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The Supply of Storage under Heterogeneous Expectations

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ABSTRACT

Expected prices for storable commodities often lie below spot prices plus interest and marginal storage charges. Recently this gap has been explained as the value of a call option held by a representative storer whenever a positive probability exists that stocks could dwindle to zero. However, the probability of an aggregate stock-out is effectively zero in most markets most of the time. This paper presents an alternative model that explains the gap as an equilibrium between fundamentals traders and noise traders. Applications of the model suggest that rational agents make up 84 percent of the U.S. copper market, and more than 95 percent of the corn and wheat markets.

Key Words: *storage, heterogeneous expectations, noise traders.*

Empirical evidence has been unable to confirm that all economic agents act in accordance with the rational expectations hypothesis. (See Irwin and Thraen for a review.) Difficulties in forming expectations arise when forecasters attempt to discriminate between fundamental market information and random uninformative noise. The human capital needed to do so is costly, and many simple approximations exist as cheap alternatives. For many agents quasi-rational (time series) expectations may be the cost-effective choice.

The noise trader literature (e.g. Black; De Long, Shleifer, Summers, and Waldman; Cutler, Poterba, and Summers) stipulates that markets can deviate from their rationally expected equilibrium for extensive periods due to the existence of quasi-rational agents, noise traders.[1] Also known as *chartists,* noise traders may be responsible for speculative bubbles, as demonstrated by Frankel and Froot (1986, 1990). They are also growing in number, in the form of commodity pool managers using positive feedback trading strategies (Sanders, Irwin, and Leuthold). If these traders have sufficient capital reserves, they may not be driven out of the market in the short run. The character of market equilibrium depends on the number of quasi-rational agents and whether fully rational agents adjust their behavior because quasi-rational agents exist.

This paper presents a theory of the supply of storage for a commodity that is traded by rational and quasi-rational agents. In the heterogeneous expectations context, rational traders must account for the equilibrium effect of noise traders on the market. The result is rational expectations that vary positively with aggregate stocks, in accord with established observation. Empirical estimation using two hundred monthly observations in the U.S. copper market validates the theory. The copper market is a useful example for testing the model because its supply of storage curve is highly variable (e.g., Thurman, Brennan (1991), Pindyck) and because good monthly stock-holding data are available. The model is

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[1] The terms "noise" and "quasi-rational" will be used interchangeably throughout when applied to trading behavior.

then applied to the market for two agricultural commodities—corn and wheat.

The model is presented as an alternative to the Kaldor-Working convenience yield hypothesis and more recent theories of storage and price dynamics. Its purpose is not to debunk those theories but to analyze heterogeneity as a source of price movement. The model is intentionally simple and restrictive to focus all the more clearly on the topic at hand.

Noise Traders and Heterogeneous Expectations

The heterogeneous storage model is best developed in contrast to the representative agent rational expectations storage model (Samuelson, Williams and Wright, and Deaton and Laroque (1992, 1996)). If all agents are fully informed, fully rational, risk neutral, and identical, then no speculative trades will ever be executed because all agents will value commodities equally. Expected price will always exceed current price by the full costs of carrying stocks (including marginal physical storage costs and interest) whenever stocks are held.

The representative agent storage model usually assumes full rationality in both spot and futures markets. Tomek and Gray and Zulauf *et al.* have provided empirical evidence that futures prices cannot be distinguished empirically from rational expectations of future spot prices. Therefore, futures prices may often be treated as rationally expected prices.

However, if some agents form quasi-rational expectations, the market's dynamics change. Quasi-rational expectations have been studied by Nerlove; Nerlove, Grether, and Carvalho; Frechette; and Chavas. These expectations have been used in econometric analysis for many years, most recently by Foster and Mwanaumo; Thijssen; White and Shideed; and Winter-Nelson.

In general, quasi-rational expectations are simple time series approximations to fully rational expectations. They are approximations in the following sense. When agents make decisions based on forecasts, they prefer unbiased forecasts; however, they are unwilling to incur the costs of generating them (assuming it is possible to do so). Instead, they use a simpler, cheaper alternative with the hope that it will not be too biased to help in the decision-making process. The unbiased forecast is the rational expectation, and the simpler, cheaper one is usually a quasi-rational expectation. When decision-makers approximate the rational expectation with a quasi-rational one, they are expecting the error to be small enough that they are unwilling to pay for a better approximation.

Conventional wisdom says that quasi-rational traders will lose money on average and exit from the market in the long run. However, in the short run these traders are known to exist in great numbers. Their pockets are deep enough and their numbers are replenished quickly enough that they are not driven out of the market, even in the long run. For example, Chavas estimated an aggregate price expectation function using a weighted average of different expectations. He estimated the proportion of quasi-rational producers in the U.S. pork industry to have averaged approximately 73 percent over the 1960–96 period. The staying power of quasi-rational agents in commodity markets means that a new model of price formation must be considered and must allow for heterogeneous expectations.

The inclusion of quasi-rational agents generates an important logical inconsistency that Chavas did not address. Assume one group of traders forms fully rational expectations and a second group forms quasi-rational expectations. Let the spot price at time t be denoted p_t, and let the rational (mathematical) expectation of p_t formed at time $t - 1$ be denoted EP_t. Let the quasi-rational expectation of p_t (formed at time $t - 1$) be denoted p_t^e. Trade occurs between the groups if $EP_t \neq p_t^e$. For the purposes of illustration, let r_{t-1} be the interest rate and K be marginal physical storage costs, and assume $EP_t < (1 + r_{t-1})p_{t-1} + K < p_t^e$.

In this case, the rational group wants to sell the commodity, which looks expensive to them. They sell the commodity to anyone who will buy, until price falls to $(EP_t - K)/(1 + r_{t-1})$. Simultaneously, the quasi-rational group wants to buy the commodity, which looks

cheap to them. They buy from anyone who will sell, until $p_{t-1} = (p_t^e - K)/(1 + r_{t-1})$. A logical inconsistency results because there is no limit to the size of the net position (stocks, plus long futures, minus short futures) taken by each trader, and no equilibrium exists.

The logical inconsistency can be eliminated by dropping the assumption of risk neutrality. As Black discussed, noise trading itself adds a source of risk to the market and all market participants must contend with it. Traders may be risk neutral toward demand shocks and other fundamental risks because they can be hedged in a straightforward manner. However, they are most probably not risk neutral toward the risks associated with noise trading. Therefore, the size of short or long positions is limited by the differential between EP_t and p_t^e, the risk preferences of traders, and the nature of noise trader risk.

Consider two representative groups of traders: one forms expectations rationally and one quasi-rationally. Assume that futures markets are efficient in the sense that stocks and long futures generate equal stochastic returns and that, therefore, stocks and futures are perfect substitutes. Pure hedgers are not considered because their net exposure to the market is minimal by definition. Speculators invest in stocks and futures based on the rate of return they expect to receive. Expected profits induce them to store the commodity and/or buy futures, but their speculative position is constrained by risk aversion.

Assume further that quasi-rational agents make up a constant fraction, a, of market participants and that rational agents make up the remaining fraction, $1 - a$, with $0 < a < 1$. If the total number of traders is N, then there are aN quasi-rational traders. Let the stocks held by quasi-rational agents be denoted s_{t-1}^{qr}, and let the size of their net long position in the futures market be denoted L_{t-1}^{qr}. By definition, each quasi-rational trader holds $(s_{t-1}^{qr} + L_{t-1}^{qr})/aN$ units of stocks and long futures, which defines his or her net long position overall in the market.

In equilibrium this quantity depends on the expected rate of return. Let K be constant marginal storage costs and r_{t-1} be the discount rate. The quasi-rational expected rate of return is then $(p_t^e - K)/(1 + r_{t-1})p_{t-1}$. As developed by Lintner (1969), let $q(\cdot)$ be a speculative demand function relating individual speculative holdings to the expected rate of return, and let $Q(x) = q(x)N$. The form of $q(\cdot)$ is based on risk preferences, and if traders are risk averse then its properties can be summarized as $q(1) = 0$; $q'(x) > 0$; and $q''(x) < 0$. In equilibrium $q[(p_t^e - K)/(1 + r_{t-1})p_{t-1}]$ units are held by each quasi-rational investor. Thus,

$$(1) \quad s_{t-1}^{qr} + L_{t-1}^{qr} = aQ[(p_t^e - K)/(1 + r_{t-1})p_{t-1}].$$

Similarly, the net position held by rational agents, $s_{t-1}^{r} + L_{t-1}^{r}$, depends on the same speculative demand function, $Q(\cdot)$, applied to the rationally expected rate of return:

$$(2) \quad s_{t-1}^{r} + L_{t-1}^{r} = (1 - a)Q[(EP_t - K)/(1 + r_{t-1})p_{t-1}].$$

That is, rational optimizing agents choose to hold stocks, long futures, and/or short futures that add up to the expression on the right-hand side of equation (2).

In equilibrium the total supply of stocks, s_{t-1}, is equal to the sum over the two sources of storage demand, $s_{t-1} = s_{t-1}^{qr} + s_{t-1}^{r}$. Every futures contract has long and short sides, and opposing sides net out to zero by definition $(L_{t-1}^{qr} + L_{t-1}^{r} = 0)$. Other ways to hedge (aside from futures) include options, contracts, indexing, etc. These methods are not considered in the paper because they do not serve to elucidate the point. They can all be combined and they all zero out as futures do.

Therefore combining equations (1) and (2):

$$(3) \quad (s_{t-1}^{qr} + L_{t-1}^{qr}) + (s_{t-1}^{r} + L_{t-1}^{r}) = (s_{t-1}^{qr} + s_{t-1}^{r}) + (L_{t-1}^{qr} + L_{t-1}^{r}) = s_{t-1}$$

yields an expression for the equilibrium level of aggregate stocks:

$$(4) \quad s_{t-1} = aQ[(p_t^e - K)/(1 + r_{t-1})p_{t-1}] + (1 - a)Q[(EP_t - K)/(1 + r_{t-1})p_{t-1}],$$

which must be nonnegative at all times. Solving equation (4) for the rational expectation, EP_t, yields a supply of storage curve representing the intertemporal heterogeneous expectations equilibrium:

$$(5)\quad EP_t = (1 + r_{t-1})p_{t-1} \times Q^{-1}\{[s_{t-1} - aQ((p_t^e - K) \div (1 + r_{t-1})p_{t-1})]/(1 - a)\} + K.$$

Expected price covers storage costs, but the expected rate of return depends on the position held by noise traders. Notice that stocks and expected price are positively related, *ceteris paribus,* in accord with earlier models and established empirical fact (Keynes, Kaldor, Working—1948, 1949; Brennan—1958; Telser; etc.).

The complete behavioral system describing the market includes equation (5), the rational expectations assumption, and four additional equations:

Current Consumption:	$c_t = \mu(p_t, \ldots) +$ shock at time t
Current Supply:	$q_t^s = S($information available at time $t - 1) +$ shock at time t
Quasi-rational Expectations:	$p_t^e = \theta($information available at time $t - 1)$
Storage:	$s_t = s_{t-1} + q_t^s - c_t$.

The complete system can be solved and simulated using the methodology of Williams and Wright (1991), Deaton and Laroque (1992, 1996), or Miranda (1998) who consider the representative agent storage model under risk neutrality.

This supply of storage curve (5) is an equilibrium relationship among utility maximizing traders. In contrast, supply of storage curves that require the existence of convenience yield are not based formally on utility maximization. Additional explanations of supply of storage curve behavior include transformation costs (Wright and Williams; Benirschka and Binkley; Brennan, Wright, and Williams) and uncertainty (Heinkel, Howe, and Hughes; Milonas and Thomadakis; and Susmel and Thompson), but their importance in price dynamics remains under debate (Frechette and Fackler). All explanations based on economic theory must be considered and their relative merits determined. To that end, the properties of this model can now be explored.

If noise traders sell stocks, then rational traders buy them and hold a net long position, which means:

$$s_{t-1} - aQ[(p_t^e - K)/(1 + r_{t-1})p_{t-1}] > 0,$$
$$\text{so } (EP_t - K)/(1 + r_{t-1})p_{t-1} > 1,$$

from equation (5). On the other hand, if noise traders buy enough stocks, then rational traders may hold a net short position, which means:

$$s_{t-1} - aQ[(p_t^e - K)/(1 + r_{t-1})p_{t-1}] < 0,$$
$$\text{so } (EP_t - K)/(1 + r_{t-1})p_{t-1} < 1.$$

In the limiting case of $a = 0$, all agents are rational, and ρ_t is zero in equilibrium. $Q(\cdot)$ is zero if $(EP_t - K)/(1 + r_{t-1})p_{t-1} < 1$ and infinity if $(EP_t - K)/(1 + r_{t-1})p_{t-1} > 1$. Equilibrium is obtained only when $(EP_t - K)/(1 + r_{t-1})p_{t-1} = 1$.

Now assume a is between zero and one. Let the rationally expected excess rate of return be denoted $\rho_t = (EP_t - K)/(1 + r_{t-1})p_{t-1} - 1$. That is,

$$(6)\quad EP_t = (1 + \rho_t)(1 + r_{t-1})p_{t-1} + K.$$

If ρ_t is negative, then expected prices do not exhibit full carrying costs. Using equation (5) the value of ρ_t is:

$$(7)\quad \rho_t = Q^{-1}\{[s_{t-1} - aQ[(p_t^e - K)/(1 + r_{t-1})p_{t-1}]] \div (1 - a)\} - 1,$$

which measures the expected rate of return earned by rational agents above and beyond interest charges and the marginal physical cost of storage. Rational agents buy stocks long when the rate is positive and sell stocks short

when it is negative. Rational agents do not participate in the market when the rate is zero.

Now consider the effect of ρ_t on the futures price profile (i.e., the constellation of futures prices). When ρ_t is positive the futures price profile slopes up. When $\rho_t < 0$, futures prices do not cover the costs of carry. When $\rho_t < -(r_{t-1}p_{t-1} + K)/(1 + r_{t-1})p_{t-1}$, prices are expected to fall; the futures price profile slopes down, causing the empirical phenomenon known as *backwardation.*

Backwardations

A backwardation occurs when the futures price is lower than the spot price. It is an extreme example of what is more commonly observed—futures prices that fail to cover full carrying costs. The latter can best be described as a *relative backwardation.* Examples of backwardation and relative backwardation are both commonly observed in the copper market. Backwardations and relative backwardations in grain crops often occur across crop years. They also tend to occur late in the crop year when stocks have dwindled substantially from their harvest time peaks. If there is no storage across crop years, then this model is inappropriate for annual grain crop models. For semi-storable commodities a shrinkage factor can be incorporated into the discount rate, as done by Deaton and Laroque.

Economists have long debated the cause of backwardations and relative backwardations in futures and forward markets. Keynes argued that they are due to a risk premium and Kaldor believed that stocks yield a benefit to stock holders to offset expected loss. Working (1948, 1949) developed Kaldor's idea into a theory of convenience yield that has been accepted widely for the last fifty years.

Recent studies (Wright and Williams; Benirschka and Binkley; Brennan, Williams, and Wright) have suggested that backwardations are an artifact of aggregation over product space or of mismeasurement due to transportation costs. Backwardations have also been treated as call options when stock-outs are possible (Heinkel, Howe, and Hughes; Milonas and Thomadakis; and Susmel and Thompson); however, backwardations and relative backwardations are observed even when the probability of an aggregate stock-out is effectively zero. Here a new theory will be developed, based most closely on Keynes', assuming that stocks and futures are perfect investment substitutes and assuming heterogeneous expectations and risk averse preferences.

Backwardations have been linked empirically to low stock levels (Working—1948, 1949; Telser, Thurman, Pindyck, Frechette). This correlation can be verified by calculating the derivative of the expected excess rate of return (ρ_t) with respect to the stock level (s_{t-1}). Starting with equation (7), and noting that $s^r_{t-1} = s_{t-1} - s^{qr}_{t-1}$, the derivative is:

$$\frac{d\rho_t}{ds_{t-1}} = \frac{(Q^{-1})'\left(\frac{s^r_{t-1}}{1-a}\right)}{1-a} \times \left\{1 - aQ'\left[\frac{p^e_t - K}{(1+r_{t-1})p_{t-1}}\right] \times \left[\frac{dp^e_t/ds_{t-1}}{(1+r_{t-1})p_{t-1}} - \frac{(p^e_t - K)dp_{t-1}/ds_{t-1}}{(1+r_{t-1})p^2_{t-1}}\right]\right\}.$$

Let c_t be consumption and $D(c_t) = p_t$ be the inverse demand curve for the commodity. Then the derivative simplifies to:

$$\frac{d\rho_t}{ds_{t-1}} = \frac{1}{(1-a)Q'(1+\rho_t)} - \frac{aQ'\left(\frac{p^e_t - K}{(1+r_{t-1})p_{t-1}}\right)\left[p_{t-1}\cdot\frac{\partial p^e_t}{\partial p_{t-1}}\cdot\frac{\partial p_{t-1}}{\partial c_{t-1}}\cdot\frac{dc_{t-1}}{ds_{t-1}} - (p^e_t - K)\cdot\frac{\partial p_{t-1}}{\partial c_{t-1}}\cdot\frac{dc_{t-1}}{ds_{t-1}}\right]}{(1+r_{t-1})p^2_{t-1}(1-a)Q'(1+\rho_t)}$$

which equals:

$$\frac{d\rho_t}{ds_{t-1}} = \frac{1}{(1-a)Q'(1+\rho_t)} - \frac{aQ'\left(\frac{p_t^e - K}{(1+r_{t-1})p_{t-1}}\right)\left[p_{t-1}\cdot\frac{\partial p_t^e}{\partial p_{t-1}}\cdot D'(c_{t-1})\cdot(-1) - (p_t^e - K)\cdot D'(c_{t-1})\cdot(-1)\right]}{(1+r_{t-1})p_{t-1}^2(1-a)Q'(1+\rho_t)}$$

and becomes:

$$(8)\qquad \frac{d\rho_t}{ds_{t-1}} = \frac{1}{(1-a)Q'(1+\rho_t)} - \frac{\left[aQ'\left(\frac{p_t^e - K}{(1+r_{t-1})p_{t-1}}\right)D'(c_{t-1})/(1+r_{t-1})p_{t-1}^2\right]\cdot\left[p_t^e - K - p_{t-1}\frac{\partial p_t^e}{\partial p_{t-1}}\right]}{(1-a)Q'(1+\rho_t)}.$$

If the inverse demand curve slopes down $[D'(\cdot) < 0]$ and agents are risk averse $[Q'(\cdot) > 0]$, then the sign of the derivative in equation (8) is positive if and only if the numerator of the second term is less than 1. This condition will be tested empirically and validated in the empirical section below. Therefore, it can be concluded that $d\rho_t/ds_{t-1}$ is positive.[2]

The model explains why price expectations are low when stocks are low, *ceteris paribus,* and rise as stock levels rise. However, it remains unclear whether the model predicts relative backwardations. The next step is to analyze stock levels in more detail when $\rho_t < 0$.

When $\rho_t < 0$, rational agents sell stocks short. Quasi-rational agents buy stocks and borrow stocks by selling futures to maintain the equilibrium described by equation (4). Rational traders can owe stocks to quasi-rational ones in a way that is disallowed by the single representative agent model. Using equation (4), this condition can be written as follows:

$$[s_{t-1} - aQ((p_t^e - K)/(1+r_{t-1})p_{t-1})] \div (1-a) < 0, \quad \text{or}$$

$$(9)\qquad s_{t-1} < aQ\left[\frac{p_t^e - K}{(1+r_{t-1})p_{t-1}}\right].$$

If inequality (9) is rewritten as an equality, it can be solved implicitly for the critical point (s_{t-1}^*) at which rational agents take a net zero position in the market and all stocks are held by quasi-rational agents:

$$s_{t-1}^* = aQ\left[\frac{p_t^e - K}{(1+r_{t-1})p_{t-1}}\right].$$

If $s_{t-1} < s_{t-1}^*$, then rational agents sell stocks short. If $s_{t-1} > s_{t-1}^*$, then rational agents take a long position in stocks. Long and short positions are not allowed in the representative agent model.

Under condition (9), quasi-rational agents have an incentive to keep a long position in stocks because p_t^e exceeds the costs of carry. At the same time, rational agents have an incentive to keep a short position in stocks because EP_t is less than the costs of carry. The equilibrium size of the short and long positions depends on how averse agents are to the risks involved. When ρ_t is negative in equilibrium, stocks are low and EP_t (and the futures price) is below the full costs of carry, in accord with established observation. Therefore, the model explains observed relative backwardations.

The opposite case, $\rho_t > 0$, is equally common and might be called *forwardation.* Forwardation is also common in the copper market. Forwardation and relative backwardation are sometimes called "contango," to distinguish them from backwardation, but the contango distinction is not meaningful. More insight is gained by classifying backwardations and relative backwardations together. The dis-

[2] Changes in stocks depend on changes in both consumption and production. The above derivation accounts only for consumption-driven changes in stocks. The second term of equation (8) equals zero for production-drive changes. The result leaves the first term unchanged and the derivative still positive, assuming $Q'(\cdot)$ is positive.

tinction between them and forwardations is drawn at the full cost of carry.

All three are determined endogenously as a consequence of risk preferences. Risk preferences are embodied in ρ_t through the speculative demand function, $q(\cdot)$. The speculative demand function approach is fully consistent with portfolio management and diversification models, such as the Capital Asset Pricing Model (Sharpe, Lintner (1965)). According to the Capital Asset Pricing Model, the $q(\cdot)$ function may shift if additional commodities with correlated prices are included in the trader's investment portfolio. Portfolios of goods are not the focus here; rather, the risk due to noise trading in a single market provides the impetus for study. Heterogeneous expectations are explored in hopes that they will facilitate a richer understanding of commodity markets and provide a realistic explanation of futures market behavior. The model's validity can be tested by econometric estimation.

Econometric Estimation

Data

The U.S. copper market serves as a good example for testing the heterogeneous expectations storage model because of the high quality and frequency of stocks data collected for the market. Data are observed monthly and 200 observations cover January 1975 through October 1991. More recent observations were subject to revision at the time of analysis and are therefore omitted. Earlier observations are contaminated by the government buffer stock holding program that ended in 1974. The quantity of copper extracted is represented by the American Bureau of Metal Statistics (ABMS) measure of U.S. refined copper extraction, calculated in kilotons, as found in the Commodity Research Bureau's *Commodity Yearbook.* Copper stocks are represented as the sum of U.S. refined copper stocks (ABMS) and Commodity Exchange (COMEX) warehouse stocks, measured in kilotons. Copper consumption is measured as new extraction plus disappearance from inventories. The measures of extraction and inventories are used in Thurman (1988) and are also reported in *Commodity Yearbook.*

Copper prices are American Metals Markets producer prices from *Commodity Yearbook* for refined wirebar copper, delivered to U.S. locations, adjusted into 1982 cents using the producer price index (PPI) for industrial commodities, which is taken from *Survey of Current Business.* The time index, t, equals one in January 1975. Two mine strikes occurred during the sample, from July–August 1977 and from July–November 1980.

Other data include the index of industrial production from *Survey of Current Business,* copper scrap prices from the United States Bureau of Labor Statistics (#u102301), and an aluminum price series spliced together at the December 1980 observation from the United States Bureau of Labor Statistics primary aluminum series (#u10220101) and primary unalloyed ingot aluminum series (#u10220117). Two observations were linearly interpolated for the first aluminum series. The price series were normalized by the PPI for industrial commodities. The interest rate series is the annualized nominal prime rate taken from *Survey of Current Business,* adjusted to a monthly real rate by the PPI for industrial commodities.

The three most important variables are price, consumption, and stocks. Price has a sample mean of 97.43 cents and sample standard deviation of 22.72 cents. Consumption has a sample mean of 125.36 kilotons per month and a sample standard deviation of 33.06 kilotons. Stocks have a sample mean of 416.95 kilotons and a sample standard deviation of 252.98 kilotons. Futures prices are not used because monthly contracts were not instituted until the mid-1990s, making monthly analysis impossible.

Quarterly data for the corn and wheat markets from December 1935 to December 1992 included prices in dollars and stocks in thousands of bushels from the USDA and interest rates on three-month T-bills from the Federal Reserve. The PPI for all commodities was used as the deflator with a 1982 base year.

Empirical Specification

For empirical purposes assume the utility function for each representative trader is of the

negative exponential form, $U(y) = -k \exp(-\theta y)$. Let $\eta = \theta\sigma^2$, a measure of risk aversion. Then following Lintner (1969) the speculative demand is:

$$(10) \qquad Q(x) = 2(x - 1)/p_{t-1}\eta, \quad \text{and}$$

$$(11) \qquad Q^{-1}(y) = 1 + \eta p_{t-1} y/2.$$

The factor of two is needed because there are two representative traders. The expression for ρ_t simplifies from (7) to:

$$(12) \qquad \rho_t = \frac{\eta p_{t-1}}{2} \cdot \left[\frac{s_{t-1} - aQ\left(\dfrac{p_t^e - K}{(1 + r_{t-1})p_{t-1}}\right)}{1 - a} \right],$$

which simplifies to:

$$(13) \qquad \rho_t = \frac{\eta p_{t-1} s_{t-1}}{2(1-a)} - \frac{\eta p_{t-1} a}{2(1-a)} \cdot \left(\frac{p_t^e - K}{(1 + r_{t-1})p_{t-1}} - 1 \right) \times \frac{2}{\eta p_{t-1}},$$

and finally to:

$$(14) \qquad \rho_t = \frac{a}{1-a} + \frac{\eta p_{t-1} s_{t-1}}{2(1-a)} - \frac{a}{1-a}\left(\frac{p_t^e - K}{(1 + r_{t-1})p_{t-1}} \right).$$

Equation (14) shows that the slope of the supply of storage curve should depend on price (p_{t-1}), the rational share of the market ($1 - a$), and risk preferences (embodied by η).

Returns to storage also depend on the quasi-rational expectation, p_t^e. For estimation purposes, quasi-rational price expectations are assumed to depend on four lags of price as follows:

$$(15) \qquad p_t^e = \theta_0 + \theta_1 p_{t-1} + \theta_2 p_{t-2} + \theta_3 p_{t-3} + \theta_4 p_{t-4}.$$

The θ_i were estimated separately by Ordinary Least Squares.[3]

The empirical specification of the supply of storage is based on the integral form of equation (6). The integral form is the appropriate one here because copper is mined nearly continuously. A discrete difference equation might impose step function transitions improperly between levels of p_t (Day, sections 3.5 and 3.9). To calculate the integral, first put (6) into continuous form:

$$Edp = [(r + \rho + \rho r)p + K]dt.$$

Then add a stochastic term, σdB, with B representing Brownian motion:

$$dp = [(r + \rho + \rho r)p + K]dt + \sigma dB.$$

Define w as a random walk with $dw = \sigma dB$, distributed Normal(0, $\sigma^2 dt$). Finally, integrate and write in discrete terms for estimation purposes:

$$(16) \qquad p_t = A \exp[(r_{t-1} + (1 + r_{t-1})\rho_t)t] - \frac{K}{r_{t-1} + (1 + r_{t-1})\rho_t} + w_t.$$

Parameter A is an unknown constant of integration, and the small order effects of differential changes in r_{t-1} and ρ_t are assumed to be negligible. By construction, the stochastic term, w_t, is I(1) with Δw_t distributed Normal(0, σ^2). The spurious regression phenomenon can generate bias in the standard errors of parameter estimates when stochastic trends

[3] The θ_i are OLS estimates from the regression of price on a constant and four lags of price. Regression results for copper are listed with asymptotic standard errors in parentheses: $\theta_0 = 1.965$ (1.474); $\theta_1 = 0.265$ (0.073); $\theta_2 = -0.371$ (0.114); $\theta_3 = -0.014$ (0.110); $\theta_4 = 0.099$ (0.069); $R^2 = 0.929$; Durbin's $h = 0.00$. The AR(4) specification is an approximation that may not represent any particular trader's expectations. A continuum of possible expectation specifications exists, and it is assumed that it can be well-approximated by the two particular structures chosen. The results were not especially sensitive to choice of lag length, first-differencing, or various transformations. Corn and wheat were treated in the same manner as copper, using a constant and four lags of price.

Table 1. Copper Market Parameter Estimates Three Stage Least Squares 200 Observations (Jan. 1975–Oct. 1991)

Expectation	Sum of Squared Errors	R-Squared	Ljung-Box Statistic
(16) Price difference	7,237	0.127	1.09
(17) Demand	138,898	0.357	0.73

Parameter	Theoretical Value	Estimate	Asymptotic Std. Error	One-tailed P-value
a—quasi-rational share	$0 < a < 1$	0.162	0.004	0.001
η—risk parameter	$\eta > 0$	5.69×10^{-5}	0.24×10^{-5}	0.001
K—marginal storage cost	$K > 0$	-0.756×10^{-3}	1.26×10^{-3}	0.726
A—constant of integration	$A > 0$	1.75×10^{-3}	1.42×10^{-3}	0.109
μ_0—demand constant		12.53	31.83	0.347
μ_1—demand slope (p_t)	$\mu_1 < 0$	−0.655	0.290	0.013
μ_2—demand shifter (pal_t)	$\mu_2 > 0$	0.117	0.097	0.116
μ_3—demand shifter ($pscrap_t$)	$\mu_3 > 0$	0.777	0.216	0.001
μ_4—demand shifter (y_t)	$\mu_4 > 0$	0.420	0.271	0.061
μ_5—demand shifter (z_t)	$\mu_5 < 0$	−57.0	13.1	0.001
AR(1)—demand shock		0.322	0.070	0.001
AR(1)—price shock		0.295	0.072	0.001

are present, so equation (16) is estimated in first-differenced form. The resulting residuals are allowed to take on an AR(1) structure to eliminate any bias in the estimated standard errors due to the inclusion of generated regressors in the form of estimated but unknown quasi-rational expectations.

The theory is valid whether or not futures markets exist. If futures prices are unbiased predictors of future spot price, then the contemporaneous one-month ahead futures price can be substituted for the realized future spot price on the left-hand side of equation (16). If futures prices are biased, then futures prices are inappropriate.

Equation (5) expresses the rationally expected price in terms of predetermined variables: interest rate, price, stocks, and quasi-rational expectations. The empirical version of equation (5) is equation (16). The derivation of (16) assumes an additive Gaussian error structure, resulting in the rational forecast error term w_t, based on Brownian motion σdB. All the variables on the right hand side of (16) are predetermined. The error structure is derived from an explicit Brownian motion assumption rather than simply added to the end of an estimating equation. The approach is therefore more rigorous than others that ignore the theory behind the error structure.

Equation (16) is estimated simultaneously with the market demand curve to improve estimator efficiency, as done by Chavas, and to test for model misspecification. Assume the demand curve is:

$$(17) \quad c_t = \mu_0 + \mu_1 p_t + \mu_2 p_t^{al} + \mu_3 p_t^{scrap} + \mu_4 y_t + \mu_5 z_t + v_t,$$

where v_t is an AR(1) demand shock. Demand shifters include the price of aluminum (p_t^{al}), the price of scrap copper (p_t^{scrap}), the index of industrial production (y_t), and a dummy variable (z_t) equal to one during a mine strike and zero otherwise. Estimation of equations (16) and (17) was performed using three-stage least squares (Zellner and Theil) to account for the endogeneity of p_t in (17).

Empirical Results

The results of estimating equations (16) and (17) are shown in Table I. Of primary interest are the estimates of a and η. Of secondary interest are the demand curve parameters. If

these estimates lie within their theoretically valid ranges, then there is no econometric evidence to contradict the claim that heterogeneous expectations are important in the United States copper market. It may be concluded that heterogeneous expectations are a cause of relative backwardations during periods of low inventories or when expectations differ widely between fundamental traders and technical traders. By extension, it may be generalized that heterogeneous expectations cause relative backwardations in other markets as well. The empirical analysis will proceed by investigating two hypotheses about the parameters of the model.

Hypothesis 1:

Null: $a \leq 0$ or $a \geq 1$

Alternate: $0 < a < 1$.

The quasi-rational share parameter (a) is estimated at 16.2 percent. The asymptotic standard error of the estimate is 0.4 percent, and the asymptotic 95 percent confidence interval for a is (15.4%, 20.0%). The asymptotic 95 percent confidence interval for the rational share (1 − a) is (80.0%, 84.6%). Both intervals lie within the (0, 1) boundary expected for share parameters. The asymptotic one-tailed p-values for the two sub-hypotheses, a = 0 & a = 1, are both less than 0.001, so both hypotheses can be rejected at the 99-percent level. Therefore, the Null side of Hypothesis 1 is rejected at the 99-percent level of confidence, and the model is validated.

Hypothesis 2:

Null: $\eta = 0$

Alternate: $\eta > 0$.

A positive value for the risk aversion parameter (η) indicates that traders are averse to the risks generated by noise trading. A negative value indicates risk loving and invalidates the theoretical results discussed previously. The empirical estimate of η is 5.69×10^{-5} which equates to 2.85 percent per billion dollars. Its asymptotic standard error is 0.12 percent, and the asymptotic 95-percent confidence interval for the parameter is (2.61%, 3.09%). The interval lies above zero, as expected. The asymptotic one-tailed p-value for Hypothesis 2 is less than 0.001. Therefore the Null side of Hypothesis 2 is rejected at the 99-percent level of confidence.

Hypothesis 3:

Null:

$$\frac{aD'(c_{t-1})}{(1 + r_{t-1})p_{t-1}^2} \cdot \left(p_t^e - K - p_{t-1}\frac{\partial p_t^e}{\partial p_{t-1}}\right) \times Q'\left(\frac{p_t^e - K}{(1 + r_{t-1})p_{t-1}}\right) \geq 1$$

Alternate:

$$\frac{aD'(c_{t-1})}{(1 + r_{t-1})p_{t-1}^2} \cdot \left(p_t^e - K - p_{t-1}\frac{\partial p_t^e}{\partial p_{t-1}}\right) \times Q'\left(\frac{p_t^e - K}{(1 + r_{t-1})p_{t-1}}\right) < 1.$$

The null hypothesis embodies the inequality condition on equation (8). If the size of the backwardation falls as stocks rise, then $d\rho_t/ds_{t-1} > 0$ and the Null side of hypothesis 3 should be rejected. Hypothesis 3 can be tested as a cross-equation parametric restriction involving a, μ_1, and K. Writing the Null in terms of the structural parameters and rearranging yields the following hypothesis:

Hypothesis 3′:

Null: $2a[\theta_0 + (\theta_2 + \theta_3 + \theta_4)p^* - K] - \mu_1\eta(1 + r^*)(p^*)^3 \leq 0$

Alternate: $2a[\theta_0 + (\theta_2 + \theta_3 + \theta_4)p^* - K] - \mu_1\eta(1 + r^*)(p^*)^3 > 0$

where asterisks (*) represent sample means.[4] The estimated value of this nonlinear combination of parameters on the left-hand side of the Null is 154.47 with an asymptotic standard error of 63.56. The one-tailed asymptotic p-value for the Null is 0.008. Therefore the Null

[4] Note that $D'(c_{t-1}) = 1/\mu_1$, from equation (17). Parameter μ_1 is negative, so the direction of the inequality switches from "<" in Hypothesis 3 to ">" in Hypothesis 3′.

side of Hypothesis 3′ is rejected at a 99-percent level of confidence, and the model is validated.

Further validation of the model comes from inspecting the parameter estimates from the demand curve.[5] The slope (μ_1) of the inverse demand curve is significantly negative at the five-percent level. The prices of two substitute goods (aluminum and scrap copper) are positive demand shifters, as is the index of industrial production. The demand shifter parameter estimates are statistically significant, jointly, at the one-percent level with a Wald statistic of 16.78. The mine strike dummy is a demand shifter representing additional substitution away from copper during periods of market unrest; its estimate is negative and statistically significant. All are in accord with economic theory.

The parameter estimates validate the theoretical model and provide interesting insight into the United States copper market. In particular, rational market participants are shown to outnumber quasi-rational market participants by five to one. This finding is very different from that of Chavas, who estimated quasi-rational agents to outnumber rational agents in the pork market by three to one. The difference between these two findings may be in (1) the data frequency (Chavas used annual data.), (2) the sample period (Chavas used data from 1960–1996.), or (3) the markets studied.

For further comparison, the model is applied to the corn and wheat markets. These markets provide a good basis for comparison to the copper market because backwardations are common in the copper market but rare in the grain markets. If backwardations are caused by heterogeneity, then one should find more homogeneity in the grain markets.

The quasi-rational share in the corn market is estimated at 3.3 percent with an asymptotic standard error of 4.1 percent, leaving the rational share at 96.7 percent. The null hypothesis that there are no quasi-rational agents cannot be rejected by the asymptotic t-test ($t = 0.81$), and the relative number of quasi-rational agents is small. The quasi-rational share in the wheat market was 1.2 percent with an asymptotic standard error of 0.065 percent, leaving the rational share at 98.8 percent. The null hypothesis that there are no quasi-rational agents can be rejected by the asymptotic t-test ($t = 18.25$), but the relative number of quasi-rational agents is small.

The risk aversion measures in these two markets are 9.41 percent per billion dollars for the corn market with an asymptotic standard error of 3.53 percent per billion dollars and 0.046 percent per billion dollars for the wheat market with an asymptotic standard error of 0.088 percent per billion dollars. The corn measure exceeds the copper measure (2.85%), but the wheat estimate is considerably smaller.

There are several implications of these results. First, the grain markets both have low estimated shares of quasi-rational participants compared to the copper market. The model predicts that such markets should experience backwardations infrequently compared to copper, and they do.

Second, the risk-aversion level in the wheat market is low, compared to that for the copper and corn markets. In addition, the share of quasi-rational traders is estimated at only 1.2 percent. The implication is that the risk neutral representative agent model may be a good approximation for analyzing the wheat market. The corn market's share of quasi-rational traders is slightly higher than that of wheat but statistically insignificant, implying that the risk neutral representative agent model may be a good approximation for the corn market also. The implication is not as strong as it is for the wheat market. The 16-percent quasi-rational share in the copper market indicates that a risk neutral representative agent model may be a poor approximation for analyzing the copper market. All three markets are much more rational than indicated by Chavas's results for the hog market (73% quasi-rational), for which the risk-neutral representative agent model may be a very poor approximation.

[5] The estimates for marginal storage costs (K) and the constant of integration (A) have relatively large standard errors and are not statistically different from zero at the ten-percent level. In theory, both parameters should be positive.

Conclusion

This paper develops a structural model of the supply of storage under heterogeneous expectations. In the model, stocks and expected returns are positively related, in accord with empirical evidence. Stock holding during a futures backwardation is explained as follows. When stocks are low, the rationally expected excess return falls below zero. Quasi-rational agents take a long position in the market, and rational agents sell stocks short to maintain equilibrium. Rational agents cannot drive up the spot-futures basis because quasi-rational agents drive it back down below the full costs of carry to the heterogeneous expectations equilibrium.

An important result of the theoretical model is that aggregate stock levels and rational price expectations are positively related. Previous empirical work on this subject has shown that price expectations fall as stock levels decrease and rise as stock levels increase. Rational speculators, therefore, expect higher returns to storage when stock levels are high. The heterogeneous expectations model offers a new explanation for this empirical phenomenon.

Despite the empirical weakness of the fully rational expectations hypothesis in some previous studies, eighty-four percent of market participants are shown to trade rationally in the copper market. Sixteen percent trade quasi-rationally. Quasi-rational traders are not driven out of the market fast enough to disappear entirely, and their behavior influences the actions of rational traders. Backwardations arise from the resulting market equilibrium.

The corn and wheat markets provide further evidence in favor of the rational representative agent approach. Estimates indicate that only 3.3 percent of corn market participants and 1.2 percent of wheat market participants are quasi-rational, leaving the vast majority as rational in both markets. The low share of quasi-rational participants in these markets corresponds to the low frequency of backwardations.

This paper considers a partial equilibrium model with only one stochastic asset price. If market participants invest in multiple related markets with correlated stochastic prices that are changing continuously, then a significant portfolio diversification element may be evident in market behavior. More theoretical work is needed to address the issue of portfolio diversification within the framework of heterogeneous expectations.

Different expectation formation processes often imply very different market behaviors. Changes in the share of quasi-rational traders in a market can change the way the market behaves. One might expect that cheap and easy access to information from public sources including Internet sites and extension services might contribute to a gradual increase in the rationality of a market. On the other hand, the prevalence of commodity pools and computerized trading may drive the trend toward noise trading. The distinction is an important one requiring more study and prescriptive applications. Further analysis is required to determine the tangible effects of heterogeneity on market participants.

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