypotheses for future research.

**Literature Review**

The early study has concerned about cooperatives’ mill price location on the net marginal and average revenue product curves (NMRP and NARP) while taking member’s supply as given. The “first-best” solution was identified by Enke (1945) where the NMRP intersects the inverse supply curve of members. However, this outcome is unstable as members would have an incentive to expand their patronage with cooperatives beyond the welfare-maximizing point for that rebates of cooperatives’ profit, known as the patronage refund, can be regarded as the windfall gain (Staatz 1987). This leads to the second-best equilibrium suggested by Helmberger and Hoos (1962), where cooperatives’ mill price should be set to just cover the operating cost while the first-best solution can still be achieved by restricting membership.

Nevertheless, the average cost pricing is challenged by Staatz (1983) and Sexton (1986) that applied the game theory to model the behavior of cooperatives. They showed that linear pricing was not enough to generate a stable equilibrium and in the case of average cost pricing, some members might have an incentive to defect from cooperatives if the average cost of cooperatives is increasing. Thus, non-linear pricing rules are needed to ensure that members have no incentive to defect. Among the feasible rules is the two-part pricing proposed by Sexton (1986) that requires membership fee besides charging the marginal cost, though this might be hard to implement due to heterogeneous members. Vercammen, Fulton, and Hyde (1996) further developed the nonlinear pricing schemes by taking into account the member heterogeneity and asymmetric information. They suggested a convex distribution of cooperatives’ profit to members based on patronage to attract really large producers. As noted by Vercammen, Fulton, and Hyde (1996) this pricing scheme raises
the issue of fairness and it may also cause member’s to defect from cooperatives if there were strategic interaction with other firms in the market.

Another strand of cooperative literature explicitly studies the behavior of cooperatives in the oligopolistic competition. Sexton (1990) modeled the price-output behavior of both IOFs and cooperatives in the context of spatial competition and supported open-membership of cooperatives as a way to limit the opportunity of IOFs exercising monopolistic power. In Sexton’s study, the producers have homogeneous production function and only differ by the farm-to-processor distance and the firms take the aggregate supply of raw product in a region as given. On the other hand, Fulton and Giannakas (2001) looked at issue of member’s commitment in a simple mixed oligopoly with one consumer cooperative and investor owned firm. The market share of a firm is no longer take as given but rather depends on member’s willingness to pay for each firm’s product. They argued that besides price, there are other activities undertaken by cooperatives can signal to members about the objective of cooperatives, thus affecting member’s commitment. However, the authors have not formally modeled how the commitment parameters are determined.

This literature review suggests that there is no published co-op theories that compare the pricing strategies of the co-op and IOF with a generalized co-op objective function under uncertainty. This comparison is necessary to advance the co-op theory and to provide more realistic hypotheses for empirical analyses given the limited data available to researchers.

**Mixed Oligopoly in Agricultural Grain Markets**

In this section, we derive the equilibrium pricing strategies of a co-op and an IOF under a two-period mixed oligopoly in the local grain market. We assume that there are unit measure of producers and each is endowed with one unit of crop sold to grain processing firms. In the first period, the co-op and the IOF engage in price competition to purchase their inputs (grain) from the producers and eventually sold in a value-added product market. In the second period, the producers optimally allocate their grain. We further compare the
pricing strategies, market share and profitability of the co-op and IOF, as the objective of the co-op moves along the spectrum from profit maximization towards maximization of price paid to the member’s crop.

Producers Preference

In this model, producers and their inputs to the firms are homogeneous. Grain can be sold to the IOF at the market price $w_p$, and to the co-op for $w_c$ with a potential patronage refund, $\pi_c$ in the future based on the co-op’s future profit. Implementing the owner-user principle of the co-op, the patronage refund, $\pi_c$ is proportional to the producer’s relative use of the co-op, or $\delta$ ($0 \leq \delta \leq 1$) and the refund comes from the co-op’s profit in that period is random because neither producers nor firms observe the price $p$ before the value-added products are sold. Producers allocate their grain by maximizing the expected utility given price offers $w_p$, $w_c$. We assume that the utility function takes a CARA form: $u(w) = -\rho \exp(-\rho w)$ and the ex ante output price is normally distributed in which $\rho$ is the producers’ coefficient of absolute risk aversion. As will be shown in the next subsection, the model implies that the profit function is linear with respect to the output price. Therefore, the producers’ wealth $w$ preserves normality and the expected utility is equivalent to $E(w) - \frac{1}{2}\rho Var(w)$ (up to a monotone transformation), where $E(\cdot)$ and $Var(\cdot)$ denote the mean and variance operators, respectively.

Formally, the producers’ problem is given by

$$\max_{\delta} E(u(w)) = \max_{\delta} (1 - \delta)w_p + \delta w_c + \beta E[\pi_c(\delta, p)] - \frac{1}{2}\rho \beta^2 Var[\pi_c(\delta, p)],$$

where $\beta$ is the temporal discount factor as the refund is received at the end of the second period. The patronage refund is proportional to the grain delivered to the co-op and depends on the realized price, hence the notation $\pi_c(\delta, p)$. The decision to sell to a co-op is a joint decision by the producers to both use and invest in the co-op. There is a disutility associated with this decision from patronizing with the co-op because of the uncertainty of the co-op’s
future profitability. This risk component is represented by \( \frac{1}{2} \rho \beta^2 \text{Var}[\pi_c(\delta, \rho)] \). The risk is increasing in \( \delta \) because investing in the co-op is a decision that deepens a producer’s financial commitment to a single firm rather than diversifying it (Staatz 1987).

We require \( E(u(w)) \geq 0 \) because the producers may make no sales and obtain a monetary utility of zero. Under this utility specification an individual producer has no commitment to do business with the co-op but rather bases the decision entirely on the individual valuation of the patronage refund, characterized by discount factor \( \beta \) and the overall return-risk trade-off. This assumption is reasonable as in reality the cost of joining an agricultural cooperative is generally low. The co-op is established as a competitive yardstick and to provide the missing service to its independent member-producers. To assess the impact of the co-op on the producers’ welfare, we need to compare the agricultural markets in the absence and presence of the cooperative processors (Sexton 1990). However, to continue serving its members a co-op must survive in the long run and engage in the activities that lead to longevity. In our model, these activities are characterized by the co-op’s effort to stay profitable and maintain a positive market share at the equilibrium.

Production and Profit

We turn to the input processing of the co-op and the IOF, as it is the basis on which they announce \( w_c \) and \( w_p \) and compete in the market for input. We assume that firms only process one grain the production function is Leontif with respect to the grain input and homogeneous of degree one with respect to other inputs such as labor and capital. That is

\[
q = \min [x_0, f(x_1, \ldots, x_k)],
\]

where \( x_0 \) is the grain purchased from producers and \( x_1, \ldots, x_k \) are other inputs whose prices are exogenous. The co-op and the IOF are identical in all aspects except for their business objectives. We assume that both firms operate at the cost efficiency. The cost minimization
implies that the total cost is given by

\[ C(q) = w_0 q + cq, \]

where \( q \) is the output quantity, and \( w_0 \) equals \( w_c \) for the co-op and \( w_p \) for the IOF. The constant \( c \) equals the minimized cost of the inputs \( x_1, \ldots, x_k \) to produce one unit of output, which is an implication of the homogeneous production function. We may interpret \( c \) as the constant marginal cost for storing, processing and marketing the final product. The corresponding profit function can be written as

\[ \pi(q, p) = (p - c - w_0) q, \]

where the output price \( p \) is random with the distribution \( N(\mu, \sigma^2) \), which is the common knowledge among producers, the co-op and the IOF. In the first period of the game, the co-op and the IOF strategically quote the grain prices and act as if they were market makers, that is, they are willing to purchase any amount at the announced prices. In the second period, producers strategically determine the grain allocation. Therefore, the output quantity of the co-op always equals to \( \delta \) and its profit function takes the form

\[ \pi_c(\delta, p) = (p - c - w_c) \delta, \]

which verifies our assertions in the previous subsection: the co-op’s profit is linear with respect to \( p \) and thus preserves normality; the patronage refund is proportional to the co-op usage \( \delta \).

Similarly, the profit of the IOF is given by

\[ \pi_l(1 - \delta, p) = (p - c - w_p) (1 - \delta). \]

Grain Supply Function

The subgame perfect Nash equilibrium of this two-period game can be solved by backward induction. In the second period, the producers take \( w_c \) and \( w_p \) as given and optimally
allocate crop. Since no firms carry private information on the output price \( p \), producers’ observation on \( w_c \) and \( w_p \) does not update the distribution of \( p \). Plug Equation (2) into Equation (1), and note that the expected utility is quadratic and globally concave with respect to \( \delta \), hence the analytic solution:

\[
\delta^* (w_c, w_p) = \frac{w_c - w_p + \beta (\bar{p} - c - w_c)}{\rho \beta^2 \sigma^2}.
\]

The no-short-sale constraints \( 0 \leq \delta^* \leq 1 \) will be verified in the equilibrium. The function \( \delta^* \) characterizes the supply curve and the market share for the co-op, from now on, these two terms are used interchangeably. Equation (3) is the standard result from the mean-variance tradeoff utility function that the optimal amount of crop to sell to the co-op is the relative gain of selling co-op over the IOF adjusted for risk. Ceteris Paribus, \( \delta^* \) is decreasing linearly in the \( w_p \), price offered by the IOF at the rate \( 1/\rho \beta^2 \sigma^2 \) as the producers would sell more crop to the IOF when it’s profitable to do so. On the other hand, the market share for the co-op is increasing in the co-op’s price offer but at a smaller rate, \( (1 - \beta)/\rho \beta^2 \sigma^2 \), as a higher mill price offered by the co-op in the first period implies a lower expected patronage refund. However, due to the time preference, i.e. \( \beta < 1 \), the overall impact of increasing \( w_c \) is positive. The intertemporal tradeoff between the cash price today and the future refund will play an important role in defining the equilibrium.

**Firm Objectives**

Consider the two agricultural grain marketing firms in the first period. Both will take advantage of the supply function \( \delta^* \), as they move prior to the producers. We assume that they simultaneously quote grain prices to producers, so that the equilibrium can be solved by the mutually best response functions.

It is conventional to assume that a private firm is a risk-neutral profit maximizer. The IOF chooses \( w_p \) to maximize its expected profit, given the mill price offer by the co-op, while subject to the non-negative expected profit constraint. By assumption, there is one
unit of raw crop in total. Given the market share for the co-op described in Equation (3),
the supply curve faced by the IOF is $1 - \delta^* \geq 0$. we can write the objective function of the
IOF as

$$\max_{w_p} (\bar{p} - c - w_p) \left[ 1 - \frac{w_c - w_p + \beta (\bar{p} - c - w_c)}{\rho \beta^2 \sigma^2} \right],$$

s.t. $0 \leq w_p \leq \bar{p} - c$

The expected profit is quadratic and globally concave with respect to $w_p$, so the first
order condition characterizes the IOF’s best response to the co-op’s price offer $w_c$:

$$w_p^* = a_p + b_p w_c,$$

where $a_p = \frac{1}{2} \left[ (1 + \beta) (\bar{p} - c) - \rho \beta^2 \sigma^2 \right],$ 

$$b_p = \frac{1}{2} (1 - \beta).$$

We impose two constraints on the IOF’s objective function: non-negative profit, $\bar{p} - c - w_p \geq 0$ and non-negative offer price, i.e. $w_p \geq 0$. Both constraints insure that the IOF plays
a positive role in the raw product market. The IOF has a linear best response function to
the co-op’s price offer with the slope given by $b_p \in [0, 1/2]$. An decrease of the price offer
by the co-op reduces the marginal payoff of the IOF, who thus will respond by a similar
price cut. In other words, the mill price offered by the co-op is a strategic complement to
the IOF’s choice of price. The intercept $a_p$ is the IOF’s price offer when the co-op offers
a zero price. In other words, $a_p$ is the certainty equivalence of the stochastic patronage
refund when $w_c = 0$. If $a_p > 0$, it implies that the producers are willing to accept a zero
cash price and exchange their crop with a future refund check. This scenario is unlikely in
practice, we therefore impose the following restriction.

**Assumption 1.** The IOF’s offer price for the raw crop is zero in the absence of the
co-op’s price, i.e.

$$a_p = 0$$
\begin{equation}
(1 + \beta) (\bar{p} - c) - \rho \beta^2 \sigma^2 = 0.
\end{equation}

Assumption 1 is in fact a parameter restriction rather than behavior constraint. Even without it, the majority of our findings in the following sections still hold, though the expressions could be substantially simplified and more intuitive under this assumption.

Assumption 1 has an immediate implication for the co-op’s price behavior in the absence of the IOF, which we put into the following corollary:

**Corollary 1** In the absence of the IOF, the market share of the co-op is 1 if and only if the co-op’s offer price is positive.

The proof is by contradiction. Suppose \( w_c = 0 \), a producer’s expected utility from selling all her crop to the co-op is given by \( \beta (\bar{p} - c - \frac{1}{2} \rho \beta \sigma^2) \). Assumption 1 implies that \( \bar{p} - c = \frac{2 \beta}{(1+\beta)} \bar{p} \beta \sigma^2 \leq \frac{1}{2} \rho \beta \sigma^2 \), i.e. \( \beta (\bar{p} - c - \frac{1}{2} \rho \beta \sigma^2) < 0 \) for \( \beta \leq 1 \), that is without cash payment the expected utility for the producer who sells to co-op is less than zero. Since a producer will not sell unless the expected utility is positive, such a market structure is not viable. Note that in this special case in which there is only a co-op, a producer can still benefit from selling part of her crop to the co-op \( \delta \), as long as \( \delta < \frac{2 \beta}{(1+\beta)} \). To facilitate the following equilibrium analysis, we assume that a market structure can only exist if the sum of market shares between the co-op and the IOF is one and a producer can generate a non-negative expected utility from selling crop.

Corollary 2 reflects the owner-user principle of the co-op in that the producer will demand a positive cash payment now from co-op if the IOF didn’t exist; whereas she is indifferent when the market price was zero if IOF was the only firm in the market because she cannot sell to anyone else. A producer’s demand for cash price now from the co-op is a result from her shared responsibility of co-op’s future profit which bears risk, so she rather receives cash payment today to offset some disutility from future uncertainty.

Let’s now turn to the objective of the co-op which is more controversial in the literature because of its unique ownership structure. In this paper, we allows for a flexible
objective such that a co-op is seeking to maximize any convex combination of producers’ cash payments and its own future profit given the co-op’s risk attitude. The co-op’s objective function can be written as:

$$\max_{w_c} \alpha \delta^* w_c + (1 - \alpha) \delta^* \left[ (\bar{p} - w_c - c) - \frac{1}{2} \theta \delta \sigma^2 \right],$$

where $\alpha \in [0, 1]$ reflects the co-op’s valuation on the cash price and the future refund, which could be used for retaining investments before it is refunded to the producers. In reality, a co-op only distributes out a portion of its profit to the member producers every year while retains the rest to keep an adequate amount of equity for business operation and future growth. The equity management issue of the co-op is beyond the scope of this paper. $\theta \geq 0$ is the risk aversion level of the co-op, may be that of risk aversion like their members (the cooperative is an extension of the farm) or it may differ from that of the producers. Unlike IOF, we allow the possibility for co-op to be risk averse as if its goal is to benefit farmers.

In the next section, we will evaluate and compare the market equilibrium of pricing and market share between the two firms under different objectives of the co-op, each characterized by a unique set of parameter values for $\alpha$ and $\theta$. 

**Equilibrium Pricing and Market Share**

*Co-op as a Risk Neutral Profit Maximizer*

We start with a simple but non-trivial case by restricting $\alpha = \theta = 0$, which implies that the co-op has exactly the same business objective as the IOF. In this situation, the co-op is identical to the IOF for all but only one aspect: producers cannot claim the profit of the IOF but the profit of the co-op is eventually refunded to the producers. Since the profit is stochastic, producers also share the risk associated with the volatility of the co-op’s profit. This case is important in that it isolates the user-owner principle of the co-op.
In this case, the co-op’s objective function is globally concave and the best response function is given by

\[ w^*_c = a_c + b_c w_p, \]

where

\[ a_c = \frac{(\bar{p} - c) (1 - 2\beta)}{2 (1 - \beta)}, \]
\[ b_c = \frac{1}{2 (1 - \beta)}. \]

The graphic demonstration of the best response functions is presented in Figure 1. Similar to the best response of the IOF, the slope of best response function of the co-op is greater than zero, meaning \( w^*_c \) and \( w^*_p \) are strategic complements to each other. An economically meaningful solution requires the two best response functions, Equations (5) and (6) to cross in the first quadrant as shown in Figure 1. Note that \( \frac{1}{b_p} > b_c \) and \( a_p = 0 \) by Assumption 1, so the necessary and sufficient condition for the existence of positive solutions is \( a_c > 0 \), which implies that \( \beta < 1/2 \).

The equilibrium grain prices take the form

\[ (7) \quad w^{**}_{c1} = \frac{2 (1 - 2\beta)}{3 (1 - \beta)} (\bar{p} - c), \]
\[ w^{**}_{p1} = \frac{(1 - 2\beta)}{3} (\bar{p} - c). \]

Plug Equations (7) back to the objective functions, we obtain the expected profit margin of the co-op and the IOF in the equilibrium:

\[ \bar{p} - c - w^{**}_{c1} = \frac{1 + \beta}{3 (1 - \beta)} (\bar{p} - c), \]
\[ \bar{p} - c - w^{**}_{p1} = \frac{2 (1 + \beta)}{3} (\bar{p} - c). \]

This verifies that the expected profit of the IOF is always positive, a necessary condition for a profit-maximizing firm to survive in the market. Furthermore, the expected patronage refund is also positive. Substituting the equilibrium prices in Equation (3) yields the market
share for the co-op: $\delta^* = 1/3$. The expected utility of the producers at the equilibrium is also positive:

$$E(u(\delta^*)) = \frac{7 - 11\beta}{18}(\bar{p} - c),$$

which implies that the existence of the co-op benefits the producers. Otherwise, the single IOF on the market could theoretically quote an infinitesimal price and the expected utility in the absence of the co-op would be zero.

The Nash equilibrium describes a status quo with no reference of the path to it. A good property of our model is that the equilibrium is stable if we interpret the two best response functions as an evolutionary bargain process: basing on an arbitrary guess of the rival’s price, the two firms quote response prices. Using the response prices, they further update the response prices, and so on. To see how the co-op influences the IOF’s price quote, we reformulate Equations (5) and (6) as an iterative process:

$$w_p^{(i+1)} = a_p + b_p w_c^{(i)},$$

$$w_c^{(i+1)} = a_c + b_c w_p^{(i)}.$$  

Then the IOF’s price quote series have the following recursion:

$$w_p^{(i+2)} = b_p b_c w_p^{(i)} + (a_p + b_p a_c).$$

Since $b_p b_c = 1/4$, this constitutes a contraction mapping and the iteration will converge to the unique fixed point $\frac{a_p + b_p a_c}{1 - b_p b_c}$, which is $w_{p1}^{**}$ in Equations (7). In other words, deviation from the equilibrium has a self-recovery mechanism through the iterative bargaining.

Figure 2 plots the equilibrium prices, profits, market shares and the expected utility as functions of the discount factor, which critically characterizes the risk-neutral equilibrium: i) the interior solution exists if and only if $\beta < 1/2$; ii) the grain prices offered by both the co-op and the IOF are decreasing in $\beta$; iii) compared with the IOF, the co-op offers a higher
cash price which implies a lower profit margin; and iv) the market share for the co-op is lower than that of the IOF.

These attributes of the equilibrium have important economic implications. The producers are benefited from the co-op because the price competition increases the crop price and the co-op’s profit is further rebated to producers. If the producers are impatient, they must be eager for a high cash price. The co-op raises the price, as it is producer-owned and operated. More importantly, to keep the market share and profitability, the IOF has to uplift the price as well. The strategic complementarity is the feature of most price competition games. This explains the inverse relation between the discount factor and the equilibrium grain prices of both the co-op and the IOF.

Overall, the discount factor \( \beta \) reflects how the producers evaluate the future patronage refund. In reality, allocated equity, referring to the retained patronage refund of producers accumulated over time could be issued to producers 10 - 30 years later, so the discount factor could take any value between zero and one. In this simple case, \( \beta \) can only take values less than \( 1/2 \), reflecting a low degree of patience of the producers for this particular objective of the co-op.

The equilibrium market share of the co-op is fixed at \( 1/3 \), regardless of the model parameter values. In fact this is the upper bound for the market share of a profit-maximizing co-op when we relax the risk neutrality restriction as will be shown in the next subsection.

**Co-op as a Risk Averse Profit Maximizer**

In this subsection, we impose the parameter restrictions \( \alpha = 0 \), and \( \theta > 0 \), so the co-op exhibits an absolute risk aversion of degree \( \theta \) and its business objective is to maximize the expected utility of the profits. Risk aversion of the co-op reflects the user-owner principle in that the risk attitude of members should have an impact on the co-op’s business operation. Such conservative management behavior may sometimes put the risk-averse co-op in a disadvantage position competing with the risk-neutral IOF (need citation). On the other
hand, a cooperative is created by producers pooling their resources so it also plays a role in
risk-sharing in that the co-op is able to take on projects which may be too risky for a single
producer. In this paper, we do not model explicitly the risk sharing mechanism of a co-op,
so to facilitate the following analysis we make the following assumption:

Assumption 2. The co-op is less risk averse than its member-producers, i.e. \( 0 \leq \theta < \beta^2 \rho \).

Assumption 2 makes intuitive sense as an agricultural grain co-op has access to the low-
cost storage technology due to economies of scale and manages the spot risk by adjusting
the inventory level intertemporally. A co-op can also hedge its risk regarding the profit
uncertainty using derivative products such as futures contracts whereas farmers may fear
to participate in such markets because of lack of knowledge. Note that the co-op’s risk
aversion coefficient is discounted by \( \beta^2 \), which reflects the timing of the uncertainty as we
compare the co-op’s objective function with the producers’ expected utility in Equation (1).

The objective function of the co-op remains globally concave under \( \theta > 0 \), and the first
order condition with respect \( w_c \) yields:

\[
\begin{align*}
\frac{w_c^*}{2} &= \tilde{a}_c + \tilde{b}_c w_p, \\
\text{where } \tilde{a}_c &= \frac{\beta (p - c) [(1 - \beta) \theta + \beta (1 - 2 \beta) \rho]}{(1 - \beta) [2 \beta^2 \rho + (1 - \beta) \theta]}, \\
\tilde{b}_c &= \frac{(1 - \beta) \theta + \beta^2 \rho}{(1 - \beta) [2 \beta^2 \rho + (1 - \beta) \theta]}.
\end{align*}
\]

Figure 1 compares the best response functions of a risk-averse co-op and a risk-neutral
co-op. Specifically, we have \( \tilde{a}_c < a_c \), and \( \tilde{b}_c > b_c \). A sufficient, but not necessary, condition
for Equations (5) and (8) intersecting in the first quadrant is \( \beta < \frac{1}{2} \). The strategic com-
plementarity of the price offering remains to hold regardless of the risk attitude. Another
feature of this pricing function is that only the relative size of the risk aversion matters (i.e.,
\( \theta / \rho \)) for the price.
Solving Equations (5) and (8), we obtain the equilibrium grain prices offered to the producers.

\[
\begin{align*}
w_{c2}^* &= \bar{p} - c - \frac{[\beta^2 \rho + (1 - \beta) \theta] (1 + \beta) (\bar{p} - c)}{(1 - \beta)[3\beta^2 \rho + (1 - \beta) \theta]}, \\
w_{p2}^* &= \bar{p} - c - \frac{[2\beta^2 \rho + (1 - \beta) \theta] (1 + \beta) (\bar{p} - c)}{3\beta^2 \rho + (1 - \beta) \theta}.
\end{align*}
\]

Again, this equilibrium is stable as \( b_p \hat{b}_c = \frac{\beta^2 \rho + (1 - \beta) \theta}{2[2\beta^2 \rho + (1 - \beta) \theta]} < 1 \), implying a contraction mapping and arbitrary starting prices will converge to the unique fixed point. Note that the co-op’s risk aversion level reduces the equilibrium price offer of both the co-op and the IOF, i.e. \( w_{c2}^* < w_{c1}^* \), \( w_{p2}^* < w_{p1}^* \). This is because the marginal utility of the co-op’s profit is reduced by the risk associated with each additional unit of good sold. In other words, the co-op needs to be compensated for the risk by a higher margin. Meanwhile, the strategic complementarity feature of the game implies that the IOF will take the opportunity to match such low grain price and earn more profit. Both \( w_{c2}^* \) and \( w_{p2}^* \) are decreasing functions of \( \theta \). Figure 3 and 4 illustrates the market equilibriums as the parameter values of \( \beta \) and \( \theta / \rho \) vary.

The equilibrium profit margin are given by

\[
\begin{align*}
\bar{p} - c - w_c^* &= \frac{[\beta^2 \rho + (1 - \beta) \theta] (1 + \beta) (\bar{p} - c)}{(1 - \beta)[3\beta^2 \rho + (1 - \beta) \theta]}, \\
\bar{p} - c - w_p^* &= \frac{[2\beta^2 \rho + (1 - \beta) \theta] (1 + \beta) (\bar{p} - c)}{3\beta^2 \rho + (1 - \beta) \theta}.
\end{align*}
\]

Both firms have positive margin regardless of the parameter values. This also verifies that the profitability constraint in the IOF’s problem is satisfied. The equilibrium market shares of the co-op equals

\[
\delta^{**} = \frac{\beta^2 \rho}{3\beta^2 \rho + (1 - \beta) \theta}.
\]
It is clear that $0 \leq \delta^{**} \leq \frac{1}{3}$ for all parameter values and thus the no-short-sale constraint is satisfied. In comparison to the situation where the co-op is a risk-neutral profit maximizer, risk aversion reduces its market share. Specifically, the co-op’s equilibrium market share is increasing as it becomes less risk averse relative to its members, attaining its maximum value $\frac{1}{3}$ if it is risk neutral. The market share of a risk-averse co-op still increases as the producers become more patient but cannot achieve market share of $1/3$ because the existence of such equilibrium requires a more strict upper bound on the temporal discount factor.

The reduction in the co-op’s market share is due to the lower co-op’s offer price at the equilibrium, a consequence of the co-op’s risk aversion as discussed earlier. The direct impact of a larger $\theta$ is a reduction of the co-op’s total payoff and marginal payoff because of the disutility of risk. In response to the reduced marginal payoff, the co-op lowers the grain price, which is echoed by the IOF due to strategic complementarity. On the one hand, the lower input price means a higher IOF profit, the beneficiary of which is the owner of the IOF. On the other hand, the low input price increases the patronage refund, but the temporal discount renders the future refund less attractive than the equivalent amount of the cash payment. The producers are hurt by both the low grain price from the IOF, and insufficiently compensated price reduction from the co-op.

The risk attitude analysis bears policy implications on the risk management of the co-op. In this paper, risk refers to the exogenous volatility of the output price. However, non-systematic risk may be diversified, and many shocks can be insured by participating the derivatives market. Therefore, the degree of risk aversion is related to the extent to which the business risk can be hedged. If less risk aversion induces more desirable allocations for both the producers and the organization, managers who operate the co-op may pay more attention to the identification and hedging the business risk.
The Dual Nature of Cooperatives Objective

The previous subsection shows that risk aversion results in a competitive disadvantage for the co-op which offers a lower mill price as the risk aversion level $\theta$ increases that in turn hurts the producers but benefit the IOF. Thus in order to improve the co-op’s competitiveness by increasing the price offer to attract more market share, the co-op must employ a dual mandates by directly taking into account the role of the cash price in the producers’ utility when making the pricing decision. In this section we relax the assumption that the co-op pursues only profit maximization by allowing for more flexible parameterization in the co-op’s objective function, i.e. $\alpha \geq 0$ and $\theta \geq 0$. The dual objectives present a challenge for the co-op to tradeoff between the producers’ cash payment and its own profitability. In other words, the value of $\alpha$ is the key to the co-op’s pricing decision and some parameterization of $\alpha$ will produce corner solutions that are not of our interest. For example, in the extreme case that $\alpha = 1$, the sole mission of the co-op is to maximize producer’s cash payment with no concern for the profit. As a result the co-op would set $w_c$ as high as possible that theoretically could be infinity. To facilitate the following analysis, we impose the following restriction on the co-op’s objective function:

**Assumption 3.** $0 \leq \alpha < 1 - \frac{2\beta^2\rho}{(1-\beta)\theta+4\beta^2\rho}$.

Assumption 3 is the necessary and sufficient condition for the co-op’s objective function being concave in $w_c$. When the co-op is risk neutral, the upper bound for the value of $\alpha$ is $1/2$ meaning the co-op cannot overweight producer’s cash price relative to its own profitability. Such restriction on the co-op’s objective function intuitively makes sense in this two period model because if co-op overweighted producer’s cash payment now, it could theoretically set the mill price to infinity regardless of the profit in the second period. As a consequence, the IOF will not stay in the market because of a negative profit. This situation is theoretically feasible but not realistic. In the rest of the analysis, we mainly focus on the interior solutions. In addition, as $\theta$ becomes greater than zero, the upper bound of $\alpha$ is relaxed upwards because at equilibrium the co-op has a higher margin under risk aversion.
than risk neutrality as shown in the previous subsections. Thus when the co-op’s objective function accounts for mill price offer, the risk-averse co-op is more capable of increasing the price offer by reducing the profit margin. To demonstrate this idea formally, we can write down the general form of the co-op’s best response function for a given \( w_p \):

\[
\begin{align*}
    w^*_c &= \hat{a}_c + \hat{b}_c w_p \\
    \text{where } \hat{a}_c &= \frac{(\bar{p} - c)\beta \{(1 - \alpha)[(1 - \beta)\theta + \beta (1 - 2\beta)\rho] + \alpha \beta^2 \rho\}}{(1 - \beta)[2\beta^2 \rho (1 - 2\alpha) + (1 - \alpha)(1 - \beta)\theta]} \\
    \hat{b}_c &= \frac{(1 - 2\alpha)\beta^2 \rho + (1 - \beta)(1 - \alpha)\theta}{(1 - \beta)[2\beta^2 \rho (1 - 2\alpha) + (1 - \alpha)(1 - \beta)\theta]}. 
\end{align*}
\]

The co-op’s best response function is still linear in \( w_p \) while maybe looking cumbersome at first, the property of \( w^*_c \) is rather simpler than the expression may suggest. In terms of the slope, \( \hat{b}_c \) is always positive but discontinuous at \( \alpha = 1 - \frac{2\beta^2 \rho}{(1 - \beta)\theta + 4\beta^2 \rho} \). So the prices offers by the co-op and the IOF are still complementaries to each other. Under assumption 3, we focus on the range of \( \alpha \) between 0 and \( 1 - \frac{2\beta^2 \rho}{(1 - \beta)\theta + 4\beta^2 \rho} \), the interval in which the co-op’s objective function is concave. Notice that \( \partial \hat{b}_c / \partial \alpha = \frac{\beta^2 \rho \theta}{[2\beta^2 \rho (1 - 2\alpha) + (1 - \alpha)(1 - \beta)\theta]^2} > 0 \), namely as the co-op increasingly focuses on producer’s cash payment today, its price decision becomes more sensitive to change in price offer made by the IOF counterpart. However \( \hat{b}_c(\theta = 0) \) is equal to \( b_c \), meaning that the slope of the best price-response function of a risk-neutral co-op is not affected by the objective-weighting parameter \( \alpha \). So we can establish the following trajectory of the slope of the co-op’s BR as the co-op changes from being a risk-neutral profit maximizer to having dual objectives:

\[
b_c \leq \hat{b}_c \leq \hat{b}_c.
\]

that is, caring about producer’s cash payment reinforces the positive impact of co-op’s risk aversion on how it responds to IOF’s price.

As for the the intercept, \( \hat{a}_c \) is required to be positive for the existence of equilibrium, which imposes an additional restriction on temporal discount factor \( \beta \) as discussed in the section 3.1. However, under assumption 3, \( \hat{a}_c \) is greater than zero.
for all $0 \leq \beta < 1$. Unlike the case for $\hat{b}_c$, $\hat{a}_c$ is continuous and increasing in $\alpha$, i.e. 
$$\frac{\partial \hat{a}_c}{\partial \alpha} = \frac{\beta^2 \rho (p-c)(2\beta \rho - \theta)}{2 \beta^2 \rho (1-2\alpha) + (1-\alpha)(1-\beta) \theta} > 0 \text{ for } \alpha \in [0,1]$$
and we can establish the trajectory of the intercept of co-op’s BR as the co-op changes from being risk-neutral profit maximizer to having dual objectives:
$$\bar{a}_c \leq \hat{a}_c < a_c.$$ 

The second strict inequality in the above expression reflects the upper bound of $\alpha$ is smaller than 1, while $\hat{a}_c$ only attains $a_c$ at $\alpha = 1$. A graphic comparison of the co-op’s best responses under different business objectives is illustrated in Figure 1. Finally, we obtain the equilibrium grain prices offered to producers by solving equation (5) and (8):
$$w_{c3}^{**} = \frac{\hat{a}_c}{1 - b_p \hat{b}_c},$$
$$w_{p3}^{**} = \frac{b_p \hat{b}_c}{1 - b_p \hat{b}_c}.$$

A numerical illustration of the equilibrium as $\alpha$ varies is presented in Figure 5. It immediately follows the trajectory of co-op’s BR that $w_{c3}^{**} > w_{c2}^{**}$ and $w_{p3}^{**} > w_{p2}^{**}$, i.e. the equilibrium offer prices by both co-op and IOF are higher when the risk-averse co-op is taking into account producer’s cash payment in its objective function. The comparison to the case of co-op being risk neutral is not straightforward analytically and we demonstrate the trajectory of price offers in Figure 5.

Substituting $w_{c3}^{**}$ and $w_{p3}^{**}$ into equation (3) yields the equilibrium market share for a co-op:
$$\delta^{***} = \frac{\beta^2 \rho [(1-\alpha) - 2\alpha \beta/(1+\beta)]}{3(1-2\alpha)\beta^2 \rho + (1-\alpha)(1-\beta) \theta}.$$ 

As the equation (9) shows, producer’s temporal discount factor, co-op’s objective weighting parameter $\alpha$ and its risk aversion level relative to that of producers play important roles in characterizing co-op’s market share. As discussed in section 3.2, the market share of co-op is decreasing in $\theta/\rho$, namely co-op’s relative risk aversion to producer’s has a negative
impact on its own market share. $\delta^{***}$ is increasing in $\beta$ as producers become more patient, ceteris paribus, they will put more values on future cash flow received from co-op thus allocate more to co-op. The impact of $\alpha$ on $\delta^{***}$ is expected to be positive as producer valuing the present more and it’s indeed the case as $\partial \delta^{***}/\partial \alpha = \frac{\beta^2 \rho (1-\beta)(3\beta \rho - 2\theta)}{[3(1-2\alpha)\beta^2 \rho + (1-\alpha)(1-\beta)\theta]^2 (1+\beta)} > 0$. However, $\delta^{***}$ is discontinuous at $\alpha = 1 - \frac{2\beta^2 \rho}{(1-\beta)\theta + 4\beta^2 \rho}$ and there is no theoretical maximum value for $\delta^{***}$ besides its capped by 1. We now know that risk-averse co-op with increasing caring for producer’s cash payment will have bigger market share, so an interesting question to ask is that if $\delta^{***}$ can exceed $1/3$, the upper bound of market share for a pure profit maximizing co-op. To answer this question, we evaluate $\delta^{***}$ at $\alpha = \frac{1}{2} < 1 - \frac{2\beta^2 \rho}{(1-\beta)\theta + 4\beta^2 \rho}$ and obtain $\delta^{***}(\alpha = \frac{1}{2}) = \frac{\beta^2 \rho}{(1+\beta)\theta} > \frac{1}{(1+\beta)}$ under assumption 2. Thus for $0 \leq \beta \leq 1, \delta^{***}(\alpha = \frac{1}{2}) > 1/2 > 1/3$ and it is feasible for co-op with dual objectives to achieve a higher market share than co-op just maximizing profit. Although, we haven’t discussed in the prior text, it’s important from the perspective of producers and the co-op to decide what is the optimal $\alpha$ such that the joint utility of co-op and producers are maximized.

**Discussions**

We now discuss some implications of this framework that may shed light on a number of issues related to farmer cooperatives. We also discuss a few testable hypotheses emerged from our model that takes into account the unique characteristics of co-ops when studying their behaviors and performances of cooperatives.

The restructuring of cooperatives including mergers, acquisitions and even bankruptcies have received much attention from researchers in 90s. While the change in landscape of agricultural businesses characterized by lower commodity prices and increasing competitions is the often cited reason for co-op consolidations, co-ops that have undergone restructuring featuring low operating efficiencies and high expenses\(^4\). The Figure 6 shows the number of farm marketing cooperatives in United States has been consistently declined.
since 1950 and despite growing sales over the years, agricultural cooperatives are still not the dominant form of agribusiness in the United States as indicated by the market share of co-ops. Cook (1995) in his seminal work discusses the evolution of agricultural cooperatives. He points out the vaguely defined property rights challenges the survival of traditional cooperatives and thus forces co-ops to restructure. Specifically, Cook (1995) lists five property rights constraints of agricultural cooperatives which includes free rider problem, horizon problem, portfolio problem, control problem and influence costs problem. This paper complements Cook’s neo-institutional approach and provides a formal model that explains why traditional cooperatives may be at competitive disadvantage.

Our model takes into account the free rider problem by default as members have no commitment to patronize the co-op but rather to take advantage of its user-owner principle for a patronage refund. We show that for a profit-maximizing co-op this creates troubles as the co-op is likely to pay a higher cash price but obtain a smaller market share than the IOF at equilibrium, the latter is consistent with the data. Although by considering the cash price paid to the members co-op may acquire a bigger market share, the member’s "commitment" to sell to co-op is weakened, reflected by the increasing sensitivity of the co-op’s price offer to that of the IOF’s.

In reality determining the optimal pricing policy and the subsequent implication for patronage refund can be tricky for co-ops because of diverse objectives represented by heterogeneous members. This is the so called influence costs problem. In this paper, we assume producers are homogeneous in their objectives and take the weighting factor $\alpha$ as given. Solving for the optimal weighting factor under a group of diverse members may be an interesting topic for future research.

The portfolio problem described by Cook (1995) justifies our analysis of a risk-averse co-op which is usually ignored in the literature. In our model, the portfolio problem faced by member-producers is to allocate their crop between the IOF that is "risk-free" because producers know the price they receive upfront, and co-op which is "risky" for the amount
of patronage refund is uncertain. Members that are risk-averse would want the co-op to exercise the similar degree of caution which may lower the risk of member’s portfolio but also result in missing good investment opportunities for the co-op, implying lower expected returns.

Our model is also well suited for understanding the horizon problem which occurs when a member expects not to fully realize returns on his investment in the co-op before retiring. In our model patronage refund received in distant future is adjusted for the producer’s time value and risk attitude which together characterize the producer’s valuation on the patronage refund as well as the equilibrium outcomes. For example, we show that for the equilibrium to exist, the discount factor cannot be too big, i.e. the producer place relatively less value on the patronage refund which is true in reality, as most of producers’ patronage refund is retained in co-ops as allocated equity that will not be distributed after 10-20 years.

Cook (1995) argues that it was the adjustments to property rights constraints by co-ops that contributed to the increase in market share of co-ops since 1988. However, co-ops’ market share in farm marketing businesses seemed to reach a top around 1996 and started to decline consistently despite the continuing consolidation of cooperatives. One possible explanation is that as agricultural cooperatives become much larger in sizes thanks to the mergers and acquisitions, individual producers may have less influence on the co-op’s management decisions and the potential conflict of interest between leadership and members creates disincentive for the members to patronize co-ops. However, it’s hard to measure the dual nature of a co-op’s objectives in studying performances of co-ops due to inaccessibility of data that results in failure of taking into account the diverse interests of a group of heterogeneous members (Soboh et al. 2009).

Given the best possible data researchers could have on co-ops is at firm level, this framework may be able to offer an alternative solution to empirically studying the co-op’s behaviors. A set of predictions emerged from our model that involve studying the co-op’s profitability and its market share. One of these predictions is that co-ops that have bigger
market share are likely to have lower profit margin after accounting for the possible monopolistic power while despite the geographic advantage of a local co-op, the barrier of entry for marketing businesses is generally low. Also the firm sizes may affect this relationship which is correlated with the co-op’s caring for members on farm profit as we discussed earlier. We demonstrate in our model if co-op act exactly like the IOF, its market share should be little affected by margins. Note this prediction should be product-specific, as nowadays co-ops are very well diversifying their business across several farm markets.

The other prediction is related to the comparison between the co-ops and the IOFs in the same business and of similar asset sizes. Our model shows that the IOFs should on average have higher profit margins than co-ops in the marketing sector. This may be one of the main drivers for the continuing restructuring of cooperatives.

Finally, it would be an interesting empirical question to compare the characteristics of co-ops in earlier 90s and now, and to understand what restructuring occurred in 90s that caused the growth of the co-op’s market share while the market share for co-ops is still not dominant nowadays despite the important economic and social roles they are deemed to play.
Notes

The claim that the patronage refund is proportional to the grain supply is a model conclusion rather than an assumption. In the next subsection, we will show that the profit function satisfies the property $\pi_{coop}(\delta, p) = \delta \pi_{coop}(1, p)$.

Service will otherwise not be provided by for-profit IOF due to the lack of profitable motive (citation)

Suppose the production function is homogeneous of degree $\gamma$ with respect to $x_1, \ldots, x_k$. Denote $\tilde{C}(q) = \min \sum_{i=1}^{k} x_i w_i$, subject to $f(x_1, \ldots, x_k) \geq q$. The cost function satisfies $\tilde{C}(q) = \tilde{C}(1) q^{\frac{1}{\gamma}}$. Denote $c = \tilde{C}(1)$, which is a function of the exogenous input prices $w_1, \ldots, w_k$, hence a constant.

Source: United States Department of Agriculture, Rural Business-Cooperative Service, RBS Research Report 180
References


The best response pricing functions of the IOF and the co-op are linear with respect to the rival’s price offer. As the business objective of the co-op varies from a risk-neutral profit maximizer, to a risk averse entity, and then to a dual nature firm, the slope and intercept of the curve shift. The numeric equilibriums are computed based on the parameter values $\alpha = 0.2$, $\theta = 0.4$, $p = 8$, $\beta = 0.95^{20}$, $\tau = 100$, and $c = 80$. Parameter restrictions $\alpha = 0$ and $\theta = 0$ apply to a risk-neutral co-op. A risk-averse co-op faces the restriction $\theta = 0$. 
Figure 2. Risk Neutral Co-op and Market Equilibrium, as Discount Factor Varies

The parameter value of the discount factor shapes the market equilibrium when the co-op is a risk-neutral profit maximizer. The numeric equilibriums are computed based on the parameter values $\alpha = 0$, $\theta = 0$, $\rho = 8$, $p = 100$, $c = 80$. In this case, the price offer of the co-op is always higher than that of the IOF, and the market share of the co-op is 1/3. Both firms have positive profits and the expected utility of the producers is also positive.
The parameter value of the discount factor and the degree of risk aversion shape the market equilibrium when the co-op is risk averse. The numeric equilibriums are computed based on the parameter values $\alpha = 0, \theta = 0.4, \rho = 8, \pi = 100, \epsilon = 80$. \[\]

\[\]

\[\]

\[\]
The parameter value of the discount factor and the degree of risk aversion shape the market equilibrium when the co-op is risk averse. The numeric equilibriums are computed based on the parameter values $\alpha = 0$, $\beta = 0.95^{0.9}$, $\bar{p} = 100$, $c = 80$. 

The parameter value of the discount factor and the degree of risk aversion shape the market equilibrium when the co-op is risk averse.
The parameter value of $\alpha$ critically determines the market equilibrium when the co-op has the dual objective on the cash price and future refund. The numeric equilibriums are computed based on the parameter values $\theta = 0.4, \rho = 8, \beta = 0.95^{20}, \pi = 100, c = 80$. 
Figure 6. Marketing Cooperatives: Sizes, Sales and Shares

The aggregate time series plots show the characteristic facts of the U.S. agricultural marketing cooperatives in 1951 - 2007. The number of co-ops series has the y-axis labeling on the left. The net sales series has the y-axis labeling on the right. The market share series has y-axis in the unit interval. Data Sources: United States Department of Agriculture, Rural Development, Cooperatives Historical Data; United States Department of Agriculture, Economic Research Service, Farm Income and Wealth Statistics.