Estimating the Variance of Food Price Inflation

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Abstract

Stochastic index theory views each commodity price change as an independent observation on the rate of inflation that can be estimated by averaging over all prices. Our methodology estimates both the overall rate of inflation and relative price changes along with standard errors.

Key Words: food prices, index numbers, inflation, standard errors of inflation

The level of food price inflation in the United States is often measured by the Consumer Price Index for Food (CPI). However, this point estimate of food cost is deficient in the sense that the variance of the estimate is unknown. Hence, social scientists and policy makers have no way of knowing how precisely inflation is measured. Nor do they have any way of knowing if the impact of inflation is the same for some demographic groups in society, such as low income households. Given a level of food price inflation for society and/or a demographic subgroup, economists would like to be able to construct a confidence interval which would contain the true but unknown rate of inflation at a given level of probability. Such an interval would allow social scientists to determine if the point estimate of inflation is significantly different from zero. For example, given an estimate of food inflation of 3.5 percent, the question arises whether or not this is statistically different from a zero rate of inflation? Confidence in a point estimate cannot be established without an estimate of its variance. In addition, a confidence interval for a point estimate can be compared to the inflation rate indicated by the CPI. If the CPI were not statistically different from the estimated level of inflation, then social scientists as well as policy makers could have confidence in the inflation rate indicated by the CPI.

Two basic approaches exist in the theory of index numbers. One approach relates index numbers to underlying utility functions (Diewert). An alternative approach is to view individual price changes as independent or stochastic observations on the underlying rate of inflation (Frisch). From the latter point of view, each independent price change is viewed as having an inflationary component and a random component. The main criticism of this approach, as expressed by Keynes, is that there is no way of accounting for changes in relative prices, and is therefore of little use to economists. However, Clements and Izan have recently improved upon the stochastic approach to inflation. Their method--which links index number theory and least squares theory--leads to the desirable result of estimating the unknown parameter of inflation and its variance as well as accounting for real relative price trends.

The purpose of this article is to apply the procedure developed by Clements and Izan and extended by Selvanathan to measure the rate of food price inflation in the United States from 1980-1988. This method allows economists to calculate both a point estimate and a variance for the rate of inflation, and assumes that the source of error is the dispersion of real relative prices from their trend rates of change. Therefore, the error of inflation is

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larger when the deviation of real prices from their trend rates of change is large. Hence, this technique provides a formal link between changes in relative prices and the measurement of inflation. To our knowledge this paper represents the first application of this technique to U.S. data.

This article is organized as follows. In section two we will present the theory and assumptions for estimating inflation based upon least squares and show how the point estimates of inflation and the trend rates in relative prices are calculated as well as their sampling variances. In addition, we will show how all expressions have clear economic interpretations. In the third section we will introduce our data and the empirical results which we estimated. The final section will contain our summary and recommendations for further research.

The Stochastic Measurement of Inflation

Let p_{it} be the price of the ith commodity ($i = 1,\ldots, n$) in period t and $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ be the price log-change. In any time period we assume that a given price change is composed of three parts: a general pervasive inflation rate common to all goods, say α_t, a relative price trend for the commodity in question, say B_i, and a stochastic error term, say ϵ_{it}. Thus, the log-change in price for good i may be written:

$$Dp_{it} = \alpha_t + B_i + \epsilon_{it}. \tag{1}$$

From equation one it is easy to show that the change in the ith relative price is:

$$Dp_{it} - \alpha_t = B_i + \epsilon_{it}. \tag{2}$$

The first term on the left side is simply the rate of change in the price of good i minus the overall rate of inflation. This is equal to the right side which is the trend in the real relative price plus an error. The error occurs if the real relative price is different from its trend rate of change. Hence, if the expected value of the error term is assumed to be zero ($E(\epsilon_{it}) = 0$) then the expectation of the right hand side of equation 2 is B_i, the trend rate of change. Clements and Izan make two important assumptions about the error term. First, they assume that the error terms are independent over time and across commodities ($E(\epsilon_{it}\epsilon_{it}) = 0$ if $t \neq t$ or $i \neq i$). Second, they assume that the variance of the error term is inversely proportional to the mean of the budget share of the commodity in question. Hence, the variability of the error term will be larger for goods with a relatively small share of the budget. For instance, we would expect the variance of the price of sugar and sweets which typically accounts for 3 percent or less of the food budget to be greater than the price variance for red meats which typically accounts for 18 percent or more of the food budget. Note also that this assumption allows the researcher to interpret the intercept as an estimator of a Divisia Index, which will be demonstrated below. Hence, we may state the variance as:

$$var\ \epsilon_{it} = \eta_t^2/\bar{w}_{it}, \tag{3}$$

where $\bar{w}_{it}$ is the average budget share of the ith good between period t and t-1, and η_t is the error term common to all commodities. It is apparent that the error term is likely to be heteroscedastic across time and commodities.

Note that equation 1 as written is not identified. This is easily seen by noting that for any good we have one normal equation but two unknowns. Hence, a restriction must be placed upon equation 1 in order to identify it. Clements and Izan suggest that an appropriate restriction would be that the sum of the average commodity budget shares multiplied by their respective relative price trends equals zero. Hence, the restriction can be written:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} \bar{w}_{it}B_i = 0. \tag{4}$$

In practice, the equations are first corrected for heteroscedasticity across commodities and then for heteroscedasticity across time. Clements and Izan suggest ignoring that the variance and budget shares are dependent upon the time subscript. This assumption greatly simplifies the calculation of the

ordinary least square estimates. The justification for this assumption is that budget shares may change slowly over time so that the average budget share over the sample period is a close approximation to the moving average budget share for any two contiguous time periods in the data set.

However, departures of the overall mean from the moving average mean will affect the efficiency of the estimated variance, and hence bias the estimated confidence intervals. Our data show that the budget share for dairy products was approximately 12.6 percent in 1980 and 12.5 percent in 1988, although it was as large as 13.1 percent in 1982. The mean value used in this study for dairy products is 12.6. Contrasted to this, the largest changes in budget shares occurred in the red meats category, where the budget share ranged from a high of 25.3 in 1980 to a low of 17.8 in 1988, with a mean value of 21.2. Again, efficiency will be sacrificed to the extent that the overall mean for a food category differs from the moving average mean between any two time periods. In general, changes in the magnitude of each budget share depends upon the price and income elasticity of the category. The assumption of a constant or slowly changing budget share is equivalent to assuming unitary or near unitary price and income elasticity. Note, also that changes in elasticities may be due to changes in tastes and preferences. However, the issue of structural change generates rich discussion in the field of economics and is far from settled (Haidacher).

Clements and Izan precede with the analysis by multiplying equation one by the square root of the average budget share of good i in order to correct for heteroscedasticity across commodities. Equation one is then equal to:

$$(\bar{w}_i)^{1/2}\, Dp_{it} = \alpha_t\, (\bar{w}_i)^{1/2} + B_i\, (\bar{w}_i)^{1/2} + \varepsilon'_{it} \qquad (5)$$

where epsilon prime is now a constant across all commodities. Least squares can be applied to equation 5 with the restriction imposed that the sum of the average budget shares multiplied by their respective price trends equals zero. When that is done the following two formulas are found for the least square coefficients for the underlying rate of inflation in time period t, and the real relative price trend over the entire sample period:

$$\alpha_t = \sum_{i=1}^{n} \bar{w}_i Dp_{it}, \qquad B_i = \frac{1}{T}\sum_{t=1}^{T} (Dp_{it} - \alpha_t) \qquad (6)$$

where T is the number of time periods in the sample.

The above two least square coefficients are first round because there is a need to correct for heteroscedasticity over time. In order to proceed, we need to derive an expression for the error term in equation 5 for each time period in order to apply weighted least squares to that equation. However, Clements and Izan again assume that the average budget share for the entire time period is approximately equal to the average budget share from any two contiguous time periods. Substituting the expression for B from equation 6 into equation 5 and solving for the error term yields the following expression:

$$\varepsilon'_{it} = (\bar{w}_i)^{1/2}\, [(Dp_{it} - \alpha_t) - \frac{1}{T}\sum_{t=1}^{T} (Dp_{it} - \alpha_t)]. \qquad (7)$$

The expression in equation 7 is equivalent to:

$$\varepsilon'_{it} = (\bar{w}_i)^{1/2}\, [(Dp_{it} - \alpha_t) - (\bar{Dp}_i - \bar{\alpha})]. \qquad (8)$$

By expanding the arguments in equation 8 over all the $i = 1...n$ commodities, the variance of ε_{it} is found to equal:

$$\Theta_t^2 = (\varepsilon'_{it})^2 = \sum_{i=1}^{n} \bar{w}_i (Dp_{it} - \alpha_t)^2 + \sum_{i=1}^{n} \bar{w}_i (\bar{Dp}_i - -2\sum_{i=1}^{n} \bar{w}_i (Dp_{it} - \alpha_t)(\bar{Dp}_i - \bar{\alpha}). \qquad (9)$$

Since the expression in equation 9 is a consistent estimator of the variance of equation 5, we can divide equation 5 by the square root of equation 9

and apply least squares. Hence, dividing equation 5 by Θ_t Clements and Izan get the expression:

$$(\bar{w}_i)^{1/2} Dp_{it}/\theta_t = \alpha_t(\bar{w}_i)^{1/2}/\theta_t + B_i(\bar{w}_i)^{1/2}/\theta_t + (\bar{w}_i)^{1/2}\varepsilon_{it}/\theta_t. \quad (10)$$

By again applying least squares with the appropriate constraint imposed, the following final parameter formulas are derived:

$$\alpha_t = \sum_{i=1}^{n} \bar{W}_i Dp_{it}, \qquad B_i = \sum_{t=1}^{T} \phi_t(Dp_{it} - \alpha_t), \quad (11)$$

where

$$\phi_t = (1/\Theta_t^2)/\sum_{t=1}^{T}(1/\Theta_t^2). \quad (12)$$

There are several points to note about the least square formulas that have been derived. First, the expression for the underlying rate of inflation, α_t, from both equations 5 and 11 is a Divisia index with the average budget share from the entire time period of the data set substituted for the average budget share for any two contiguous time periods. As noted above, a key assumption in making this substitution is that budget shares change slowly over time, and that the average over the entire sample period is approximately equal to the average from any two contiguous time periods. This assumption also facilitates the estimation of the ordinary least square estimates, but creates problems of efficiency when the overall mean diverges from the moving average mean. Second, the first round estimator of the parameter for the relative price trend variable, B, is the simple average of the change in this variable over the entire data set in equation 5, whereas in the final derivation, equation 11, this variable is a weighted average. The weights for this price trend variable, as seen in equation 12, are inversely proportional to the variance at each time period. Hence, the largest weight is placed upon those observations with the least error, i.e., when the observed relative prices are closest to their trend values (see equations 2 and 8). Note, that like other price indexes, the stochastic approach cannot account for a change in quality other than reflecting the price change in the relative trend or in the inflation rate.

Having derived the expressions for the least square coefficients, Clements and Izan present, with proof, the formulas for the sampling variances. These are:

$$var\ \alpha_t = \Theta_t^2/(n-1),$$
$$var\ B_i = [1/(n-1)\sum_{t=1}^{T}(1/\Theta_t^2)]\ (1/\bar{w}_i - 1). \quad (13)$$

It is easily seen in equation 13 that the variance for the underlying rate of inflation, α_t, will increase as real relative prices deviate from their trend rates of change. In turn, the variance of the real relative price trend depends upon the deviation of all relative prices from their trend rates of change, which is a constant for all commodities over the sample period, and more importantly, the size of the budget share for the good in question which is expected to vary from one item in the budget to another. The final formula that can be derived is for the average rate of inflation over the entire sample period. It is simply the average of the sampling variance from each time period:

$$var\ \bar{\alpha} = 1/T^2\sum_{t=1}^{T} Var\ \alpha_t \quad (14)$$

Note that the above discussion pertains to estimating the rate of inflation and its standard error using the prices of all "n" goods unconditionally. However, Selvanathan has shown that all of the unconditional results can be carried over to the conditional case with "n" replaced by n_g, where n_g is the number of goods in subgroup S_g. In the following we extend the unconditional results to the subgroup of food consumed at home.

Data and Empirical Results

Cost-of-food indexes were constructed from data taken from the Continuing Consumer Expenditure Survey (CCES) for the years 1980

through 1988. The CCES contains two components, each with its own questionnaire and sample. The first is an interview panel survey in which approximately 5,000 households are surveyed every 3 months over a 1 year period. The second is a diary survey of approximately the same sample size in which households keep an expenditure diary survey for two consecutive weeks. This latter survey obtains data on small, frequently purchased items that are normally difficult to recall, including food and beverages.

By using this survey we considered eleven food categories: cereal and bakery products, red meats, poultry, fish, eggs, dairy, fruits and vegetables, sugar and sweets, fats and oils, nonalcoholic beverages, and miscellaneous prepared foods. The individual CPI's which correspond to the above 11 food groups and which comprise the CPI for food at home were used as proxies for price. Budget shares were calculated from the survey data for several demographic groups in order to determine if food cost inflation has impacted any particular group more than others. The demographic groups which we looked at were: Total population, the population residing in the northeast, northcentral, south, and western regions, whites, blacks, and the population with the lowest income (first income quintile).

The results of our estimations of the base rate of food inflation are contained in table 1. According to the CPI, the change in the rate of food-at-home inflation ranged from a low of 1.0 percent in 1983 to a high of 7.0 percent in 1981, and averaged 3.5 percent from 1980 to 1988. This is very similar but lower than our point estimates for the total population, most of which are highly significant. Our point estimates were 1.2 and 7.3 percent for the same two years noted above. Note also that for 1983 and 1985 our point estimates are not significantly different from zero. Like the CPI, we found inflation for food-at-home to average 3.5 percent for the total population over the sample period. Using the estimated standard error, a 95 percent confidence interval for 1983 for the total population would range from -0.14 to 2.54 percent. Likewise, a 95 percent confidence interval for 1981 would range from 5.85 percent to 8.75 percent. Over the entire sample period a 95 percent confidence interval would range from 2.95 percent to 4.05 percent. Each of these intervals obviously contains the point estimate of the CPI quite comfortably. Indeed, the null hypothesis that the estimated rate of inflation is equal to the rate indicated by the CPI cannot be rejected in any year at a 5 percent level of significance. This should be a reassuring finding for all users of the CPI.

In looking at the different demographic groups and comparing them to the population as a whole no clear pattern emerges and none are statistically different from the inflation rate indicated by the CPI. Hence, it appears that the national food-at-home CPI gives a very good indication of the food inflation rate for all regions and demographic groups.

In order to put the estimated rates of change into a familiar context, the corresponding indexes with 1980 = 100 are shown in table 2. This conversion shows that those in the northeast and those with the lowest income in the population have had higher at-home food cost inflation than the rest of the population. But as noted above, these are not statistically significant changes.

Table 3 contains our estimates for the trend rates of change in real relative prices. Several, but not all, of these estimates are significantly greater or less than zero. In looking at the total population those categories which have trend rates of change which are significant at a 5 percent level of confidence are: cereals and bakery products with a trend rate of change of 1.59 percent per year, meats with a trend of -1.47 percent per year, fruits and vegetables with a trend rate of 2.14 percent per year, and beverages with a trend rate of change of -1.87 per year.

These results lend some credibility to the idea that the food sector is price volatile and should not be included in estimating the "core rate" of inflation. Only 4 categories out of eleven had any statistically significant trend rates. The prices of goods in the other 7 categories either experienced fluctuations from year to year to such an extent that no trend pattern could be statistically established or were flat over the sample period. For instance, for the population as a whole, the change in the real relative price of eggs ranged from -19.6 percent in 1985 to 10.2 percent in 1987, thereby resulting in no discernable price trend. Note, that while trend rates of price changes are presented for various

Table 1. Yearly Rates of At Home Food Inflation by Demographic Group 1981-88[a]

Year	CPI	Total Population	Region N.E.	N.C.	S.	W.	Race White	Black	Lowest Income
				Percent					
1981	7.00	7.30	7.30	7.29	7.31	7.47	7.40	7.10	7.38
		(0.63	(0.64)	(0.63)	(0.64)	(0.61)	(0.63)	(0.66)	(0.63)
1982	3.50	3.40	3.80	3.43	3.36	3.37	3.40	3.30	3.64
		(0.61)	(0.61)	(0.60)	(0.62)	(0.60)	(0.61)	(0.62)	(0.65)
1983	1.00	1.20	1.20	1.17	1.18	1.23	1.20	1.10	1.20
		(0.58)	(0.62)	(0.57)	(0.59)	(0.59)	(0.58)	(0.60)	(0.66)
1984	3.70	3.90	4.20	3.72	3.86	3.92	3.80	4.00	4.00
		(0.85)	(0.95)	(0.82)	(0.86)	(0.84)	(0.86)	(1.00)	(0.88)
1985	1.50	1.40	1.40	1.44	1.40	1.51	1.50	1.20	1.36
		(0.82)	(0.83)	(0.78)	(0.84)	(0.82)	(0.94)	(0.90)	(0.92)
1986	2.80	2.80	2.80	2.71	2.82	2.72	2.70	3.00	2.78
		(0.89)	(0.92)	(0.87)	(0.90)	(0.91)	(0.89)	(0.93)	(0.92)
1987	4.20	4.20	4.20	4.17	4.14	4.18	4.10	4.30	4.08
		(1.02)	(0.99)	(1.02)	(1.02)	(1.01)	(1.01)	(1.07)	(1.04)
1988	4.10	4.00	4.10	3.89	3.95	4.02	4.00	4.00	4.04
		(0.41)	(0.43)	(0.40)	(0.43)	(0.40)	(0.41)	(0.40)	(0.42)
Average Rate									
	3.48	3.50	3.60	3.50	3.50	3.60	3.50	3.50	3.60
		(0.24)	(0.24)	(0.26)	(0.26)	(0.26)	(0.30)	(0.30)	(0.30)

[a]Figures in parentheses are standard errors

Table 2. Implied Cost of Food Indexes

Year	CPI	Total Population	Region N.E.	N.C.	S.	W.	Race White	Black	Lowest Income
1980	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1981	107.2	107.6	107.6	107.6	107.6	107.8	107.7	107.4	107.7
1982	111.0	111.3	111.8	111.3	111.3	111.4	111.4	111.0	111.7
1983	112.1	112.6	113.1	112.6	112.6	112.8	112.7	112.2	113.0
1984	116.3	117.1	118.0	116.9	117.0	117.3	117.1	116.8	117.6
1985	118.0	118.8	119.6	118.6	118.7	119.1	118.9	118.2	119.2
1986	121.4	122.1	123.0	121.8	122.1	122.4	122.1	121.8	122.6
1987	126.6	127.4	128.3	127.0	127.2	127.6	127.4	127.1	127.7
1988	131.9	132.6	133.7	132.1	132.3	132.9	132.4	132.3	132.9

Table 3. Trends in Real Relative Food Prices for Food at Home, 1981-88[a]

Group	Cereal-Bakery	Meat	Pol.	Fish	Eggs	Dairy	Fruits-Vegs.	Sugar-Sweets	Fats-Oils	Bev	Misc.
Total	1.59	-1.47	0.03	1.48	-2.05	-1.22	2.14	-0.80	-0.46	-1.87	0.78
Pop	(0.58)	(0.44)	(1.08)	(1.24)	(1.74)	(0.60)	(0.51)	(1.17)	(1.26)	(0.72)	(0.66)
N.E.	1.45	-1.68	0.24	1.50	-2.38	-1.31	2.08	-0.90	-0.59	-2.05	0.60
	(0.57)	(0.44)	(1.00)	(1.12)	(1.74)	(0.61)	(0.50)	(1.22)	(1.28)	(0.75)	(0.72)
N.C.	1.65	-1.41	0.05	1.51	-2.14	-1.17	2.21	-0.78	-0.43	-1.81	0.84
	(0.56)	(0.42)	(1.16)	(1.40)	(1.81)	(0.60)	(0.52)	(1.13)	(1.23)	(0.70)	(0.62)
S.	1.61	-1.45	0.00	1.52	-2.05	-1.21	2.16	-0.79	-0.45	-1.84	0.80
	(0.60)	(0.45)	(1.07)	(1.24)	(1.72)	(0.62)	(0.53)	(1.29)	(1.28)	(0.70)	(0.66)
W.	1.56	-1.50	-0.04	1.44	-2.14	-1.26	2.16	-0.85	-0.49	-2.00	0.75
	(0.58)	(0.46)	(1.01)	(1.21)	(1.72)	(0.58)	(0.49)	(1.16)	(1.25)	(0.72)	(0.62)
Wh.	1.58	-1.49	-0.02	1.49	-2.16	-1.22	2.13	-0.81	-0.49	-1.88	0.78
	(0.62)	(0.48)	(1.20)	(1.40)	(1.93)	(0.63)	(0.55)	(1.25)	(1.33)	(0.77)	(0.70)
Blk	1.66	-1.48	0.19	1.55	-1.82	-1.15	2.15	-0.70	-0.34	-1.86	0.81
	(0.64)	(0.43)	(0.94)	(1.16)	(1.66)	(0.75)	(0.55)	(1.32)	(1.41)	(0.85)	(0.83)
Low	1.54	-1.53	0.05	1.34	-1.93	-1.26	2.12	-0.83	-0.48	-1.98	0.72
Inc.	(0.63)	(0.51)	(1.12)	(1.45)	(1.74)	(0.67)	(0.56)	(1.32)	(1.38)	(0.81)	(0.77)

[a]Figures in parentheses are standard errors

demographic groups, none of these would be statistically different from those for the total population since the underlying rates of inflation for the demographic groups are not statistically different from those of the total population.

Summary and Recommendations

This article has applied a new technique to U.S. food expenditure data in order to estimate the rate of food cost inflation and the real relative price trends in food commodities, along with their respective standard errors. Results indicate that food cost inflation over the period of 1980-1988 was adequately reflected in the CPI for food-at-home. In addition, no statistically significant difference was found between regions, race, or the lowest income group when compared to the inflation rate indicated by the CPI. Four out of eleven commodity groups were found to have real relative price trends which were significantly different from zero. These included cereal and bakery products, meats, fruits and vegetables, and nonalcoholic beverages. No statistically significant difference was found to exist between the regions, races, or the lowest income group.

For the population as a whole the trend rate of cereal and bakery products increased an average of 1.59 percent per year while fruits and vegetables increased at an average rate of 2.14 percent per year. Meats, dairy, and nonalcoholic beverages were found to decline by 1.47, 1.22, and 1.87 percent per year, respectively.

The technique developed by Clements and Izan and presented in this paper can be applied to a wide variety of problems. We have applied the technique only to food items, but one could just as easily look at inflation over all items. Likewise, while we have applied the technique to price indexes it could be applied to quantity indexes or used in the measurement of real income across demographic groups.

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