The Optimal Length of an Agricultural Carbon Contract

Sumeet Gulati and James Vercammen

July 21, 2005
Working Paper Number: 2005-02

Food and Resource Economics, University of British Columbia
Vancouver, Canada, V6T 1Z4
http://www.agsci.ubc.ca/fre

The Optimal Length of an Agricultural Carbon Contract♣

by

Sumeet Gulati and James Vercammen*

July 21, 2005

Abstract. In this paper we present the economic determinants of the optimal length of a carbon offset contract. We find that because of a declining capacity of the soil to sequester carbon, the optimal length of the carbon contract is finite (the marginal benefit of remaining in the contract is declining over time, whereas marginal opportunity cost is rising). We also explore the effect of varying key parameter values on the optimal length in the contract. If the contract requires the farmer to sequester at a higher rate, the farmer chooses the contract for a shorter length of time, and this may decrease rather than increase social welfare. If society places a higher value on carbon accumulation, the contract is chosen for a longer length of time. Finally, if both the farmer and society have a higher discount rate, the model provides a somewhat surprising result. The overall time in the contract, and benefits from carbon accumulation are higher when the common discount rate is higher.

Keywords: Carbon Offset Contracts, Greenhouse Gas Policy, Soil Carbon

JEL Classification: Q200, Q580

♣ Financial support from SSHRC-BIOCAP Canada is gratefully appreciated. We would like to thank two anonymous referees for valuable comments. All errors are ours alone.

* Sumeet Gulati is an Assistant Professor, Food and Resource Economics, and Forest Resources Management, University of British Columbia, Phone: (604) 822-2144, Email: sumeet.gulati@ubc.ca, James Vercammen is a Professor, Food and Resource Economics, and Strategy and Business Economics, University of British Columbia. Phone (604) 822-8475, Email: james.vercammen@sauder.ubc.ca.

1. Introduction

According to OECD (1997), from 1993 till 1997, fourteen countries in Europe paid 11 billion dollars to divert 20 million hectares of agricultural land into land easement, or forestry. In the US, the Conservation Reserve Program (CRP) spends $ 1.5 billion annually to contract 12-15 million hectares (Ferraro and Simpson (2002)). Similarly, carbon offset contracts resembling the programs above are likely to become more common once carbon trading and offset systems for control of greenhouse gas emissions develop (see Thomassin 2003 for a discussion). What are the incentives faced by agricultural landowners when they participate in such a carbon offset contract?

On being presented a carbon contract, some landowners may make permanent changes to land use (e.g., convert marginal cropland to hayland). These farmers would generate a permanent increase in the level of sequestered carbon. However, it is anticipated that most landowners would sign the contract for a pre-determined length of time, in which case both the length of the contract and land use after the expiration of the contract are important determinants of the social value of the contract. Consider for example if the landowner reverts back to pre-contract land use. In this case, eventually all of the carbon sequestered during the contract is released back into the atmosphere. As discussed by Lee and McCarl (2003) this type of behaviour provides only temporary benefits, and the magnitude of these benefits are highly dependent on the farmer's chosen length of the contract.[1]

In this paper, we present a simple dynamic model to analyze a one-time fixed-term carbon offset contract. Our aim is to evaluate the economic determinants underlying

[1] The usefulness of temporary sequestration benefits are examined in detail by Lecocq and Chomitz (2001), Wilman and Mahendrarajah (2002) and Cacho et al. (2003).

the farmer's choice of contract length, and the associated benefits to society from a temporary carbon contract. We also analyze changes in the optimal length of the contract and the social benefits that accompany variations in key contract parameters such as the sequestration effectiveness of the technology in place while under contract, the price of carbon and the discount rate.

In the model presented, a landowner signs a contract for *T* periods (where *T* can be infinite). Upon expiration of the contract, the land reverts back to pre-contract use (assumed to reflect a long-run no-contract steady state).[2] If the farmer chooses a finite *T*, there are only temporary sequestration benefits. The size of these temporary benefits depends positively on the length of the contract (*T*), and the effectiveness of the land in sequestering carbon. The landowner is offered a menu of time-contingent contracts (i.e., a unique payment is provided to the landowner for each chosen value of *T*), but only one land use option is available under the contract.[3]

Payment for signing the contract is priced by a fully rational agent (in our case the government) with perfect foresight. As the landowner chooses alternative lengths for the carbon contract, the menu of payments provided to the landowner equal the discounted market value of the carbon initially sequestered and eventually released.[4] Specifically, if a landowner with a known carbon stock requests a contract of length *T*, the government calculates the net present value (NPV) of the period-by-period increments to soil carbon

---

[2] We assume that land returns to pre-contract use to simplify the model. A more complete model would allow the farmer to choose the most efficient post-contract land use. However, such an alteration is not likely to alter the main intuition presented in the model. One could potentially justify this assumption by assuming prohibitively high private investment costs of changing land use. While the carbon contract might pay for the cost of transition to a new technology, the prohibitive cost of changing to a third technology might prevent the landowner from choosing the post-contract efficient technology.
[3] Endogenizing the choice of land use within the contract is outside the scope of this analysis.
[4] See Chomitz and Lecocq (2003) for a discussion of how the environmental integrity of a carbon sequestration project can be maintained by discounting the price of carbon when sequestration is temporary versus permanent.

(assumed nonstochastic) while the contract is in place (the market price per unit of sequestered carbon is assumed constant over time). However, the government also perfectly anticipates the future release of the carbon during the post contract period when the land reverts back to its traditional use. The NPV of these period-by-period reductions in the stock of sequestered carbon during the post-contract period is calculated by the agent and subtracted from the NPV of the sequestered carbon during the contract period.[5]

Through our analysis we wish to explain the economic determinants of contract length, *T*.[6] We find that although the net present market value of the sequestered carbon rises continuously with *T*, farm profits generally reach a maximum at some unique value of *T*. The optimal value of *T* occurs when the marginal benefit of extending the contract by one more period is equal to the marginal profits foregone by not operating under pre-contract land use (i.e., marginal opportunity cost). The marginal benefit of extending the contract diminishes over time. This is because the rate of accumulation of carbon slows as its stock approaches the soil's carrying capacity. The marginal opportunity cost of staying in the contract rises over time. This is because, as soil carbon rises, the contracted land becomes more productive under the pre-contract technology.

The literature on carbon markets has provided little discussion regarding the economic determinants of optimal contract length. It is well known that the economic incentives for a landowner to enter a carbon offset contract depends on the opportunity cost of changing production practices relative to the rate of soil carbon sequestration (Antle et al. 2001, 2003). However, the fact that marginal costs and benefits of

[5] This net value of sequestered carbon is set equal to the NPV of the payment provided to the landowner to reflect the perfectly competitive market for carbon contracts.
[6] Pautsch et al. (2001) use survey data to empirically examine the determinants of a farmer's soil carbon conservation decision.

sequestration are changing over time and will thus give rise to an optimal contract length has apparently not been discussed. A somewhat analogous problem to the one considered here is how the optimal rotation length for forestry is impacted by carbon offset contracts (van Kooten et. al. 1995). The analogy between the two situations is apparent if we consider creating the sink capacity for soil for subsequent sequestration as similar to the growing of the trees, and consider the harvesting of trees as similar to filling the sink with sequestered carbon.[7]

The rest of the paper is structured as follows. In section 2 we present our simple dynamic soil carbon model. We also derive the marginal benefit and marginal cost schedules that determine the optimal contract length, and then present an expression for the optimal length of contract. Numerical simulation results, and comparative static results, are presented in Section 3. Two of the comparative static results are rather intuitive (e.g., sequestration effectiveness and optimal contract length are negatively related whereas the price of carbon and optimal contract length are positively correlated). The relationship between the discount rate and optimal contract length is shown to be more complex. Concluding comments are contained in Section 4.

2. The Model

We present a simple model in which a representative farmer enters a carbon sequestration contract. The farmer chooses the term of the contract in which she is required to sequester soil carbon. However, at the end of the contract the farmer reverts back to pre-contract land use and releases the carbon sequestered. The farmer values carbon due to

[7] To simplify the analysis we restrict the landowner to only one carbon contract. Unlike a tree rotation problem we do not allow the landowner to enter into an alternating rotation of contracting and no contracting periods.

the productivity associated with it (i.e., more soil carbon implies higher crop yields). Society values carbon accumulation due to its benefits to the environment. The societal benefit of each unit of sequestered carbon is assumed to be fixed over time at level P.

Let the scalar x represent an index of land use and cultivation intensity (hereafter referred to as farm technology). For a given stock of soil carbon, the rate of accumulation of soil carbon is negatively related to x. Prior to signing contract, $x = x_n$ where x_n maximizes the long-run profits of the farm. The contract specifies that $x = x_c$ with $x_c < x_n$. This implies that soil carbon accumulates through the contract and rises above the level maintained by the farmer prior to the carbon contract.

Optimal Technology in the Absence of a Contract

Without the option of a contract, farm profits are given by a simple reduced form profit function

$$\pi_t = \pi_0 + kxC_t, \qquad (1)$$

where π_0, and k are positive parameters, and C_t denotes the stock of carbon in the soil. At any time t, farm profits are a sum of a fixed profit parameter (π_0) and a portion related to the level of carbon stored in the soil (or the soil's productivity).

If the farmer adopts a higher x current profits are higher, but future profits are lower. This is because an increase in x also reduces the carbon available in the next period (denoted C_{t+1}). The carbon growth equation is

$$C_{t+1} - C_t = a\left(M - C_t\right) - bx, \qquad (2)$$

where $0 < a < 1$, and M and b are positive parameters.[8] If we rewrite the carbon growth equation as $C_{t+1} - (1-a)C_t = aM - bx$, and let $H = M - \frac{b}{a}x$, the solution to the linear first order carbon difference equation is

$$C_t = (C_0 - H)(1-a)^t + H, \tag{3}$$

where C_0 is the initial stock of carbon. The steady state stock of carbon that emerges from this equation is H. As can be seen from the $H = M - \frac{b}{a}x$ expression, if the farmer chooses a more intensive technology, the steady state stock of carbon is lower.

Using equation (1) we can calculate the net present value of farm profits over $\bar{T}$ periods

$$\pi(C_0, \bar{T}, x) = \sum_{t=1}^{\bar{T}} \delta^t \pi_0 + kx \sum_{t=1}^{\bar{T}} \delta^t C_t \tag{4}$$

where $\delta = \frac{1}{1+r}$ is the farmer's (and also society's) discount factor, and r is the common discount rate. Using the solution to the carbon growth equation (2), and the formula for the sum of a finite series, we get

$$\pi(C_0, \bar{T}, x) = (\pi_0 + kxH)\left(\frac{1-\delta^{\bar{T}+1}}{1-\delta} - 1\right) + kx(C_0 - H)\left[\frac{1-\left[\delta(1-a)\right]^{\bar{T}+1}}{1-\left[\delta(1-a)\right]} - 1\right]. \tag{5}$$

Throughout this paper we assume that the farmer cannot vary technology continuously over time. We assume that in the absence of a contract, the farmer chooses

[8] Assuming a linear form for the carbon growth equation allows us to derive simple solutions for key variables in our model. This assumption is made for simplicity, and as biological growth functions are usually non-linear, it is not likely to be an accurate representation of reality. However, it should be noted that making this assumption will not qualitatively affect the main points presented in the paper. This is more evident once the marginal benefits and costs from being in the contract are explored later in the paper.

a level of technology at the beginning of her time horizon and then persists with it.[9] For a farmer with an infinite time horizon and no contract, the objective is to choose technology x to maximize $\pi\left(C_0,\infty,x\right)$:

$$\max_x\left\{\pi\left(C_0,\infty,x\right)=\left(\pi_0+kxH\right)\left(\frac{\delta}{1-\delta}\right)+kx\left(C_0-H\right)\frac{\left(1-a\right)\delta}{1-\left(1-a\right)\delta}\right\}. \tag{6}$$

The solution is

$$x_n=\frac{a}{2b}\left[M+\frac{1-a}{1-\delta\left(1-a\right)}\left(\frac{1}{1-\delta}-\frac{\left(1-a\right)}{1-\delta\left(1-a\right)}\right)^{-1}C_0\right]. \tag{7}$$

Equation (7) is an expression for the optimal technology choice for the farmer in the absence of a contract. This implied value for x_n is the technology which is assumed to be in place when the farmer is offered a contract, and is also the technology that the farmer reverts back to after the contract expires.

Notice that the optimal technology, x_n, in equation (7) is conditioned on the starting value for the carbon stock, C_0. To simplify our analysis, we focus on the steady state choice of technology; i.e, we assume $C_0=H_n=M-\frac{b}{a}x_n$. If this expression is substituted into equation (7), the revised expression is

$$x^{**}=\frac{a}{b}\left[\frac{2}{1-\delta}-\frac{\left(1-a\right)}{1-\delta\left(1-a\right)}\right]^{-1}\frac{M}{1-\delta}. \tag{8}$$

[9] This is consistent with the earlier discussion of a prohibitive private cost for varying technology. It also implies that even though technology x is continuous, we effectively observe only two technology choices.

Thus, if the farmer starts with carbon stock $C_0 = M - \frac{b}{a}x^{**}$ and the optimal technology is chosen, then the carbon stock does not change and the system remains in steady state forever.

The Carbon Contract

Society also values carbon sequestered in the soil due to its global warming benefits. We assume that this additional benefit can be valued at a constant price *P* per unit of carbon accumulated. Let $\Gamma_t = C_{t+1} - C_t$ denote the carbon accumulated (or depleted) in time *t*. Society's valuation for carbon accumulated in time *t* is $P\Gamma_t$. The farmer does not internalize society's benefits while deciding on her choice of technology. The government corrects this externality by offering a carbon contract.

Assuming that the farmer is at a steady state level of soil carbon ($C_0 = M - \frac{b}{a}x_n$, where $x_n = x^{**}$ from above), the farmer is offered a one time carbon contract. Under the terms of the contract, the farmer can choose the length of the contract (denoted *T*), but cannot choose the technology (which is specified by the contract to be x_c). Intuitively, this is similar to assuming that the farmer loses control over the plot of land covered by the contract. On signing the contract, the farmer commits to growing alfalfa, or timber, or any other carbon preserving crop on the plot of land. Once the farmer exits the contract she automatically reverts back to the pre-contract technology.[10]

[10] Consistent with the earlier discussion this could be because on regaining control over her plot of land, the farmer is constrained by long term investments made prior to the contract.

On being offered the contract, the farmer can choose to not sign the contract ($T = 0$). She can also choose to sign the contract and stay in it forever ($T = \infty$). Or she could choose a finite time span for being in the contract ($0 < T < \infty$). The farmer receives a payment for signing the contract. This payment equals the social benefits from carbon accumulated by the farmer's actions. Specifically, it is related to the length of time the farmer chooses to stay in the contract, and the anticipated time path of soil carbon de-accumulation on leaving the contract.

We assume complete information, and perfect foresight for both the farmer and the agency offering the contract. The contract stipulates that the farmer receives a payment equal to the discounted social value of carbon sequestered through the infinite time horizon. This payment includes the positive social value of carbon accumulated during the contract period, and the negative social value of the carbon released to the atmosphere during the post contract period. Based on this payment scheme the farmer announces the length of time she will stay in the contract (we assume that time in the contract is fully enforceable at zero cost).[11] This payment scheme allows the farmer to fully internalize the carbon sequestration benefits she provides to society.[12]

Given that the farmer reverts to earlier technology on exiting the contract, payments in the contract can be calculated as follows. The value of carbon accumulated during the contracting period is denoted $\Phi\left(C_0, T, x_c\right) = \mathrm{P}\sum_{t=1}^{T}\delta^t\Gamma_t$, where $C_0 = M - \frac{b}{a}x_n$

[11] If the farmer could renege on the contract at some cost, it would then be necessary to specify the distribution of payment over the life of the contract. As it now stands, any distribution of payment is acceptable as long is it provides the farmer with the same net present value.
[12] Note that the farmer only signs the contract if the value of payments is larger than the opportunity cost of being in the contract. Clearly, she would never sign if the payment was negative.

($x_n = x^{**}$) is the steady state carbon stock. Using the solution for carbon growth (equation (3)), and recalling that $\Gamma_t = C_{t+1} - C_t$, the expression for $\Phi\left(C_0, T, x_c\right)$ is

$$\Phi\left(C_0, T, x_c\right) = -a\mathrm{P}\left(C_0 - H_c\right)\sum_{t=1}^{T}\delta^t\left(1-a\right)^t, \tag{9}$$

where $H_c = M - \frac{b}{a}x_c$. Equation (9) can be further simplified to

$$\Phi\left(C_0, T, x_c\right) = a\mathrm{P}\left(H_c - C_0\right)\left[\frac{1-\left[\delta\left(1-a\right)\right]^{T+1}}{1-\left[\delta\left(1-a\right)\right]} - 1\right]. \tag{10}$$

Note that $H_c - C_0 = \frac{1}{a}\left(C\left(T\right) - C_0\right) = \frac{b}{a}\left(x_n - x_c\right) > 0$, where $C(T) - C_0$ is the carbon accumulated during the contracting period. We can similarly construct the value for carbon depleted after the contract expires. This is

$$\Phi\left(C(T), \infty, x_n\right) = a\mathrm{P}\left(H_n - C(T)\right)\left[\frac{\delta\left(1-a\right)}{1-\delta\left(1-a\right)}\right], \tag{11}$$

where $H_n = M - \frac{b}{a}x_n$, and $C\left(T\right)$ is the stock of carbon at the end of the contract.

Using equations (10) and (11), the payment offered to the farmer at the start of the contract is

$$\xi\left(C_0, \infty\right) = \Phi\left(C_0, T, x_c\right) + \delta^T\Phi\left(C(T), \infty, x_n\right). \tag{12}$$

The farmer's problem is to choose T to maximize the net present value of profits that include payments from the contract:

$$V = \max_T\left\{\pi\left(C_0, T, x_c\right) + \Phi\left(C_0, T, x_c\right) + \delta^T\left[\pi\left(C\left(T\right), \infty, x_n\right) + \Phi\left(C(T), \infty, x_n\right)\right]\right\} \tag{13}$$

After considerable manipulation, an expression for the optimal value of *T* can be written as

$$T^{*}=\frac{\ln(K_{1})-\ln(K_{2})}{\ln(1-a)} \tag{14}$$

where:

$$K_{1}=\frac{\delta}{1-\delta}\left[k\left(x_{c}H_{c}-x_{n}H_{n}\right)+\frac{\delta(1-a)}{1-\delta(1-a)}\frac{b}{a}\left(kx_{n}-aP\right)\left(x_{c}-x_{n}\right)\right]\ln(\delta)$$

and

$$K_{2}=\frac{\delta(1-a)}{1-\delta(1-a)}\left(C_{0}-H_{c}\right)\left(x_{n}-x_{c}\right)k\ln\left(\delta(1-a)\right)$$

The Marginal Benefit and Marginal Cost of Staying in the Contract

While choosing the optimal *T* the farmer weighs the benefits from staying an extra period in the contract, against its opportunity cost. Let $\pi^{c}=\pi\left(C_{0},T,x_{c}\right)$ be the NPV of profit earned while in the carbon contract, and let $\Phi^{c}=\Phi\left(C_{0},T,x_{c}\right)$ be the NPV of the carbon accumulation benefits that accrue while the farmer is in the contract. Similarly, let $\pi^{n}=\pi\left(C\left(T\right),\infty,x_{n}\right)$, and $\Phi^{n}=\Phi\left(C(T),\infty,x_{n}\right)$ be the corresponding levels out of the contract. Note that $\Phi^{n}<0$ because soil carbon de-accumulates after time *T*.

The marginal benefit of staying in the contract as of time *T* is given by

$$MB_{T}=\left(\frac{d\pi^{c}}{dT}+\frac{d\Phi^{c}}{dT}\right)\delta^{-T} \tag{15}$$

The marginal benefit of staying in the contract is the sum of the change in farm profits in the contract, and the change in the value of carbon accumulation benefits from staying in the contract. As carbon accumulation falls with an increase in the carbon stock (see equation (2)), the marginal benefit of staying in the contract declines with time.

However, the marginal opportunity cost of staying in the contract rises with time. This is because, as the carbon stock rises, the productivity of the soil rises. In other words, with a higher stock of carbon, the difference in profits from operating with the carbon conserving technology in the contract versus the pre-contract technology is larger. The marginal opportunity cost of staying an extra period in the contract at time T is

$$MC_T = -\left[\left(\frac{d\pi^n}{dC(T)} + \frac{d\Phi^n}{dC(T)}\right)\frac{dC(T)}{dT} + \ln(\delta)\left(\pi^n + \Phi^n\right)\right]. \tag{16}$$

The term $\ln(\delta)\left(\pi^n + \Phi^n\right)$ in equation (16) reflects the effect of changing T via the discount factor δ^T. Explicit expressions for the derivatives contained in the marginal benefit and marginal cost equations are given in Appendix. These expressions are used to plot the marginal benefit and marginal cost schedules presented in Figure 1.

3. Numerical Simulation

In this section we present simulation results for the optimal time in the contract as chosen by the farmer. These results should be viewed as an illustrative example rather than a comprehensive analysis. After presenting and discussing the base case, we present some comparative static results for the choice of T. The parameter values we assume for the base case simulation are given in Table 1. Equation (14) is used to calculate the optimal value of T^* given these parameter values.

Table 1: Baseline Parameter Values

Parameter	Description	Value
π_0	Constant profit parameter	5
K	Profit parameter	0.075
M	Carbon growth parameter	50
A	Carbon growth parameter	0.1
B	Carbon growth parameter	0.2
R	Common discount rate	0.05
P	Price for accumulated carbon	10
$\bar{x}$	Upper limit on technology	20
x_c	Technology in contract	10

As indicated above, we assume that the stock of soil carbon at the time the contract is signed (x_n) is equal to the steady state value associated with technology choice x^{**}, as given by equation (6). Given the parameters in Table 1, $x_n \text{ or } x^{**} = 14.71$ and $C_0 = 20.59$.

Base Case Results

The optimal time period to exit the contract is where the marginal benefit of staying in the contract equals its marginal opportunity cost. Using the assigned parameter values and the expressions for the MB_T and MC_T schedules that were derived above, the optimal exit time can be illustrated graphically. Specifically, the point where the two curves intersect is the optimal time to exit the contract. The MB_T and MC_T schedules corresponding to the base case parameters are presented in Figure 1.

Figure 1: Simulated MB and MC of Contract Length

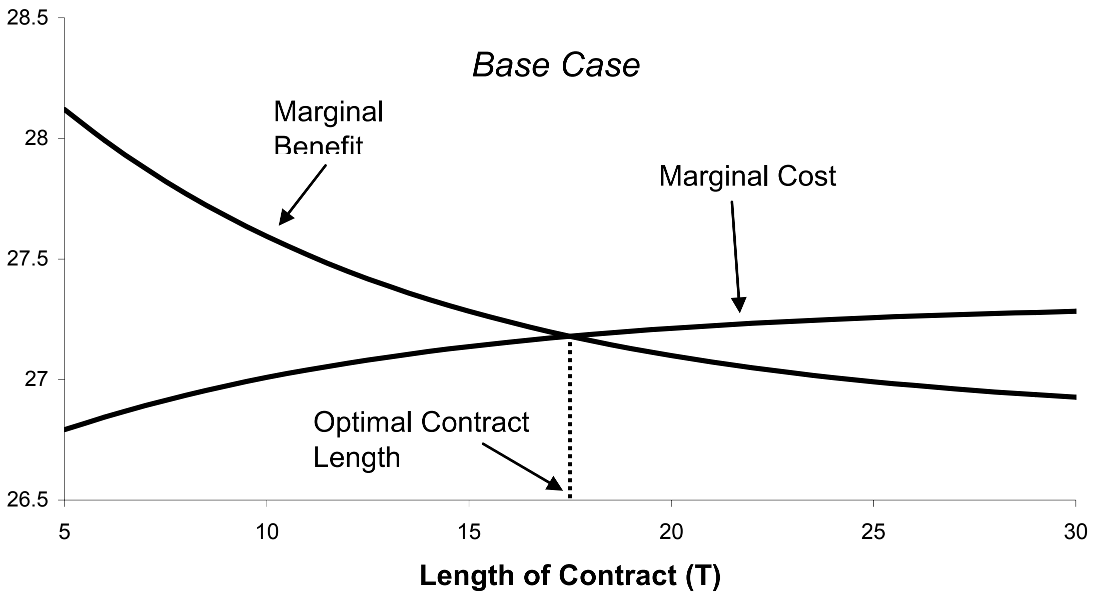


Figure 1 shows that the two curves intersect somewhere between 17 and 18 periods. The precise time period where it is optimal to pull out of the contract is 17.50 periods. Given the 5% discount rate which comprises the base case, it may be useful to consider each time period to be a year, which implies that a contract of between 17 and 18 years is optimal for the farmer.

The remaining solution values associated with the base case simulation are shown in Table 2. Note that the stock of soil carbon rises from 20.59 to 28.58 due to the carbon contract. The stock of carbon eventually returns to 20.59 given a sufficiently long length of time after the contract expires. Despite the temporary nature of the carbon contract, the net present value of the carbon accumulation and de-accumulation cycle is 32.42. This number is useful as a benchmark in the comparative static results presented below.

Table 2: Results based on Baseline Parameter Values

Variable	Description	VAlue
T	Time in initial contract	17.50
ξ	Net present social value of sequestrated carbon	32.42
V	Net present value of farm profits and contract payments	567.0
$C(T)$	Carbon stock after contract terminates	28.58
C_0	Carbon at the start of contract	20.59

Comparative Static Results

In this section we examine how some of the key parameters alter the optimal time chosen by the farmer in the contract.

Technology Imposed by the Contract

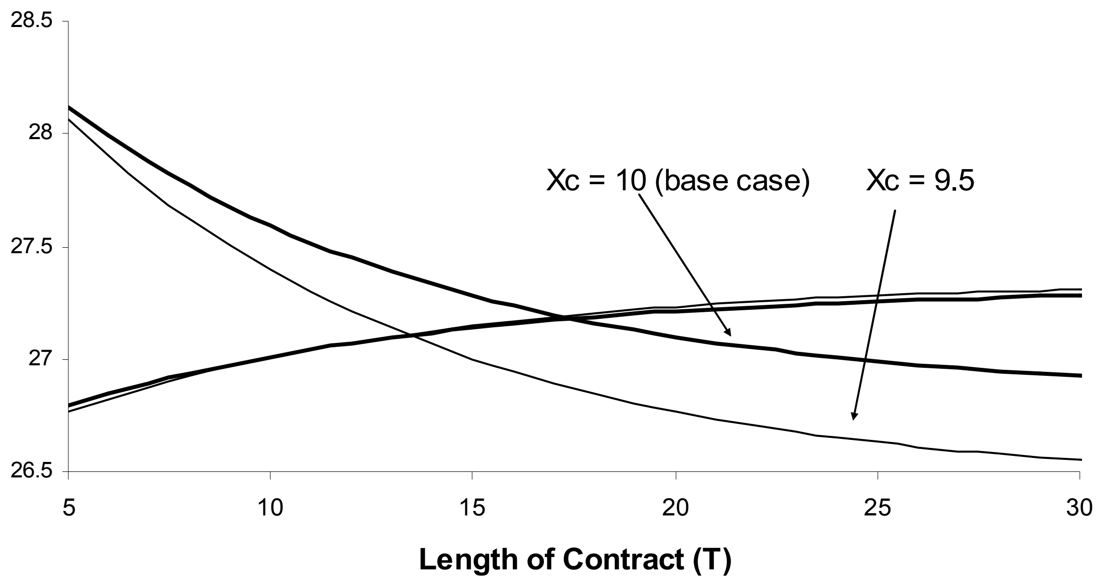
Consider the effect of changing the technology imposed by the contract (x_c). A higher x_c implies a less conserving technology (or a more exploiting technology). The marginal benefit of staying an additional period in the contract has two components. The first is the farm profit component. A higher value of x_c raises short-term farm profits (instantaneous farm profits are $\pi_t = \pi_0 + kxC_t$) and thus *MB* is higher. Further, as x_c is higher, carbon is accumulated more slowly during the contract period. This implies that the fall in carbon accumulation benefits as time increases is also slower. This lowers the *MB* curve, and also implies that the curve declines more slowly over time as compared to the base case.

Similarly, the marginal cost has two main components. A higher x_c in the contract implies that the terminal stock, *C(T)*, is smaller at any given time period. Consequently, profit outside the contract is also smaller. However, for the same reason as the stock of carbon is smaller, society's loss from carbon harvest is also smaller. This raises the MC curve, and also implies that the curve rises faster than earlier. Simulation results reveal that the overall effect of an increase in x_c is primarily to shift up the *MB* curve, which implies a longer optimal time to remain in the contract (see Figure 2 for the case of a decrease in x_c). This result is intuitive in the sense that the greater the similarity between the technologies used in and out of the contract, the longer the farmer will remain with the contract.

The agent offering the contract may choose to decrease x_c (i.e., require the farmer to sequester more carbon each period) in an attempt to raise the present value of the temporary benefits from sequestration. Given the above results, we know that lowering x_c will induce the farmer to choose a shorter contract, which will necessarily reduce the present value of the temporary benefits. Consequently, reducing x_c has an ambiguous impact on the level of temporary sequestration benefits.

Starting with base case parameter values, suppose x_c is decreased from 10 to 9.5. Figure 2 shows that in this case the optimal length of the contract for the farmer decreases from 17.5 years to about 13.4 years. Recall from Table 2 that in the base case with x_c = 10, the present value of the temporary sequestration was 32.42. With x_c = 9.5, this value falls to 29.94. Thus, in this particular case, shifting to a more conserving technology reduces the temporary benefits from sequestration. The relatively high sensitivity of the farmer's choice of *T* prevents the implementation of a technology that conserves a relatively high level of carbon. It is useful to point out that the decrease in x_c from 10 to 9.5 also reduces the farmer's welfare, so a reduction in x_c can not be justified.

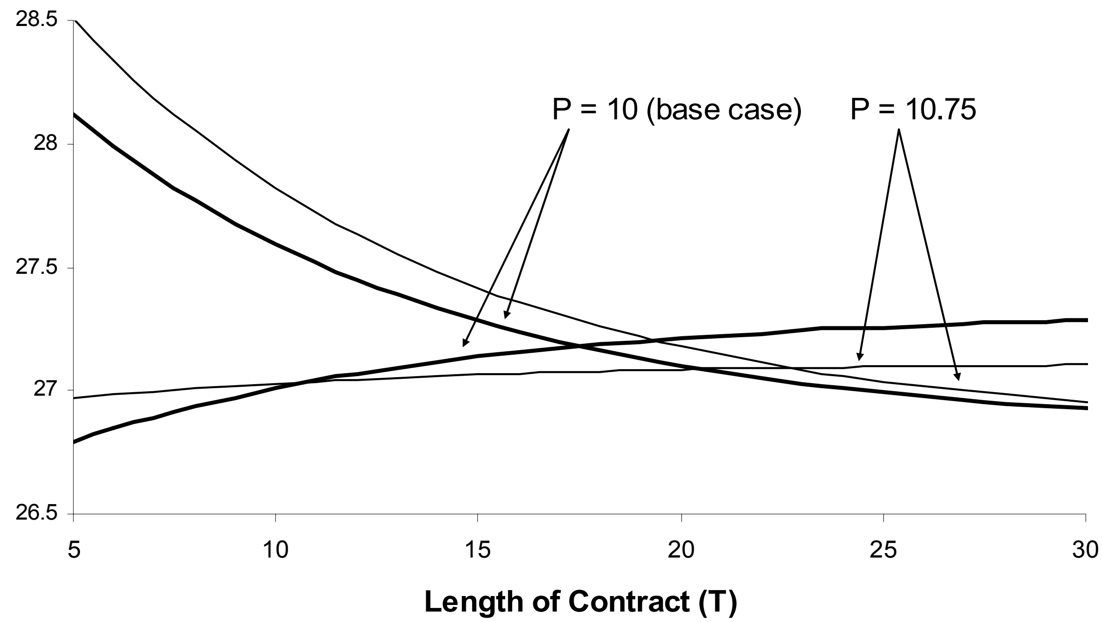
Figure 2: Impact of More Conserving Contractual Technology



Price for Accumulated Carbon

If the market price of carbon increases, the marginal benefit of extending the contract must necessarily increase. This is shown in Figure 3 where *P* is raised from its base case value of 10 to 10.75. Figure 3 also shows that the marginal cost changes with an increase in *P*. The marginal cost increases for relatively small values of *T* and then decreases for relatively high values of *T*. A higher value of *P* increases the accumulation of carbon and thus increases its stock at any given point in time. This raises the opportunity cost of farming with the contract (i.e., the non-contract technology is relatively more profitable), and causes the marginal cost to shift up. However, a higher stock also implies that the de-accumulation of carbon is more rapid. As de-accumulation imposes a cost to the farmer the marginal cost also shifts down. Figure 3 illustrates that this de-accumulation disincentive to quit the contract dominates as the stock of carbon (and *T*) increases. The overall impact of an increase in P from 10 to 10.75 is to increase the optimal length of the contract from 17.5 years to 22.75 years.

Figure 3: Impact of Higher Carbon Price



Discount Rate

Consider the case when the common rate of discount (r) is raised. An increase in the discount rate affects several variables in our model. A new discount rate changes the technology chosen by the farmer before and after the contract ($x_n = x^{**}$ changes). This implies that the steady state value of carbon before and after the contract also changes. A change in the common rate of discount also alters the way farmers and society view future benefits and costs. Future benefits from carbon accumulation are valued lower. Further, current losses from carbon de-accumulation are also valued lower.

With a higher common rate of discount, farm profits in the future are less valuable. This implies that in the steady state before the contract is offered, the farmer chooses a higher carbon exploiting technology ($x_n=x^{**}=15$). The carbon stock on joining the contract due to the higher technology is now lower (C_0=19.93). Given a lower carbon stock at the start of the contract, there is a higher rate of accumulation in the contract. This can shift the *MB* curve upwards. Coupled with the fact that de-accumulation occurs at a later date (on exiting of the contract) the marginal cost of being in the contract at any point in time is also lower. In other words, unlike what one usually expects, a higher discount rate can make it worthwhile for the farmer to stay in the contract longer.

We now present a more detailed explanation for the change in the *MB* and *MC* curves. Recall that the marginal benefit of staying in the contract for the farmer comes from two components. Firstly, as the farmer stays longer in the contract she has higher future profits from a higher carbon stock. However, given a higher discount rate, this future benefit is valued less (thus the *MB* is lower). Secondly, as the farmer stays longer in the contract she can raise her payments from carbon accumulation at time *T*. Given a higher discount rate, and a lower starting value for carbon stock, society values current accumulation higher. Payments rise and the *MB* curve rises.

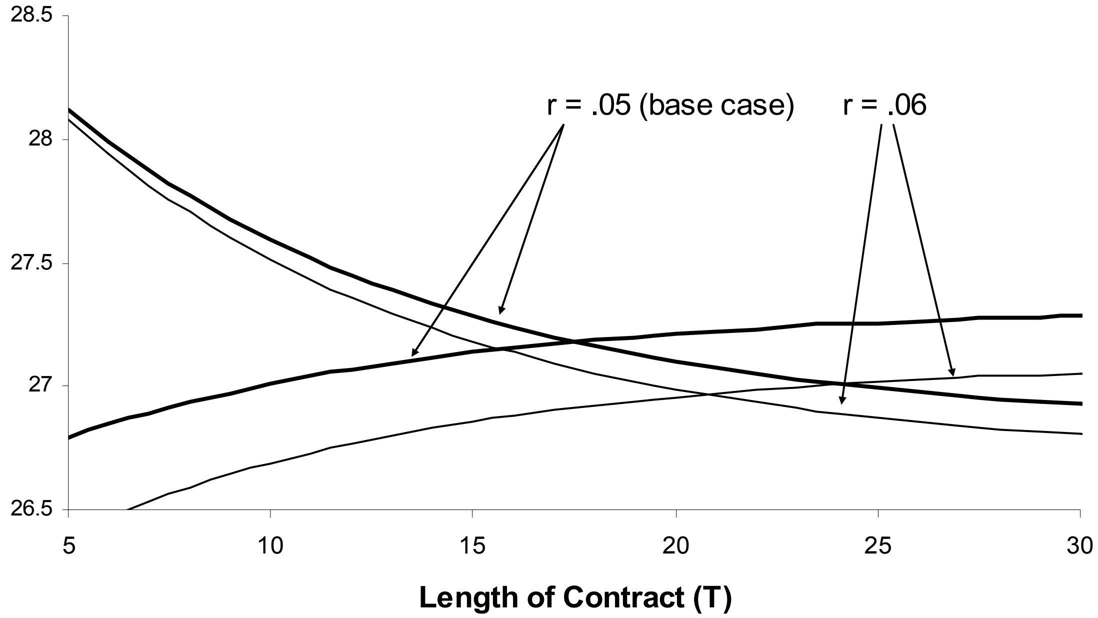
The *MC* of staying in the contract results from three components. Firstly, as the farmer stays in the contract longer, she delays the high profits that could accrue from reverting to her old technology. With a higher discount rate, she wishes to gain higher current profits as soon as possible. In other words, the *MC* of staying in the contract is now higher.[13] The second component is the social loss from de-accumulation. Given that society is also more impatient, it values current carbon gains/losses higher. In other words, given greater impatience, the social loss from carbon de-accumulation rises. This lowers the *MC* of staying in the contract.[14] The final component includes the changes in profits and payments as carbon stocks rise due to an increase in *T*: $\left(\frac{d\pi^n}{dC(T)}+\frac{d\Phi^n}{dC(T)}\right)\frac{dC(T)}{dT}$. This term is ambiguous and we look to the simulation results to sign the overall effects.

Given our parameter values, we find that both the *MB* and *MC* of staying in the contract fall as the common rate of discount is increased. We also find that the *MB* falls relatively less than the *MC*, and as discussed earlier an increase in the common rate of discount implies that the farmer exits the contract later. Figure 4 illustrates the case, where the rate of discount is increased from 5% to 6%. The optimal length of the contract for the farmer increases from 17.5 years to about 20.728 years. Recall from Table 2 that in the base case with *r* =5%, the present value of the temporary sequestration was 32.42. With *r* =6%, this value rises to 39.74.

[13] The component relevant to this argument is the loss in interest from having to wait another period to obtain farm profits outside the contract ($\ln(\delta)\left(\pi^n\right)$).

[14] The component relevant to this argument is the gain in interest from having to wait another period to incur social losses from de-accumulation outside the contract ($\ln(\delta)\left(\Phi^n\right)$).

Figure 4: Impact of Higher Discount Rate



4. Conclusion

In this paper, we present economic determinants of the optimal length of a one time carbon offset contract. We find that given a carrying capacity of carbon in the soil, and fixed prices, the optimal length of the carbon contract is finite. Intuitively, this optimal length is found by equating the marginal benefit and marginal opportunity cost of staying in the contract.

Parameter variations affecting the optimal contract length yield some expected and some unexpected results. When the contract requires the farmer to use greater carbon conserving technology the farmer chooses the contract for a shorter period of time. Assuming our baseline parameters the farmer's response to the contractual requirements for more carbon conservation can decrease rather than increase aggregate carbon benefits for society. Next, if society places a higher value on carbon accumulation (*P* rises), the contract is chosen for a longer length of time, as is expected.

Finally, if both the farmer and society have a higher discount rate, the model provides a counter-intuitive result. From standard intuition we know that with a higher discount rate the farmer places a higher value on instantaneous profits outside the contract. This encourages the farmer to choose a shorter contract. However, this ignores the impact of the discount rate on starting values. A higher discount rate lowers the steady state carbon stock outside the contract. This implies that there is greater potential for carbon accumulation, and higher payments are possible within the contract. Further, on returning to exploitative behaviour at the end of the contract, penalties are also higher. Under the parameter values assumed, the effect of the discount rate on starting values dominates the impact on incentives to quit early. We find that a higher discount rate implies increased time in the contract, and higher benefits from carbon accumulation.

This paper is an initial exploration of the incentives faced by landowners choosing carbon contracts (note that the basic intuition is also valid for most renewable resource contracts). Several extensions could be employed to further the realism and policy relevance of this paper. While we only consider the case where the contract is offered to a farmer at a long run steady state, different starting values are likely to have an impact. One extension would be to evaluate the effects of different starting carbon values on the optimal length of the contract chosen. Another extension would be to include a fixed cost borne by the farmer for changing land use. Then instead of assuming a return to pre-contract land use, the farmer could endogenously implement the optimal level of technology on exiting the contract. Finally, one could also explore the effects of uncertain carbon accumulation on the optimal choice of contract length.

Appendix

The expressions for the derivatives used to define the marginal benefit and marginal cost schedules are as follows:

$$\frac{d\pi^c}{dT} = \frac{-\left(\pi_0 + kX^c H^c\right)}{1-\delta}\ln(\delta)\delta^{T+1} - \frac{kX_c\left(C_0 - H_c\right)}{1-\delta(1-a)}\ln\left(\delta(1-a)\right)\left(\delta\left(1-a\right)\right)^{T+1}$$

$$\frac{d\Phi^c}{dT} = \frac{aP\left(C_0 - H_c\right)}{1-\delta(1-a)}\ln\left(\delta(1-a)\right)\left(\delta\left(1-a\right)\right)^{T+1}$$

$$\frac{d\pi^n}{dC_T} = \frac{\delta(1-a)}{1-\delta(1-a)}kX_n$$

$$\frac{d\Phi^n}{dC_T} = \frac{-\delta(1-a)}{1-\delta(1-a)}aP$$

$$\frac{dC_T}{dT} = \left(C_0 - H_c\right)(1-a)^T \ln(1-a)$$

References

Antle, J., S. Capalbo, S. Mooney, E. Elliot and K. Paustian, "Economic Analysis of Agricultural Soil Carbon Sequestration: An Integrated Assessment Approach", *Journal of Agricultural and Resource Economics* 26 (2001): 344-67.

Antle, J. S. Capalbo, S. Mooney, E. Elliot and K. Paustian, "Spatial Heterogeneity, Contract Design, and the Efficiency of Carbon Sequestration Policies for Agriculture", *Journal of Environmental Economics and Management* 46 (2003): 231-250.

Cacho, O. J., R.L. Hean and R.M. Wise, "Carbon-Accounting Methods and Reforestation Incentives" *Australian Journal of Agricultural and Resource Economics* 47 (2003): 153-179.

Chomitz, K.M. and F. Lecocq, "Temporary Sequestration Credits: An Instrument for Carbon Bears", *World Bank Policy Research Working Paper 3181* (December 2003).

Ferraro, P.J. and R.D. Simpson, (2002), "The Cost-effectiveness of Conservation Performance Payments," *Land Economics* 78(3): 339-353.

Lecocq, F. and K. Chomitz, "Optimal Use of Carbon Sequestration in a Global Climate Change Strategy: Is there a Wooden Bridge to a Clean Energy Future?", World Bank, Development Economic Research Group (July 2001).

Lee, H.C. and B. A. McCarl, "The Dynamic Competitiveness of U.S. Agricultural and Forest Carbon Sequestration", Department of Economics, *University of Western Ontario, Working Paper* (November 2003)

OECD (1997), *The Environmental Effects of Agricultural Land Diversion Schemes,* Paris, OECD.

Pautsch, G.R., L.A. Kurkalova, B.A. Babcock and C.L. Kling, "The Efficiency of Sequestering Carbon in Agricultural Soils", *Contemporary Economic Policy* 19 (2001): 123-134.

Thomassin, P.J., "Canadian Agriculture and the Development of a Carbon Trading and Offset System", *American Journal of Agricultural Economics* 85 (2003): 1171-1177.

van Kooten, G.C., C.S. Binkley and G. Delcourt, "Effect of Carbon Taxes and Subsidies on Optimal Forest Rotation Age and Supply of Carbon Services" *American Journal of Agricultural Economics* 77 (1995): 365-374.

Wilman, E.A., and M.S. Mahendrarajah, "Carbon Offsets", *Land Economics* 78 (2002): 405-416.