Dynamics of Advertising and Demand for Milk in the United States Delineated by Milk Fat Type

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Introduction

Per capita fluid milk consumption has been on a decline in the United States for more than a decade (ERS, 2013; USCB, 2012, 2010, 2001). However, per capita consumption of all dairy products and milk supply has been on a rise (ERS, 2013). In the year 2000, per capita consumption of fluid milk was approximately 21 gallons per year. By 2011, that total dropped to 18 gallons (a 15% drop) a year (Nielsen Scantrak, 2012; USCB, 2012, 2010, 2001). During the same period, milk advertising funds decreased from $321 million to $240 million (a 25.2% decline) per year (Dairy Management Inc, 2013; MilkPeP, 2013; Qualified Programs, 2013).

Some of the decline in milk consumption may be attributed to the correlation of the rise in obesity and milk consumption, particularly whole milk (Berkey et al., 2005; Wiley, 2010). For instance, the Surgeon General’s 2010 report stated: “by age 2, children should be drinking low-fat or non-fat milk”, which is reemphasized by the supplemental nutrition assistance for Women, Infants and Children (WIC) program requirements (2012) of only allowing parents whose child(ren) is(are)less than two years as being eligible to use WIC to purchase whole milk. This likely has an effect on the consumption of liquid milk products in the United States, particularly milk products with a higher fat content, such as whole milk. There are four ‘main’ types of fluid milk which are differentiated by their fat content; those include whole milk (~3.25% milk fat), two-percent, one-percent, and skim (less than 0.5% milk fat) milks (Agricultural Marketing Service, 1995).

Multiple studies have been conducted reflecting the potential relationship between milk consumption and weight gain, particularly among adolescents. Some studies, such as Berkey et al. (2005) and Wiley (2010), linked milk consumption to an increased body mass index (BMI) among children and adolescents. However, Chen et al. (2012) and Mozaffarian et al. (2011) were unable to connect drinking milk to gaining weight. Though there are numerous studies on this topic, it appears there is little proof that milk increases or decreases one’s BMI. Regardless, America’s obesity problem is no secret – and fluid milk is unique in that it provides four products all at different fat (calorie) levels. However, milk advertising typically does not differentiate among the four. Suppliers and processors of fluid milk products contribute to the pool of advertising funds. Instead of appropriating specific advertising amounts to types of milk, in general, milk is generically advertised. If generic advertising was known to separately affect
consumption of the four types of milk, we may be able to advise advertising firms to cater to specific methods of advertising. In turn, public officials may be able to use this information to increase overall health status for individuals in the United States.

Previous work has been done on examining the effects of advertising on various products (Funk et al., 1977; Brester and Schroeder, 1995; Schmit et al., 2002) including beverages and fluid milk (Gould, 1996; Kaiser and Reberite, 1996; Kinnucan et al., 2001; Zheng and Kaiser, 2008). Numerous models have been used, and findings for advertising effects are not uniform. Funk et al. (1977) used a simple demand model which included competitors’ (grocery chains) advertising and found that an increase in beef advertising is associated with a relatively small increase in beef sales. Brester and Schroeder (1995) used a Rotterdam (Theil, 1965) model to measure the impacts of brand and generic advertising on meat demand finding mixed results for the effect advertising has on various meat products. Schmit et al. (2002) used a probit model and incorporated a polynomial distributed lag advertising (Almon, 1965) variable. Again, mixed results were found for advertising effects (elasticities in this example).

Most of the previous work in extant literature aggregates milk types (Kinnucan and Forker, 1986; Kaiser and Reberite, 1996; Gould, 1996; Zheng and Kaiser, 2008; Kinnucan et al., 2001). Also, some previous works focused on data from particular regions, namely New York, rather than the entire US (Kaiser and Reberite, 1996; Kinnucan and Forker, 1986). Additional works such as Kinnucan et al. (2001) and Zheng and Kaiser (2008) looked at advertising for non-alcoholic beverages, including milk, across the United States using annual time-series data. However, to the best of our knowledge, no paper has analyzed monthly time series data and per capita milk consumption representative of the entire United States when trying to measure advertising effects.

**Methodology**

The overall purpose of this study is to measure the impact of generic milk advertising has on the demand for the four major types of fluid milk (whole milk, 2% milk, 1% milk, and skim milk). To do so, we estimate a complete demand system, assuming milk as a weakly separable group.¹

¹ A complete system of demand equations describes a household’s allocation of expenditure among some exhaustive set of consumption categories (Pollack and Wales, 1978), and is theoretically plausible if it is derivable from a well-behaved utility function, or equivalently if the demand equations are homogeneous of degree zero in prices and expenditure, and the implied Slutsky matrix is symmetric and negative semi-definite (Pollack and Wales, 1978).
Various models have been executed in measuring effects of advertising on fluid milk such as a double logarithmic model (Kinnucan and Forker, 1986; Kaiser and Reberte, 1996), a Rotterdam model (Kinnucan et al., 2001), and an Almost Ideal Demand System (AIDS) model (Zheng and Kaiser, 2008). Following Zheng and Kaiser (2008), we begin with an AIDS model (Deaton and Muellbauer, 1980) which takes the following form:

\[ w_{it} = \alpha_i + \beta_i \ln \left( \frac{m}{P} \right) + \sum_j \gamma_{ij} \ln(p_j) + e_{it} \]  

(1)

where \( w_{it} \) is the budget share for good \( i \) at time \( t \), \( \alpha_i \) is a constant for milk product \( i \), \( m \) is total expenditures, \( P \) is a price index which will be further defined below, \( p_j \) is the price of good \( j \), \( \beta_i \) and \( \gamma_{ij} \) are parameters to be estimated and \( e_{it} \) is the error term. Because the LA/AIDS and other approximations to the AIDS do not have satisfactory theoretical properties as they can violate consumer demand theory restrictions such as symmetry for most combinations of prices and expenditures (Hahn, 1994), the following restrictions on the AIDS models are imposed.

\[ \sum_i \alpha_i = 1, \quad \sum_i \beta_i = 0, \quad \sum_i \gamma_{ij} = 0 \]  

(Adding Up)

\[ \sum_i \gamma_{ji} = 0 \]  

(Homogeneity)

\[ \gamma_{ij} = \gamma_{ji} \]  

(Symmetry)

and \( \ln(P) \) is:

\[ \ln(P) = \alpha_0 + \sum_k \alpha_k \ln(p_k) + \frac{1}{2} \left( \sum_k \sum_i \gamma_{ki} \ln(p_k) \ln(p_i) \right) \]  

(3)

which is referred to as the Translog Price Index (TPI). As Banks et al. (1997) noted, incomes vary considerably across individuals and income elasticities vary across goods; therefore, the income effect for individuals at different points in the income distribution must be fully captured in order for a demand model to predict responses to tax reform (or other policy areas) usefully. To meet these criteria, Banks et al. (1997) developed a new demand system that has log income as the leading term in an expenditure share model and additional higher order income terms. In other words, the model allows for Engel curves that are potentially non-linear in the log of expenditure (Cranfield, 2012). This model is referred to as the Quadratic Almost Ideal Demand System (QUAIDS), and takes the following form:

\[ w_{it} = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln \left( \frac{m}{a(p)} \right) + \frac{\lambda}{b(p)} \left( \ln \left[ \frac{m}{a(p)} \right] \right)^2 + e_{it} \]  

(4)
where variables are defined above as before, the $Ln(a(p))$ is the TPI, and $b(p)$ is the simple Cobb-Douglas price aggregator defined as:

$$ b(p) = \prod_{i=1}^{n} p_i^{\beta_i} $$

and $\lambda$ is defined as:

$$ \lambda(p) = \sum_{i=1}^{n} \lambda_i \ln(p_i), \quad \text{where } \sum_{i} \lambda_i = 0. $$

However, $\lambda(p)$ is assumed to be independent of prices, which makes the underlying indirect utility function where QUAIDS is derived to be observationally equivalent to PIGLOG class (price independent generalized logarithmic; see Banks et al., 1997). The demands generated are rank three (maximum possible rank for any demand system that is linear in functions of income (see Gorman, 1981)), exactly aggregable, are derived from utility maximization, and permit goods to be luxuries at some income levels and necessities at others (Banks et al., 1997). The QUAIDS model is advantageous because it embodies very flexible price and income effects (Cranfield, 2012). Note, when $\lambda_i = 0$ for all $i$, QUAIDS collapses to the previously mentioned AIDS model; also, QUAIDS only has local monotonicity and curvature properties (Cranfield, 2012).

Notice the previously mentioned model does not include an advertising variable. In order to model the effects for advertising, we incorporate a polynomial distributed lag model (or PDL) (Almon, 1965) into the QUAIDS model as follows:

$$ w_{it} = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta \ln \left( \frac{m}{a(p)} \right) + \frac{\lambda_i}{b(p)} \left[ \ln \left( \frac{m}{a(p)} \right) \right]^2 + \sum_{k=0}^{k} \theta_{ik} \ln A_{t-k} + e_{it} $$

where $A$ represents advertising expenditures at time $t-k$ and $e$ is an error term, and other variables are described above. It is assumed that $\theta_{ik}$ can be represented with a polynomial of degree $m$, where $m = 0, 1, 2 \ldots , m$ such that:

$$ \theta_{ik} = \varphi_0 + \varphi_1 k + \varphi_2 k^2 + \varphi_3 k^3 + \ldots + \varphi_m k^m $$

Suppose that a lag length of four is chosen for the advertising variable. This would imply that we have $t-1, \ldots t-4$. Now, assuming a second degree polynomial for $\theta_{ik}$, (i.e., $m = 2, k = 1, 2, 3, 4$), we reach the following:
\[ \theta_k = \varphi_0 + \varphi_1 k + \varphi_2 k^2, \text{ for } m = 0, 1, 2 \]  

(9)

By imposing head and tail restrictions of no effects before \( k = 0 \) and after \( k = 4 \), we have the following:

\[ \theta_{t,-1} = \varphi_0 - \varphi_1 + \varphi_2 = 0, \text{ for } k = -1 \]

\[ \theta_{t,5} = \varphi_0 + 5\varphi_1 + 25\varphi_2 = 0, \text{ for } k = 5 \]  

(10)

Combining like terms, we reach:

\[ \varphi_0 = \varphi_1 - \varphi_2 = 0 \]

Now, substituting in \( \varphi_1 - \varphi_2 \) for \( \varphi_0 \):

\[ \varphi_1 - \varphi_2 + 5\varphi_1 + 25\varphi_2 = 0 \Rightarrow 6\varphi_1 + 24\varphi_2 = -4\varphi_2 \]

Consequently, \( \varphi_0 = -5\varphi_2 \Rightarrow \) we need only to estimate \( \varphi_2 \).

However, since the lag length is not generally known in advance, we must estimate the distribution using varying numbers of periods, then choose the best among them (Almon, 1965). In this sense, the carryover effects of advertising can be captured in a dynamic setting (Dharmasena et al., 2012). The optimal lags lengths can be chosen based on the Schwarz or Akaike Information Criteria. While using demand systems to model the effects advertising has on various products is not a new idea, we model the effects of advertising expenditures using a quadratic AIDS (QUAIDS) model. Further, the QUAIDS is modified to include a PDL equation – which has not been done previously to the best of our knowledge.

Model

A modified QUAIDS model was employed in this analysis. By construction, the model exhibits endogeneity issues with both expenditure shares and total expenditure. To adjust for total expenditure endogeneity, an instrumental variable (IV) approach was used. Following Capps et al. (1994) and Dharmasena and Capps (2012), the following IV regression was calculated:

\[ \ln m_t = \tau_0 + \sum_{i=1}^{4} \tau_i \ln p_{it} + \tau_5 \ln inc_t + v_t \]  

(12)

where \( m_t \) is the expenditure at time \( t \), \( \tau_0 \) is a constant, \( p_{it} \) is the price of good \( i \) at time \( t \), \( inc \) is per capita income at time \( t \), and \( v_t \) is the error term. From equation (12), \( m_t \)-hat was calculated which

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2 Serial correlation for equation (12) was tested and rejected, thus no AR(p) correction was needed; for estimates, please contact the authors.
replaced \( m \) in equation (7). Seasonality was also accounted for in the model; thus, the following model was estimated:

\[
w_{it} = \alpha_i + \sum_j \gamma_j q_j \ln p_j + \beta \ln \left( \frac{\hat{m}}{a(p)} \right) + \frac{\hat{\lambda}_i}{b(p)} \left\{ \ln \left[ \frac{\hat{m}}{a(p)} \right] \right\}^2 + \sum_{k=0}^k \theta_{it-k} \ln A_{i-k} + \sum_{q=1}^{3} \pi_q D_q + e_{it}
\]

where parameters and variables remain as discussed above, \( D_q \) is a seasonal dummy, \( \pi \) is the parameter associated with the seasonal dummy and \( \hat{m} \) is the estimated total expenditure from the IV regression represented by equation (12). In addition to the conditions stated in equation (2), because of the modifications, the following additional adding up restrictions must be met:

\[
\sum_{i=1}^{4} \theta_{it} = 0
\]
\[
\sum_{q=1}^{4} \pi_{iq} = 0
\]

Serial correlation of the errors for equation (13) was another concern with the model’s current form. Thus, after running the model specified in equation (13), the system errors were calculated.\(^{3}\) Equation (13) was further modified for serial correlation using an AR(1) process. Following Hatanaka (1974), the model is corrected for serial correlation using a lagged difference and its rho estimator. Thus, the final model includes a serial correlation coefficient, \( \rho \):\(^{4}\)

\[
w_{it}^* = \alpha_i + \sum_j \gamma_j q_j \ln p_j + \beta \ln \left( \frac{\hat{m}}{a(p)} \right) + \frac{\hat{\lambda}_i}{b(p)} \left\{ \ln \left[ \frac{\hat{m}}{a(p)} \right] \right\}^2 + \sum_{k=0}^k \theta_{it-k} \ln A_{i-k} + \sum_{q=1}^{3} \pi_q D_q - \rho (w_{it})_{t-1}
\]

**Data**

Previous papers have used varieties of data including yearly, monthly, and area specific (see above). For this analysis, we utilize monthly price data of the four milk types, per capita consumption, as well as advertising expenditure for fluid milk products. Prices are obtained from the Nielsen Scantrack reports on refrigerated milk for the four milk types from January 2000 through December 2012. They are averaged across 52 Scantrack markets (U.S. cities and regions) defined by Nielsen.\(^{5}\) In the Scantrack data, monthly quantities were reported in terms of millions of pounds and represent the consumption of the entire United States. Per capita quantity

\(^{3}\) An AR(1) model was needed to adjust for serial correlation. For estimates, please contact the authors.

\(^{4}\) All data analysis was conducted in STATA v12.1

\(^{5}\) We found strong correlations between the Nielsen price data and that of the BLS (2013) for whole milk.
values are calculated using the monthly quantity data described above, population estimates (United States Census Bureau, 2012; 2010, 2001), and a pounds of milk to gallons of milk conversion of 8.6 (Dairy Facts, 2008).

Advertising data are gathered from Dairy Management Inc, MilkPeP, and Qualified Programs and are reported quarterly. Monthly advertising data for all milk types are imputed using these quarterly advertising expenditures. All milk prices and advertising were deflated using the Diary Consumer Price Index (DCPI)\(^6\) with 2011 as the base year. Real median per capita income was used, in 2011 dollars (USCB, 2013). Table 1 reports the summary statistics for all data used in this paper.

Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertising(\ast)</td>
<td>25,100,000</td>
<td>4,543,458</td>
<td>18,300,000</td>
<td>38,400,000</td>
</tr>
<tr>
<td>Median Income/US Population</td>
<td>46,537</td>
<td>3,355</td>
<td>41,990</td>
<td>50,303</td>
</tr>
<tr>
<td>Whole Quantity</td>
<td>1,369.72</td>
<td>131.94</td>
<td>1,101.07</td>
<td>1,617.20</td>
</tr>
<tr>
<td>2% Quantity</td>
<td>1,499.48</td>
<td>86.34</td>
<td>1,336.41</td>
<td>1,742.49</td>
</tr>
<tr>
<td>1% Quantity</td>
<td>554.30</td>
<td>54.50</td>
<td>420.40</td>
<td>708.15</td>
</tr>
<tr>
<td>Skim Quantity</td>
<td>674.12</td>
<td>27.06</td>
<td>612.60</td>
<td>747.97</td>
</tr>
<tr>
<td>Dairy CPI</td>
<td>185.16</td>
<td>16.86</td>
<td>160.70</td>
<td>212.75</td>
</tr>
<tr>
<td>Whole Price(\ast)</td>
<td>3.49</td>
<td>0.24</td>
<td>2.92</td>
<td>4.17</td>
</tr>
<tr>
<td>2% Price(\ast)</td>
<td>3.36</td>
<td>0.23</td>
<td>2.82</td>
<td>4.00</td>
</tr>
<tr>
<td>1% Price(\ast)</td>
<td>3.33</td>
<td>0.23</td>
<td>2.79</td>
<td>3.95</td>
</tr>
<tr>
<td>Skim Price(\ast)</td>
<td>3.29</td>
<td>0.23</td>
<td>2.73</td>
<td>3.90</td>
</tr>
<tr>
<td>PCC All Milk</td>
<td>1.61</td>
<td>0.09</td>
<td>1.41</td>
<td>1.87</td>
</tr>
<tr>
<td>PCC Whole</td>
<td>0.54</td>
<td>0.07</td>
<td>0.41</td>
<td>0.67</td>
</tr>
<tr>
<td>PCC 2%</td>
<td>0.59</td>
<td>0.03</td>
<td>0.53</td>
<td>0.66</td>
</tr>
<tr>
<td>PCC 1%</td>
<td>0.22</td>
<td>0.02</td>
<td>0.17</td>
<td>0.26</td>
</tr>
<tr>
<td>PCC Skim</td>
<td>0.26</td>
<td>0.01</td>
<td>0.23</td>
<td>0.32</td>
</tr>
</tbody>
</table>

\(\ast\) Adjusted for inflation in 2011 dollars using Dairy CPI; / in $2011 dollars (Census derived); PCC is per capita consumption. Quantities are reported in gallons.

The main takeaway from this table is the high correlation among all four milk type prices. Though these prices are typically related to fat content, there has been a tighter price range among the milk types in recent months. We see that two percent and whole milks have the highest per capita consumption, with skim and one-percent milks having the least. Though not clear in the table, per capita consumption of these four milk types has been on a decline. This is

\(\ast\) For dairy and related products.
likely related to milk advertising funds decreasing as well as more dairy options becoming available on the market.

Results

A modified quadratic AIDS model was used to estimate the demand relationships discussed previously. Endogeneity and serial correlation were taken into account and corrected for. Table 2 provides the parameter estimates from equation (15).

Table 2: Equation Results

<table>
<thead>
<tr>
<th></th>
<th>Whole</th>
<th>Two-percent</th>
<th>One-percent</th>
<th>Skim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.337**</td>
<td>2.757***</td>
<td>-0.098</td>
<td>0.678</td>
</tr>
<tr>
<td></td>
<td>(1.168)</td>
<td>(0.897)</td>
<td>(0.527)</td>
<td>(0.941)</td>
</tr>
<tr>
<td>Beta</td>
<td>-1.495**</td>
<td>1.707***</td>
<td>-0.075</td>
<td>-0.137</td>
</tr>
<tr>
<td></td>
<td>(0.731)</td>
<td>(0.564)</td>
<td>(0.332)</td>
<td>(0.592)</td>
</tr>
<tr>
<td>Gamma i1</td>
<td>3.741</td>
<td>-4.239</td>
<td>0.146</td>
<td>0.351</td>
</tr>
<tr>
<td></td>
<td>(3.437)</td>
<td>(2.807)</td>
<td>(0.754)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>Gamma i2</td>
<td>-4.239</td>
<td>4.910</td>
<td>-0.220</td>
<td>-0.451</td>
</tr>
<tr>
<td></td>
<td>(2.807)</td>
<td>(3.025)</td>
<td>(0.908)</td>
<td>(1.639)</td>
</tr>
<tr>
<td>Gamma i3</td>
<td>0.146</td>
<td>-0.220</td>
<td>-0.029</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>(0.754)</td>
<td>(0.908)</td>
<td>(0.087)</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Gamma i4</td>
<td>0.351</td>
<td>-0.451</td>
<td>0.102</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(1.314)</td>
<td>(1.639)</td>
<td>(0.104)</td>
<td>(0.260)</td>
</tr>
<tr>
<td>Lambda</td>
<td>-0.243**</td>
<td>0.273***</td>
<td>-0.011</td>
<td>-0.019</td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td>(0.089)</td>
<td>(0.052)</td>
<td>(0.093)</td>
</tr>
<tr>
<td>Theta^</td>
<td>0.030**</td>
<td>-0.013</td>
<td>-0.001</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.009)</td>
<td>(0.004)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Quarter i1</td>
<td>-0.007***</td>
<td>-0.002**</td>
<td>0.004***</td>
<td>0.006***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Quarter i2</td>
<td>-0.005***</td>
<td>-0.003**</td>
<td>0.003***</td>
<td>0.004**</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Quarter i3</td>
<td>-0.002</td>
<td>-0.000</td>
<td>0.002***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Quarter i4</td>
<td>0.014***</td>
<td>0.005*</td>
<td>-0.009***</td>
<td>-0.010**</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

Asterisks represent the following p-values: *** p<0.01, ** p<0.05, * p<0.1; standard errors presented in parentheses.

^: First, we estimated the Phis, then we recovered the Thetas; more detail is provided below.

Though parameter estimates provide information regarding relationships, we are more interested in the resulting elasticities. Elasticities can be calculated by differentiating the QUAIDS model with respect to \( \ln m \) and \( \ln p_{ij} \), respectively, to obtain the following:

\[
\begin{align*}
\mu_i & = \frac{\partial w_i}{\partial \ln m} = \beta_i + \frac{2\lambda_i}{b(p)} \left[ \ln \left( \frac{m}{a(p)} \right) \right] \\
\mu_{ij} & = \frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \mu_i \left( \alpha_j + \sum_k \gamma_{jk} \ln P_k \right) - \frac{\lambda_i \beta_j}{b(p)} \left[ \ln \left( \frac{m}{a(p)} \right) \right]^2 \\
\rho_i & = \frac{\mu_i}{w_i} + 1, \quad \epsilon_{ij}^u = \frac{\mu_i}{w_i} - \delta_{ij}, \quad \epsilon_{ij}^c = \epsilon_{ij}^u + \epsilon_{ij}^w
\end{align*}
\]

where \( \epsilon_i \) is income elasticity with respect to good \( i \), \( \epsilon_{ij}^u \) is uncompensated own price (\( i=j, \delta_{ij}=1 \) if \( i=j \), \( 0 \) otherwise) or cross price elasticity (\( i\neq j \)) where \( \delta_{ij} \) is the Kronecker delta, and \( \epsilon_{ij}^c \) is
compensated own price \((i=j)\) or cross price elasticity \((i \neq j)\) (Banks et al., 1997). Uncompensated elasticities are shown in Tables 3, and uncompensated elasticities are shown in Table 4.

**Table 3: Uncompensated Elasticities**

<table>
<thead>
<tr>
<th>Budget</th>
<th>Whole</th>
<th>Two-percent</th>
<th>One-percent</th>
<th>Skim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity i1</td>
<td>-0.363*</td>
<td>-0.614***</td>
<td>-0.220</td>
<td>0.206</td>
</tr>
<tr>
<td>Elasticity i2</td>
<td>-0.662***</td>
<td>-0.079 (0.238)</td>
<td>-0.148 (0.283)</td>
<td>-0.540**</td>
</tr>
<tr>
<td>Elasticity i3</td>
<td>-0.093 (0.106)</td>
<td>-0.048 (0.103)</td>
<td>-1.280*** (0.245)</td>
<td>0.543*** (0.108)</td>
</tr>
<tr>
<td>Elasticity i4</td>
<td>-0.180 (0.166)</td>
<td>-0.015 (0.117)</td>
<td>0.657*** (0.153)</td>
<td>-1.124*** (0.379)</td>
</tr>
</tbody>
</table>

Asterisks represent the following p-values: *** p<0.01, ** p<0.05, * p<0.1; standard errors are in parentheses.

All budget elasticities are positive and significant. Because this is not income elasticity, it cannot be fully interpreted that milk is a ‘necessity’ or normal good. Rather, within this budget set, as total expenditure increases, each milk type’s expenditure increases as well. Another interpretation is that expenditure elasticity reveals the percentage change in the consumption of a given milk type given a one percent change in the expenditure on the set of all milk types (Dharmasena, 2010). We see that all own-price elasticities are negative with one-percent and skim milks having high significance. This is not surprising as it conforms to demand theory.

We are also able to infer some purchasing relationships among the milk types. Notice that whole and two-percent milks have negative, significant relationships with each other and negative (not significant) relationships with one-percent and skim milks, indicating they are gross complements. Perhaps this can be interpreted as persons who purchase the more high in fat milks are not willing to significantly substitute to the lower in fat milks. Another way to interpret may be that households who purchase the more high in fat milks choose to purchase the whole and two-percent milks together, to provide options in the household, but not necessarily sacrifice taste. We see that one-percent and skim milks are significant substitutes, likely related to their fat content and lower prices (though milk prices have been merging recently).

**Table 4: Compensated Elasticities**

<table>
<thead>
<tr>
<th>Compensated</th>
<th>Whole</th>
<th>Two-percent</th>
<th>One-percent</th>
<th>Skim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity i1</td>
<td>0.084 (0.238)</td>
<td>-0.191 (0.210)</td>
<td>0.080 (0.107)</td>
<td>0.027 (0.162)</td>
</tr>
<tr>
<td>Elasticity i2</td>
<td>-0.354* (0.192)</td>
<td>0.195 (0.240)</td>
<td>0.053 (0.103)</td>
<td>0.106 (0.114)</td>
</tr>
<tr>
<td>Elasticity i3</td>
<td>0.121 (0.279)</td>
<td>0.212 (0.282)</td>
<td>-1.149*** (0.246)</td>
<td>0.815*** (0.146)</td>
</tr>
<tr>
<td>Elasticity i4</td>
<td>0.521 (0.329)</td>
<td>-0.208 (0.238)</td>
<td>0.664*** (0.110)</td>
<td>-0.978*** (0.373)</td>
</tr>
</tbody>
</table>

Asterisks represent the following p-values: *** p<0.01, ** p<0.05, * p<0.1; standard errors are in parentheses.

The compensated elasticities do not provide as clear of a picture. While own price elasticities are still negative and significant for one-percent and skim milks, both whole and two-percent
loose significance and are positive. However, the significant cross-relationships among the four milk types seem to correspond to uncompensated elasticities listed above.

One of the main purposes of this paper was to extract optimal advertising lags for the different milk types. As of now, the four milk types all had the same optimal lag, though more work is needed here. The optimal lag was four lags (months) for all four milk types. Noting the above derivations, we estimated phi-2 for all for milk types. Then, we recover thetas by substitution (derived above). The following table provides the theta estimates for all four milk types and five periods (four lags, and the current period). Recalling that we assumed a second degree polynomial, with end point restrictions, we have four theta estimates for each milk type and an additional theta for the current period (t = 0).

Table 5: Recovered Advertising Thetas

<table>
<thead>
<tr>
<th></th>
<th>Whole</th>
<th>Two-percent</th>
<th>One-percent</th>
<th>Skim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theta i0</td>
<td>-0.152**</td>
<td>0.063</td>
<td>0.006</td>
<td>0.083</td>
</tr>
<tr>
<td>Theta i1</td>
<td>-0.243**</td>
<td>0.101</td>
<td>0.009</td>
<td>0.132</td>
</tr>
<tr>
<td>Theta i2</td>
<td>-0.274**</td>
<td>0.114</td>
<td>0.011</td>
<td>0.149</td>
</tr>
<tr>
<td>Theta i3</td>
<td>-0.243**</td>
<td>0.101</td>
<td>0.009</td>
<td>0.132</td>
</tr>
<tr>
<td>Theta i4</td>
<td>-0.152**</td>
<td>0.063</td>
<td>0.006</td>
<td>0.083</td>
</tr>
</tbody>
</table>

Asterisks represent the following p-values: *** p<0.01, ** p<0.05, * p<0.1; standard errors are in parentheses.

Notice that whole milk has all five thetas being negatively valued. This is likely due to the adding up restriction required by complete demand systems. Though all other milk types have positive thetas, none are significant. Further, one can see that thetas from particular periods are equal (period zero and period four; period one and period three). This is due to the substituting and recovering formulas shown above. Though advertising is generic for milk, we see there are different effects for each milk type. However, as of now, we can only conclude that each milk type had the same optimal lag of four periods.

Conclusion

This paper measures the effects of a polynomial distributed lag advertising variable on fluid milk types using a modified quadratic Almost Ideal Demand System (QUAIDS). We analyze the effects on per capita consumption of each of the four major types of milk, namely whole milk, two-percent milk, one-percent milk, and skim milk. If there are different advertising effects for various types of milk, advertising expenditures may be spent accordingly able to cater to those

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7 Each milk type was separately regressed on all four advertising lags; all four lags had positive relationships with each milk type.
differences, increasing sales of fluid milk and/or positively affecting America’s weight problem. The authors intend to spend more time focusing on the lag differences (if any) among the four milk types. Also, in regards to advertising, alternative models with fewer restrictions may be ideal so as to avoid a negative advertising coefficient. Finally, this paper only looks at four types of milk, although there are other dairy options available for consumers. Future research could incorporate these products.
References


