ENVIRONMENTAL RESTORATION OF INVADERED ECOSYSTEMS: HOW MUCH VERSUS HOW OFTEN?

By

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Environmental Restoration of Invaded Ecosystems: How Much Versus How Often?

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Abstract

This paper derives the optimal level of restorative efforts required to restore environments degraded by invasive species invasion. Specific attention is focused upon a case when the restoration efforts face the risk of failure through relapse of the restored environment. The level of restored environment may also play a role in its future improvement or susceptibility to failure. The tradeoff between the optimal level of environmental quality and number of restorative efforts required to attain that given environmental quality is analyzed.

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Introduction

Invasive species are a noticeable source of biodiversity degradation (Glowka et al. 94). Lately, invasive species have become a subject of widespread concern due to the enormous economic and environmental damages they inflict upon society (Pimentel et al. 1999, 2000). For instance, certain invasive species such as cheat grass cause destruction of grasslands, forests and the biodiversity within by inducing frequent fires. While a number of options exist to prevent the advent of invasive species, none of them are foolproof. Once the species have invaded a given eco-system, steps could be taken to either control them in part or eradicate them. However, it is rarely economically or physically viable to eradicate them. Yet, in most cases the invaded environment could be restored to a certain extent in order that society can continue to derive economic and environmental services from it.

Recent studies on the economics of invasive species management include those by Shogren (2000), Knowler and Barbier (2000), Olson and Roy (2002), Eiswerth and Van Kooten (2002), Perrings (2003), etc. While these studies focus mostly on the optimal combination of prevention and control options, one possible option is also to take restoration measures to bring the invaded eco-system close to it’s pre-invaded state. This paper looks at the important issue of the extent of optimal restoration of an invaded environment that provides economic amenities to the society. The extent of restorative efforts is analyzed for an environment that exhibits ‘hysteresis’ in environmental quality and is faced with continuous risk of future invasions. Further, the risk of re-invasion is considered that might lead to failure of the restoration project, causing the restored environment to relapse back to its initial invaded state. In light of these limitations of the
invaded environment, the optimal extent of restoration is analyzed and policy implications are derived.

Current work on restoring invaded ecosystems has been mostly confined to the field of restoration ecology. Yet, there are significant issues of economic importance that come into play while deciding the extent of restoration. Total restoration may neither be feasible nor desirable in most cases of invaded ecosystems due to high costs involved in achieving and maintaining them. Further, restored eco-systems face the risk of falling back into their degraded states from repeated invasions. Therefore, restoration efforts that do not incorporate this possibility of failure are bound to lead to inefficient outcomes. Most restoration efforts after the initial investment require substantial subsequent efforts to constantly monitor and fight the invasives for sustained periods of time. This is an essential feature of restorative efforts that are specifically targeted against invasions. The restored environment may face continuous threats from invasion even as restoration efforts are undergoing\(^1\).

There are numerous cases of biodiversity restoration where restoration efforts need to be sustained for long periods of time and despite that chances exist of reversal of the restored ecosystem back into degraded states. One case is that of grasslands of the Great Basin region in the US, which have been invaded by an alien species of grass, *Bromus Tectorum* (cheat grass). This grass is 500 times more likely to catch fire and lead to destruction of grassland as compared to the native grass of the region (BLM, 2000). As a result grassland fires have been occurring every 3 to 5 years instead of their natural wild land fire/annual grass cycle of 60 to 100 years (Kaczmarski 2003). Another

\(^1\) For example, invasive plant species may survive through the next season through their seeds, which may be hard to eliminate.
example is that of invasion of wetlands from the Pacific coast to Saskatchewan to
Arkansas by invasive weeds (aquatic macrophytes) *Typha Spp.* (cattails) that cause
significant loss of biodiversity (Milkovic 2003). Restoration efforts include flooding,
mowing, drainage, burning, chemical and biological control. However, due to their fast
reproduction rate and colonizing skills, these species re-establish themselves in restored
ecosystems time and again.

A crucial economic issue is then over the extent of restorative efforts to be
undertaken per period when risks of failure of restoration projects are real. For instance,
invasive species that lead to frequent fires may be countered by planting other species
that compete with them and are fire resistant. However, in case of a fire break-out
species of both kinds would get eliminated, therefore, negating all the previous efforts of
restoration. Another related issue is over the level of restorative efforts when risks are
stock-dependent. In the above case, the more the species of fire-resistant kind are
planted; the lower would be the risks of failure of restorative efforts. Further, higher
stock of fire-resistant species may exhibit stock-dependent resilience, i.e. once a
threshold level of fire-resistant species has been reached, there may be a sharp decline in
the level of other restorative efforts required to preserve the level of restored
environment. Experimental work on restoration ecology has revealed that degraded eco-
systems may be resilient to restoration efforts owing to changes in landscape connectivity
and changes in native species pools from invasion by exotics (Suding et al. 2004).

Restoration and resiliency improving measures under risk have been found to be
at the center of issues that deal with invaded ecosystems in the ecology literature.
However, these issues also make the economic analysis fairly complicated, as the non-
linear attributes of the ecological processes must be included in a traditional cost-benefit approach. Currently there are no known applications of restoration risks in the economics literature on invasives species and restoration, however, there has been some work related to threshold effects, such as hysteresis, in the recent past (Maler et al. 2000) that may be similar to the approach adopted in this paper.

In this paper, a model of environmental restoration is designed that incorporates the risk and resiliency effects associated with environmental restoration. The issue of how much restoration effort to undertake is then looked at in an inter-temporal cost-benefit analysis setting. When risks of failure may be stock dependent, the question of how much restoration versus how often becomes relevant, as the costs of continual but lower restoration must be weighed against the costs of less frequent by larger restorative efforts leading to a higher environmental quality. This also determines under what circumstances a more resilient state is desirable given the higher costs associated with its attainment. Numerical simulations reinforce the analysis.

The paper first starts with a deterministic model, where restoration efforts are not faced with the threat of failure, in order to understand the role of resiliency associated with environmental restoration. The analysis delves over the existence of multiple equilibriums with respect to environmental restoration. Next, risk of failure is introduced into the model. Finally, the trade off between the level of restoration and the frequency of failure of restoration is taken up in the above setting.
Basic Model

Consider a degraded environment that could provide recreational and environmental benefits upon restoration. There may be multiple options available for its restoration; however, in order to simplify things, here we assume that it is possible to combine these options together into a single restoration variable \( l \). The environmental quality \( q \) improves due to restoration efforts net of any natural rate of decay given by \( \delta \). The amount of environmental quality lost to decay increases as the level of environmental quality improves. This assumption is made in order to make unlimited improvements in environmental quality difficult. Perhaps, a more realistic assumption would be where the environmental quality stabilizes beyond a certain level; however, incorporating such dynamics may add unnecessary complexity to the model.

\[
q = \alpha l + \eta \frac{q^a}{q^a + b} - \delta q
\]

The second term in equation (1) leads to a sharp upward jump in the environmental quality once a threshold level has been crossed. This term captures the resiliency aspect of degraded ecosystems. Conventionally, resilience has been defined in two ways in the ecology literature. First one, termed as the ‘engineering resilience’ defines it as the speed of bouncing back of any perturbed system (Pimm 1984). The other one, termed the ‘ecological resilience’, is about the amount of stress that the system can tolerate before flipping from its original state to another stable but degraded state (Holling 1995, Carpenter and Cottingham 1997). In this paper we follow the ‘ecological resilience’ definition to model the impact of restoration. Parameters \( \eta \), \( a \) and \( b \) define the rate and magnitude of this effect. This functional form is associated with the process of
hysteresis in environmental literature and is characterized by a sharp jump (but not irreversible) in the states of the ecosystem that make it costlier to revert back to. For instance Maler et al. (2000) use this formulation to study the process of eutrophication of lakes where a lake turns from a clean state into a turbid state with an increase in the Phosphorous content. However, in this paper restoration induced jump in environmental quality is defined in a positive sense, as beyond a certain threshold of environmental restoration the environment shifts into a better state and is more responsive to restoration efforts. Alternatively, this formulation mandates that a willful restorative perturbation in the environmental quality would not lead a system out of its degraded state unless some threshold is crossed².

Note that the restorative efforts do not necessarily have to add in more of the environmental stock from outside. In most cases restorative efforts are simply about removing the cause of trouble. In most cases, even the degraded environments may have a capacity to grow back to their full potential, but are overshadowed by the negative forces such as pests that cause its degradation through a complex interaction involving natural forces such as fire, droughts, floods, diseases etc. One particular example is the case of Buffel grass invasion in Queensland, Australia on the native species such as the Brigalow and Gridgee. Buffel grass pastures increase the risk of fires amongst these native species, and the more fire-infested the surrounding gets, the higher is the density of the Buffel grass over time. Thus, in a positive feedback relationship with the fire and the native species, Buffel grass has been able to wipe out a large chunk of these species over

² This way to define resiliency may be taken as a cross between the conventional definitions of resiliency and hysteresis.
time (Butler and Fairfax, 2003). Other examples of models involving resiliency in grasslands can be found in Perrings and Walker (1997, 2004).

Benefits $m(q)$ are derived per period from environmental quality\(^3\). The cost of restoration $c(l)$ is convex in restorative efforts, thus making unlimited restoration prohibitive. Let $\mu$ be the shadow price of the environmental quality and $r$ the social discount rate. Society maximizes benefits from improved environmental quality net of restorative costs:

$$\text{(2)} \quad \text{Max} \int_{0}^{\infty} \{m(q) - c(l)\} e^{-rt} dt$$

subject to the constraints posed on environmental restoration by equation (1). The current value Hamiltonian is written as:

$$\text{(3)} \quad cvh = m(q) - c(l) + \mu(\alpha d + \eta \frac{q^a}{q^a + b} - \delta q)$$

First order condition with respect to restorative efforts implies that the per unit cost of restoration must be equated to the shadow value of that marginal unit of restoration.

$$\text{(4)} \quad \frac{c'(l)}{\alpha} = \mu$$

Co-state variable $\mu$ evolves as:

$$\text{(5)} \quad \dot{\mu} = -m'(q) + (r + \delta)\mu - \mu \frac{\eta ab q^{a-1}}{(q^a + b)^2}$$

From (4) and (5), the time path of restorative efforts could be derived as:

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\(^3\) These benefits are ecological benefits that do not deplete from public consumption. Ecosystems such as grasslands, forests and fisheries are also subjected to direct harvests that lead to a reduction in the environmental stock. This has not been modeled here as the primary goal of restoration may not be immediate consumption in most cases.
In a steady state, restorative efforts and the environmental quality are held constant. From (1) and (6) we get:

\[ \alpha l + \frac{\eta q^a}{q^a + b} = \delta q \]  

\[ \frac{cm'(q)}{c'(l)} = \frac{c'(l)}{c^*(l)} (r + \delta - \frac{\eta abq^{a-1}}{(q^a + b)^2}) \]

Equations (7) and (8) define a relationship between environmental quality and restorative efforts, which could be solved to derive their steady state values. The isoclines for which the levels of restorative effort and the environmental quality are constant are represented in figure 1 below.  

Note that there exist three possible equilibriums \( L, U, \) and \( R \), the low, middle and the high environmental qualities respectively. Of the three, the low and the high equilibriums are the stable ones with the middle one being unstable. The resiliency effect is depicted by a jump in the environmental quality once the environmental quality crosses the threshold given by the crest in the \( \dot{q} = 0 \) curve. The state below this threshold is the degraded state. Also notice that the \( R \) is the resilient equilibrium as environmental quality can be reduced significantly without letting the system flip to the low quality steady state. The threshold below, which the environmental quality falls into the ‘degraded’ state, is given by the trough in the \( \dot{q} = 0 \) curve. The state above this threshold is the high-quality state or the resilient state. Also notice that the ‘high equilibrium’, which is the resilient state, may not be possible to reach from a degraded state in some

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4 The shapes of the cost and benefit curves are assumed to be non-linear and the relevant parameters are shown in the Appendix.
cases. If the benefits from environmental restoration are lower than the costs incurred, or if the discount rate is high, or if the resiliency effect is not very significant, the high equilibrium may not be desirable. Figure 2 depicts a case where the benefits of restoration exceed their costs, thus leading to the high equilibrium as the only possibility.

The effect of varying levels of discount rate is depicted in figure 3 below. As the discount rate increases, only equilibrium that is possible is the low quality one, on the other hand, with low discounting high resiliency equilibrium is the only possible equilibrium. Consequently, time preferences play an important role in deciding the level of environmental restoration.

**Restoration with Relapse**

One issue that restoration projects are faced with is the relapse of restored ecosystems into their original degraded states. This could be caused by a number of factors such as renewed infestations which could be seasonal, climate-induced or man-made. Further, once the system flips back into the degraded state, one has to start all over again as the environmental quality built up in the past is gone. Therefore, the manager is faced with the challenge of incorporating such possibilities into her optimization framework. The manager’s task is to maximize her long term value:

\[
V(q) = \max \int_0^\infty (m(q) - c(l) + pV(q))e^{-\lambda(t)\tau} dt
\]

subject to (1), where \( p \) is the constant hazard rate of invasion characterized by a Poisson process. The equation of motion of the hazard rate given by:

\[
\dot{\lambda}(t) = p
\]
In equation (9), the third term represents the expected value from the system flipping back into the original state and the manager having to start all over again. \( V(q) \) represents the value function from starting all over again from the initial level of environmental quality \( q_0 \). Equation (9) in its extended form can be re-written as:

\[
V(q_0) = \max_{0}^{\infty} (m(q) - c(l))e^{-\lambda(t)-\alpha t} dt + \int_{0}^{\infty} (pV(q_0))e^{-\lambda(t)-\alpha t} dt
\]

which can be further re-written after integrating the second integral on right hand side as:

\[
V(q_0) = \max_{0}^{\infty} (m(q) - c(l))e^{-\lambda(t)-\alpha t} dt + pV(q_0) \frac{1}{r + p}
\]

given that \( \lambda(t) = pt \), the above relation can be further simplified as:

\[
V(q_0) = \frac{r + p}{r} \max_{0}^{\infty} (m(q) - c(l))e^{-\lambda(t)-\alpha t} dt
\]

Setting up the current value Hamiltonian for the above problem, we get:

\[
\begin{align*}
(m(q) - c(l))e^{-\lambda(t)} & \frac{r + p}{r} + \xi(\alpha d + \frac{\eta q^a}{q^a + b} - \delta q) \\
\end{align*}
\]

where \( \xi \) is the shadow price of quality.

The first order condition with respect to restorative effort yields:

\[
\xi = \frac{1}{\alpha} c'(l)e^{-\lambda} \frac{r + p}{r}
\]

Let \( \xi e^\lambda = \beta \), be the adjusted shadow price of quality. The rate of evolution of the shadow price is determined by the no-arbitrage condition as:

\[
\dot{\xi} = -m'(q)e^{-\lambda} \frac{r + p}{r} + \xi(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r)
\]

Therefore, the rate of change of the adjusted shadow price \( \beta \) is given by:
In steady state, \( \dot{\beta} = 0 \), implying:

\[
\beta = \frac{m'(q)}{(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r + p)} \frac{r + p}{r}
\]

Substituting for \( \beta \) from (15) above, we get the steady state relationship between restoration efforts and environmental quality as:

\[
c'(l) = \frac{m'(q)\alpha}{(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r + p)}
\]

Notice that in the no-risk case derived before, the steady state evaluation of equation (6) would yield:

\[
c'(l) = \frac{m'(q)\alpha}{(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r)}
\]

Equation (20) is similar to equation (19) except for the extra term \( p \) in the denominator of equation (19). When the restoration efforts are faced with an ever present constant exogenous risk of invasion, the risk acts as an additional discounting term. Consequently steady state restorative efforts are lower in the case when there is a risk of relapse as compared to no-risk case.

Notice that in the above equations (19 & 20), the increment in the resiliency from a change in stock serves as an adjustment to the discount rate which is also augmented by the natural rate of decay of the environmental quality. From the way this resiliency effect has been specified in the model some interesting implications can be deduced for the
optimal restoration path. The environmental quality shows a sharp jump upwards once a certain threshold level has been reached. Due to this reason, as long as the environmental quality is lower than this threshold, the resiliency effect will not be that significant. Therefore, the discounting effect brought by a change in resiliency due to environmental stock, kicks in only beyond that threshold level of stock. As a consequence, the change in the optimal steady state level of restoration effort and the environmental quality from some external disturbance in parameters would be significantly higher if the steady state is closer to this threshold. In lay terms, the incentives for restoration efforts are higher; the closer is the system to the threshold.

**Stock Dependent Risk**

In the stock-independent risk case, the relationship between the hazard rate and environmental quality is given by:

\[ \dot{\lambda} = p(q) \]  

(21)

The value function can be specified as before as:

\[ V(q) = Max_{t=0}^{\infty} \int (m(q) - c(l) + p(q)V(q)) e^{-\lambda t} dt \]

(22)

which can be further expanded for a starting level of environmental quality as:

\[ V(q_0) = Max_{t=0}^{\infty} \int (m(q) - c(l)) e^{-\lambda t} dt + Max_{t=0}^{\infty} (p(q)V(q_0)) e^{-\lambda t} dt \]

(23)

Rewriting above we get:
subject to the equations of motion for the hazard function as given by (21) and the environmental stock as given by (1).

It is not very straightforward to analytically perform dynamic optimization on the above problem using the Pontryagin’s maximum principle; therefore, we take recourse to numerical simulations.

Figure 4 below shows the time paths of restorative efforts for two starting levels of environmental quality when there is no risk of project failure ($q_0 = 2.8$ & $7.8$). Notice that the higher quality steady state is reachable only when the starting value of environmental quality is high. This is because the hysteresis effect in environmental quality is not very significant, thus requiring higher restorative efforts in order to maintain the high steady state level of environmental quality. This, however, may not be

\[ V(q_0) = \max \frac{\int_0^\infty (m(q) - c(l))e^{-\lambda - \eta} dt}{1 - \int_0^\infty (p(q))e^{-\lambda - \eta}} \]

subject to the equations of motion for the hazard function as given by (21) and the environmental stock as given by (1).

In order to reduce the problem into a standard framework one may define two more state variables as $z_1$ and $z_2$, whose rate of change is defined as:

\[ \dot{z}_1 = (m(q) - c(l))e^{-\lambda - \eta} \quad \text{and} \quad \dot{z}_2 = p(q)e^{-\eta}e^{-\lambda - \eta} \]

Now the above problem in equation (23) reduces to:

\[ V(q_0) = \max \frac{z_1}{(1 - z_2)} \]

\[ \dot{q} = ad + \eta \frac{q''}{q'' + b} - \delta q, \quad \dot{\lambda} = p(q), \quad \dot{z}_1 = (m(q) - c(l))e^{-\lambda - \eta} \quad \text{and} \quad \dot{z}_2 = p(q)e^{-\eta}e^{-\lambda - \eta} \]

The current value Hamiltonian of the above problem that would maximize $V(q_0)$ is defined as:

\[ \frac{z_1}{(1 - z_2)} + \gamma_1 (ad + \eta \frac{q''}{q'' + b} - \delta q) + \gamma_2 p(q) + \gamma_3 (m(q) - c(l))e^{-\lambda - \eta} + \gamma_4 (m(q) - c(l))e^{-\lambda - \eta} \]

The first order conditions along with the equations of motion for the co-state variable would yield a time path for the restorative efforts and the environmental quality.
feasible when the discount rate is high or the benefits from environment do not exceed their costs in the long run.

However, with a slight increase in the hysteresis effect of $\eta = .06$ (as shown below in Figure 5) and with a lower discount rate of $r = .03$, we can see that the higher steady state equilibrium is attainable even when the starting level of environmental quality is lower ($q_0 = 4.8$).

Next we compare the time paths of restorative efforts under constant and quality-dependent risk of failure with the no risk case. In the constant risk case, the hazard rate is assumed to be 0.1, where as in the quality dependent risk case the hazard function is defined as:

$$p = p_0 (1 - \theta * q(t)),$$

where $p_0 = .01$, and $\theta = .1 & q(t) < 10$

In the above equation $p_0$ is constant component of the hazard rate that is capable of falling further with an increase in environmental quality. Figure 6 shows the time paths of restorative efforts and environmental quality under the three cases for a starting level of environmental quality of 0.8. Notice that the highest level of environmental quality is attained when there is no risk of failure. Under a constant risk of failure, the environmental quality attained is the lowest, whereas the endogenous risk case has a higher environmental quality. The effect of risk is primarily to discount the future benefits from environmental quality. However, when the risk is stock dependent, environmental quality is increased to capitalize on its risk reducing impacts.

How Much Versus How Often

When restoration projects are faced with the risk of collapsing back into a degraded state, the question of how much effort to put in becomes important. If the
ecosystem keeps collapsing into the degraded state time and again, it may take a long
time before the desirable level of environmental stock is attained. Therefore, it may
happen that systems that require a low level of restorative effort but are faced with high
risks of reversal may take a longer time to reach their steady states as compared to
systems that may require a higher level of restorative effort but are faced with a lower
level of risk. Note that the risk of project failure has been accounted for in the above
models. However, the above models do not say anything about the number of times the
project would fail before a steady state is reached. The time taken to reach the steady
state in the above formulations of the problem is the one when there are no setbacks to
the restoration project. However, the actual time taken to reach a desirable level of
environmental restoration would also depend upon the number of times the relapses
happen during restoration. This concept is explored further in the setting of the model
described above.

Let $t^*$ be the time it takes for the ecosystem to reach the steady state level of
environmental quality without collapsing when there is a constant risk of reversal to the
initial degraded level\(^6\). In presence of a constant risk of reversal, the expected time $E(t)$
taken to reach the steady state would be given by:

\[
E(t) = \int_0^t \{s + E(t)\} pe^{-ps} \, ds
\]

Notice that in the above formulation, once the system reverts back into the degraded state
it has to start all over again and therefore, would take the same amount of expected time
thereafter. $s$ is the time at which the restoration effort fails, thus sending the system

\(^6\) Analytically, in most steady state problems it may take an infinite amount of time for the system to reach
the steady state. However, for practical purposes, $t^*$ can be decided to be the time taken to reach a point
very close to the target.
back to its initial level. $s$ ranges from 0 to $t^*$ and the probability of failure is exponentially distributed with hazard rate $p$. Moreover, the system faces risks of reversal even after the steady state has been reached, however, by the optimal nature of the steady state it would mean that restorative efforts and environmental stock are optimally chosen at that level of risk. Integrating the above term we get:

$$E(t) = \left[ -spe^{-ps} - \frac{e^{-ps}}{p} + E(t)(-e^{-ps}) \right]_0^t$$

Solving the above equation one derives the expected time as:

$$E(t) = \frac{e^{pt^*}}{p} - \frac{1}{p} - t^*$$

The figure below plots the contours for $E(t^*)$ for a range of values for the hazard rate $p$ and $t^*$. Notice that the expected time it takes is much higher when either $t^*$ or $p$ are higher.

The case of stock-dependent risks is slightly complicated. Note that the time taken to reach the steady state without any interruptions is a function of the rate of discount, the marginal benefits and costs of restoration, the rates of decay of environmental stock and the resiliency parameter. For example, a high rate of discount would require a lower stock and thus would take less time to reach as compared to a case when the benefits from environmental stock are high or the costs of restoration are low. Whereas, a low rate of discount would make the resilient state more desirable thus requiring more time to traverse. Similarly, a lower level of $p_0$ (the constant component of the hazard rate) would make a higher environmental quality feasible. This is shown in figure 8 below, where maximum possible level of environmental quality falls with an
increase in $p_0$. However it can be numerically verified that the expected time to steady state $t^*$ actually is lower for the case when $p_0$ is 0.1 (420 time units) as compared to the case when $p_0$ is 0.5 (37279 time units). Using a time horizon of 250, the time to reach the steady state without relapses is 211 units for $p_0 = .1$ and 162 units for $p_0 = .5$. This is because the steady state level of environmental quality falls as $p_0$ rises. However, an increase in $p_0$ also increases the number of relapses, thus increasing the total expected time.

If the hazard rate falls quickly with an increase in the environmental stock, it would reduce the expected number of relapses over the same period, as the expected duration before for a single relapse increases. This would have an effect of reducing the expected time to reach the steady state. However, the negative effect of environmental quality hazard rate would also make it beneficial to strive for a higher environmental quality as the hazard rate comprises one of the elements of the adjusted discount rate as derived in equation (14) above. This is shown in figure 9 below where an increase in $\vartheta$, the parameter that influences the impact of stock of quality on hazard rate, leads to an increase in the maximum possible environmental stock. It can also be numerically verified that the expected time to steady state falls as $\vartheta$ increases even as the time taken to reach the steady state without relapses is higher for higher levels of $\vartheta$.

A reduced discount rate would mean that future benefits from environment get a higher weightage than before and therefore more environmental quality would be strived for. As a consequence, whether the stock dependant resiliency effect leads to higher expected time to steady state than the stock independent one would depend upon whether
the effect of reduction in discounting achieved through lower hazard rate (which leads to an increased time to steady state) dominates the effect of a reduction in the expected number of relapses through a reduced hazard rate. The net effect could go in either direction. The dilemma in this case when stock of environmental quality could have a negative influence on project completion time is obvious. On one hand it offers the incentive to attain higher stock of environmental quality, as a higher quality yields direct utility and also reduces the risk of relapse. However, on the other hand a higher quality also means that a higher restoration effort is required to reach there. If costs are convex in restoration efforts, restoration efforts may need to be stretched over a longer period of time. However, the more time that is required for reaching the steady state, the higher would be the expected number of relapses. Therefore, a trade off between how much quality to strive for and how many failures in order to reach it is highlighted in the case of stock dependent risks of restoration.

The issue of expected time to steady state is important to policy makers as one important goal of restoration projects is to bring the system back to a level at which it could be exploited for direct economic uses. In the case when consumption of environmental quality leads to a reduction in its stock, additional restorative efforts will need to be taken in order to maintain the optimal steady state level.

Conclusion

In this paper, the role of restoration measures in improving environmental quality was looked at through the application of the concept of resiliency. Optimal restoration efforts were derived when environmental quality impacts the risks of failure of the restoration
projects. It was shown that the environmental and economic parameters determine the desirability of the level of resiliency, and a highly resilient environment may not be always desirable.

The tradeoff between the extent of restoration and the number of restorations was derived. It was shown that the expected time to reach the desirable state in the event of multiple relapses is a function of both the hazard rate \((p)\) and the time taken to reach the steady state under no relapse \((t^*)\). This relationship between \(p\) and \(t^*\) is convex, implying that the expected time to steady state under the possibility of relapses could be same for high risks of collapse but lower \(t^*\) and low risks of collapse but a higher \(t^*\). Note that \(t^*\) could be low due to several factors such as the discount rate, benefits and costs restoration, etc. It also turned out that no straightforward derivation of expected time to steady state is possible when risks are stock dependent.

There exist several other challenges to restoration projects. Some even oppose the idea of human interferences in degraded environments. Holling and Meffe (1996) in an influential paper argue in favor of natural disturbances that help build the resiliency of a system rather than human interventions that shield it against them. Conflicting opinions exist towards the choice of restoration tools, with some even claiming that exotic species themselves may play beneficial roles in restoration of the environment as human interferences lead to further disturbances (Antonio and Meyerson 2002). However, when restorative options are available and their advantages are clear, it may be worthwhile to apply them, especially when the benefits from their restoration span economic and environmental goods. In case of environments invaded by alien species, the need for restoration is an urgent one, as invasive species pose serious threats of extinction of
valuable native ecosystems. It must also be kept in mind that restoration projects need to incorporate longer time horizons and utilize the resiliency effects offered by higher levels of environmental quality in order to be able to ward off current and future threats of invasion.
References


Table 1: Parameters used for Simulation

<table>
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<th>Parameters</th>
<th>$a$</th>
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<th>$\delta$</th>
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</table>

Table 2: Functional forms of Cost and Benefit Functions

- $m(q) = q^\gamma$
- $c(l) = l^n$
Figure 1: Steady State Levels of Environmental Quality and Restoration
Figure 2: A Case of Resilient Steady State (high stock benefits)
Figure 3: Restorative Effort Isoclines for various Discount Rates

- \( \dot{q} = 0 \)
- \( i = 0, r = .005 \)
- \( i = 0, r = .05 \)
- \( i = 0, r = .08 \)
Figure 4: Time paths of restorative Efforts from two Starting Levels of Environmental Quality

Note: Restorative efforts fall to zero even before they reach the steady state due a higher discount rate that reduces the time horizon for optimization.
Figure 5: Time paths of restorative Efforts from two Starting Levels of Environmental Quality When Hysteresis Effect is Substantial

Note: $\eta = .06$, $r = .03$ and $q_0 = 0.8$ & 4.8
Figure 6: Restoration and Quality Levels under no-risk, constant-risk and quality stock dependent-risk
Figure 7: Contours of Expected Time to Steady State in Presence of Risk
Figure 8: Environmental Quality Levels under $p_0$
Figure 9: Environmental Quality Levels under $\varphi$