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Nutrition and the Economics of Swine Management

Michael A. Boland, Kenneth A. Foster, and Paul V. Preckel

ABSTRACT

Current methods of formulating animal rations lead to excess nutrient excretion which can potentially lead to excess manure nutrients and an increase in economic costs. These methods do not recognize the impact of diminishing returns. The objective is to simultaneously optimize feed ration composition and replacement. The results, when compared against results from a survey of feed companies, indicate that using a profit maximization rather than live weight growth maximization criterion targets nutrients to an animal's actual needs and, hence, fewer nutrients are excreted and higher returns for producers are obtained.

Key Words: nonlinear growth modeling, pigs, replacement, swine.

The terms *prescription feeding, eco-nutrition,* and *target formulation* have appeared frequently in the popular press and policy discussions regarding nutrient composition of swine rations (Gadd; Howie). The terms are used to describe feeding programs which avoid feeding excess nutrients to animals. In a survey of 21 feed companies and seven universities, Cromwell reported that nutrient allowances by those surveyed were higher than those recommended by the National Research Council, suggesting that excess nutrients are being used in feed rations. Howie notes that, "... data continue to pile up that indicate some nutrients, particularly protein and phosphorus, have simply been fed in too great of quantities." The results are higher producer feed costs and excess nutrients in the manure which increases soil phosphorus and nitrogen. Gadd suggests that producers can reduce feed costs by using only targeted amounts of nutrients required for growth.

Gahl, Crenshaw, and Benevenga recognized the limitations of existing feeding methods when they wrote, "The impact of diminishing returns on economic performance is an old concept but should be considered an important component of modern diet formulation ... an economic evaluation system should be developed to use curvilinear relationships to estimate the concentration and source of nutrient intake that would maximize economic returns rather than maximize animal growth."

Regarding diets, USDA (1995) reported that over 60% of producers feed two or three rations (phase feeding) to grower and finisher pigs. Information on animal growth response from different levels of nutrients and stages of

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Michael Boland is an assistant professor in the Department of Agricultural Economics at Kansas State University. Ken Foster and Paul Preckel are associate professor and professor, respectively, in the Department of Agricultural Economics at Purdue University. The authors thank Allan Schinckel and the animal and feed scientists who participated in the Purdue Swine Lean Growth Project. A previous version of this research was presented at the annual meetings of the American Agricultural Economics Association in San Antonio, Texas on July 29, 1996 (Contribution No. 98-508-J of the Kansas Agricultural Experiment Station, Manhattan, Kansas).

growth is required to determine when to switch rations in a phase feeding program. Phase feeding typically decreases the nutrient levels over time and alternately underfeeds nutrients at phase start and overfeeds nutrients at phase end. This is a step function approximation of the optimal feeding program that varies nutrient density over time.

One of the assumptions in the National Research Council's nutrient recommendations is that live and lean weight growth exhibits no diminishing marginal returns in response to nutrients such as lysine. Gahl, Crenshaw, and Benevenga reported that they observed diminishing marginal returns in live weight growth over six levels of lysine. In addition, the NCR-42 Committee on Swine Nutrition (Cromwell et al.) suggested that lean weight growth also has diminishing marginal returns in response to different levels of lysine. Agricultural economists as far back as 1924 recognized that diminishing returns existed in swine production (Spillman). Obviously, the assumption of diminishing marginal returns in lean and live weight growth has implications for the pork producer's decision to switch rations in a phase feeding program.

A study by USDA (1996) reported that producers ranked animal weight, market price, and the need for space to accommodate incoming animals as the most significant factors in marketing. USDA (1995) also reported that over 40% of pork producers used all-in, all-out production practices. These studies suggest that a significant portion of pork production is characterized by a continuous process with finished market hogs being replaced by younger hogs from the nursery unit or purchased feeder pigs. Consequently, the opportunity cost of facility space is an important factor in determining slaughter weight and feeding program.

Currently, most recommendations for switching rations are based upon the maximum biological response of the animal's live or lean weight to a nutrient. Simulation models are widely used by animal scientists to model growth processes and for helping to determine the ration composition used in swine diets. However, the simulation models discussed by Powers, and Murphy and Shurson do not include a method to handle the opportunity cost of replacement and joint optimization of feed ration composition for phase feeding. Our research addresses these limitations. The objective of this study is to estimate the timing of ration changes and their nutrient composition for phase feeding based on different numbers of phases, and to determine the optimal slaughter weight for the alternative feeding programs.

Background Economic Theory

Alternative methods to model the swine feeding problem have been demonstrated by Townsley (quadratic programming); Sonka, Heady, and Dahm (deterministic model without replacement); Crabtree (deterministic model with replacement); Glen (dynamic programming); Fawcett and Chavas, Kliebenstein, and Crenshaw (optimal control); and Burt (stochastic dynamic programming, 1965 and 1993). Because we assume a continuous operation, a deterministic swine model with instantaneous replacement with an identical, but younger, animal is used in this analysis. A mathematical representation of the pork producer's profit maximization problem without phase feeding is

$$\max_{z_j, x_i} \pi = \frac{P_{live} g(z, t) - \sum_{i=1}^{n} r_i x_i f(t)}{t}$$

subject to

$$z_j = \sum_{i=1}^{n} x_i h_{ji} \qquad \forall j, \qquad j = 1, \ldots, m.$$

$$x_i \geq 0 \qquad \forall i$$

$$z_j \geq 0 \qquad \forall j$$

where π denotes returns per unit of time, P_{live} denotes the price per pound of live weight, z_j denotes the nutrient levels (j = protein, amino acids, etc.), $f(t)$ denotes the cumulative feed intake in pounds of feed, r_i is the ith price for the ith ingredient (i = corn, soybean meal, etc.), x_i denotes the use of ingredient i, and h_{ji}

is the proportion of the *j*th nutrient in a unit of the *i*th ingredient. The first constraint states that the quantity of nutrient *j* in a unit of feed is equal to the sum of the nutrient contributions made by each ingredient. Each of the ingredients potentially could contain any combination or all combinations of the nutrients. For example, corn contains methionine, lysine, and other nutrients while synthetic lysine contains lysine but not methionine.

Optimizing the Lagrangian form for this problem (dropping the non-negativity constraints for ease of notation) with respect to ingredient *a* determines that profit maximization occurs when the marginal feed ingredient cost per unit of time equals the marginal value product of that ingredient per unit of time for $x_a > 0$. A similar result can be found for x_b. When both x_a and x_b are positive, these conditions can be rearranged to get the result that the ratio of the *a* and *b* ingredient prices is equal to the ratio of marginal products of ingredients *a* and *b*. Optimizing the Lagrangian function with respect to z_j shows that the value of marginal nutrient product per unit of time is equal to its Lagrangian multiplier cost, and growth must have diminishing marginal returns. Finally, differentiating with respect to time yields the decision rule for replacement which is determined when marginal profit per unit of time is equal to average profit per unit of time and marginal profit must be decreasing.

Animal Growth Modeling

Two approaches, "curve fitting" and "feed modeling", are used to model animal growth in a static framework. The "curve fitting" approach is widely used by physical scientists to fit curvilinear functions to live weight. These functional forms are single argument functions of time (for an example using broilers, see Talpaz et al.). Six exponential functional forms are typically used to model live weight in the curve fitting approach: Gompertz (Grosenbaugh); Robertson (the logistic); Brody (referred to as Mitscherlich's function by economists); Bertalanffy; Parks; and Bridges et al.[1]

The "feed modeling" approach most often used by economists estimates growth as a function of the ingredients used to feed an animal as shown by Spillman and Heady et al. Several of these studies applied to swine are described in Heady and Dillon. These studies used Cobb-Douglas, quadratic, square root, and similar functional forms for estimating live weight growth (Dent; Dent and English). Corn and soybean meal, percentage of protein supplement, or energy are some of the ingredients used in these studies.

In general, there is no consensus about which approach to use in modeling live weight growth. The functions employed by the curve fitting approach have some appeal to a researcher in determining optimal timing for slaughter. For an example using swine, see Boland, Preckel, and Schinckel. Because this approach uses exponential functions, live weight growth approaches an asymptote and does not decrease over time. However, they do not incorporate nutrition information. One of the motivations for this research is to jointly optimize the ration and replacement decisions in order to determine the optimal level of nutrients in the live weight growth function. To do so, live weight must be a function of nutrients and time. A compromise between these two approaches is considered in this research. The constant *k* in the Gompertz, Robertson, Brody, and Bertalanffy functions and the parameter *m* in the Bridges et al. function are made functions of nutrients by setting

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[1] The functional forms are:

Gompertz: $g(t) = Ae^{-e^{-k(t)}}$

Robertson: $g(t) = \dfrac{A}{1 + e^{-k(t)}}$

Brody: $g(t) = A(1 - be^{-kt})$

Bertalanffy: $g(t) = A(1 - be^{-kt})^3$

Bridges et al.: $g(t) = A(1 - e^{-mt^a})$

where *t* is time; *A*, *b*, and *k* are constants.

$$k(z) = \alpha_0 + \sum_{j=1}^{m} \alpha_j z_j$$

$$m(z) = \alpha_0 + \sum_{j=1}^{m} \beta_j z_j.$$

Parks' live weight growth function is modified to include information on the nutrients in the feed equation as *f(z, t)* rather than *f(t)* or

$$(1) \qquad g(z, t) = A(1 - b \exp^{[f(z,t)]/A}).$$

Methodology

Schinckel indicates that numerous studies have reported that phase feeding speeds up live weight growth and reduces the number of days to slaughter weight in many genotypes. These studies are frequently developed using feeding trial data which is obtained by feeding animals various diets which may include nutrients that are limiting in order to measure live weight growth in response to these nutrients. In order to optimize a phase feeding program rather than a single ration, the concept of "transitional growth" is introduced. Transitional growth occurs when an animal switches to a ration with a lower nutrient density. The concept is based on the idea of inertia, and suggests that for a period of time after the ration switch animals grow at a rate which is a convex combination of the rate associated with the previous ration and the rate associated with the new diet. The period of deceleration (or acceleration) is called *transitional growth.* The length of time for acceleration or deceleration will be denoted by Δ and the rate of acceleration or deceleration will be denoted by γ. Note that transitional growth is not the same as compensatory growth because the optimal ration might not include nutrients which are limiting live weight growth.

The production function with phase feeding for a single ration is *g(z,t).* For example, one diet might have two rations, another three rations, etc. A two-ration (two phases) diet is denoted as $G_2\ (z_1, z_2, t_1, t)$ where G_2 is the function operator for the two ration production function, z_1 and z_2 are vectors of nutrient levels in the first and second rations respectively, t_1 is time to switch from a ration containing z_1 to a different ration containing z_2, and t is the optimal slaughter weight which corresponds to the end of the second ration. In order to model live weight growth, the following equations are used

$$(2) \qquad G_2(z_1, z_2, t_1, t) = \begin{cases} g(z_1, t) & t \leq t_1 \\ G_1(z_1, z_2, t_1, t) + (t - t_1)\left(\gamma \dfrac{\partial g(z_2, t_1)}{\partial t} + (1 - \gamma)\dfrac{\partial g(z_1, t_1)}{\partial t}\right) & t_1 < t \leq t_1 + \Delta \\ G_2(z_1, z_2, t_1, t_1 + \Delta) + g(z_2, t) - g(z_2, t_1 + \Delta) & t > t_1 + \Delta \end{cases}$$

In this specification, the marginal rate of live weight growth is modified by the rate of transitional live weight growth proportion (γ). Three cases may occur. Each possibility corresponds to one of the three equations. The first right-hand side equation in (2) is a single-ration diet appropriate for the case when the optimal weight occurs before switching rations (i.e. $t \leq t_1$). The second equation in (2) corresponds to the two-ration diet case where the slaughter age t occurs after the onset of transitional growth period but before the end. The last equation in (2) represents live weight growth in the case where slaughter occurs after the point where transitional growth has finished (i.e. $t_1 + \Delta$). The amount of growth from $t_1 + \Delta$ to t is added to the live weight growth from the previous time period. Thus, growth is modeled as a continuous but differentiable function of time. Similar notation and equations can be derived for diets with three or more rations.

Data

The data used in this paper are from the 1991 Purdue Cooperative Swine Lean Growth Trial (Thompson et al.). Seven genotypes representing 443 hogs were serially slaughtered at different weights, and completely disassembled into fat-free and dissected components. These data are separated into two groups. The first group contains the observations which are used to estimate the production functions while the second group is set aside for validation purposes. These latter data are randomly selected at different live weights for comparison against the estimated growth functions. Statistical tests for equality of mean and variance between the two groups reveal no significant differences between any variables in the genotypes.

The rate of transitional live weight growth is also required for modeling a phase feeding program and is assumed to be the same for each unit of time. The observations for live weight growth and feed intake were used to determine values for the rate and the duration of transitional growth. The duration of transitional growth was limited by the experimental design to a maximum of 35 days (i.e. all animals were switched rations at varying intervals of up to 35 days). Schinckel and Einstein plotted live weight growth for each day against the number of days to determine the duration of live weight growth whose average value was determined to be 14 days in this study. At that point, live weight appeared to grow at the same rate as pigs who had been fed the identical ration in excess of 14 days. Using this figure, Schinckel and Einstein used a subset of observations from animals whose switching points occurred near the end of the maximum 35-day duration. The difference in live weight between the subset of animals (representing a "predicted" weight) and the actual observations from those animals whose ration switched was calculated. These weight differences were regressed on the number of days (no intercept was included because both live weight curves began at the same point). The parameter on days represents the estimated rate of transitional live weight growth. Schinckel and Einstein reported that the average value for the first 14 days was .54 which was used in this analysis. The in-sample (out-of-sample) data for the single genotype used in this research consists of 112 (104) observations of barrows and 119 (108) observations of gilts (Table 1).

A 1985 to 1995 average of live weight prices are taken from Indiana and Illinois direct prices in *Livestock, Meat, and Wool Market News* (USDA). The ingredients analyzed in this research are corn, soybean meal, synthetic lysine, synthetic methionine, synthetic threonine, synthetic tryptophan, and di-calcium phosphorus. The nutrients considered are protein, lysine, methionine, threonine, tryptophan, and phosphorus. The other variable and fixed costs are adapted for a feeder pig finishing producer marketing 1,107 head per year (the weighted U.S. average number found by McBride) using Foster, Hurt, and Hale's cost estimates. Profits are expressed as a return to management and operator labor. The percentage of the *j*th nutrient in the *i*th ingredient is taken from Tables 6-2 and 6-3 in the National Research Council. This information is denoted h_{ji} in the model. The prices ($/pound) are presented in Table 2.

Econometric and Optimization Considerations

The growth data are cross-sectional, corresponding to the live weights at which the animals were slaughtered. For this data the cross-sections are defined in live weight categories corresponding to the approximate grouping of the serial slaughter. These groupings are defined as 60 to 100, 101 to 130, 131 to 175, 176 to 220, 221 to 250, and 251 to 300 pounds (i.e. six cross-sections). Bartlett's test rejects the presence of homoskedasticity among the errors.[2] To correct for heteroskedasticity, the live weight data are transformed by dividing each observation by the reciprocal of the variance for each cross-section. Due to its use in prior swine research (Dent; Dent and English; Heady, Sonka, and Dahm; Glen; Chavas, Klie-

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[2] Due to space limitations, all test statistics for these hypotheses tests may be found in Boland (1996).

Table 1. Selected In- and Out-of-Sample Statistics, by Sex[a]

	Gilts		Barrows	
Variable	In-Sample Mean (Std)	Out-of-Sample Mean (Std)	In-Sample Mean (Std)	Out-of-Sample Mean (Std)
Live body weight, lbs	199.14	203.26	201.94	202.11
	(62.65)	(61.23)	(60.58)	(64.67)
Carcass weight, lbs	147.24	150.34	147.78	150.34
	(49.69)	(47.65)	(48.49)	(51.04)
Carcasss fat, lbs	25.91	27.34	31.27	27.34
	(12.32)	(13.06)	(15.52)	(15.75)
Lean, lbs	65.95	62.49	61.30	62.49
	(21.61)	(22.45)	(21.47)	(22.09)
Loin lean, lbs	19.32	19.43	17.81	19.43
	(6.38)	(5.61)	(5.57)	(5.51)
Ham lean, lbs	23.52	23.95	21.81	23.95
	(7.27)	(7.01)	(6.36)	(6.21)
Picnic lean, lbs	8.46	8.41	7.97	8.41
	(2.57)	(2.39)	(2.22)	(2.49)
Butt lean, lbs	10.58	10.70	9.85	10.70
	(3.44)	(3.05)	(2.97)	(3.15)
Other lean, lbs	4.07	4.13	3.86	4.13
	(2.02)	(2.25)	(2.03)	(1.96)
Third/fourth from last rib backfat	0.89	0.91	0.98	0.91
	(0.26)	(0.28)	(0.32)	(0.32)
Number of observations	119	108	112	104

[a] Source: Thompson et al.

benstein, and Crenshaw), protein is chosen as the single nutrient in this analysis.

In order to determine the appropriate production functions, non-nested tests are used to test two competing nonlinear models (production functions). Because Davidson and McKinnon's J-test may yield conflicting results (accepting or rejecting both models), Pollak and Wales' LDC test (the likelihood dominance criterion) is used to test the competing models against each other. The hypotheses test results indicate that the modified Parks equation (1) is the most appropriate model of those considered for estimating live weight growth in this data.

Table 2. Average Prices for Feed Ingredients, \$/pound[a]

Ingredient, x_i	\$/lb
Corn	.055
Soybean meal	.133
Synthetic lysine	1.230
Synthetic methionine	1.650
Synthetic threonine	1.570
Synthetic tryptophan	10.000
Di-calcium phosphorus	.180
Other feed additive prices, P_O	.048
Live weight price, P_L, \$/lb	.490
Feeder pig price, P_F, \$/head	55.000
Other production input prices, P_V	.130

[a] Source: Foster, Hurt, and Hale.

The models are estimated using weighted nonlinear least squares in a Gauss-Newton algorithm which uses a Taylor's series approximation to the normal equations. One equation for both sexes is estimated and then separate equations are estimated by sex. Chow's test for equality of regression parameters rejects the assumption that the parameters are equal in both regressions; the separate equations for barrows and gilts are used in the model.

The parameter estimates and asymptotic standard errors for the modified Parks equation are presented in Table 3. All parameter

Table 3. Parameter Estimates and Asymptotic Standard Errors for the Modified Parks Equation for Estimating Live Weight Growth

Variable	Barrows	Gilts
Asymptote, A	709.238	2992.291
Exponential coefficient, B	.941* (.046)[a]	1.158* (.075)
Protein coefficient	0.621* (.168)	1.158* (.278)
Time coefficient	2.280* (.140)	1.208* (.391)
R^2	.981	.988

* The parameter is significant at $\alpha = .05$ and the $t_{05,n}$ critical value is 1.645.

[a] The number in parentheses is the asymptotic standard error.

estimates are significant and the signs on each parameter are positive, indicating that the marginal product of live weight with respect to protein and time is positive. As Burt (1978, 1993) suggested, the second-order condition is also examined and diminishing marginal returns with respect to protein are found within the range of the experimental data.[3] The cumulative feed intake functions for each sex were previously estimated and presented in Table 2 of Boland, Preckel, and Schinckel. The constrained profit maximization problem for the feeder pig finishing producer is the following:

$$\max_{x_i, z_j, t} \pi = \frac{1}{t}((P_L + D(H_P) + L(H_P))H_P - C)$$

subject to

$$(3) \qquad H_P = G_P(z_1, \ldots, z_P, t_1, \ldots, t_{P-1}, t)$$

$$(4) \qquad C = P_F + \sum_{p=1}^{P} \sum_{i=1}^{n} x_{pi} r_i (f(t_p) - f(t_{p-1})) - P_O O f(t) - P_v \dot{H}_p$$

$$(5) \qquad \sum_{i=1}^{n} x_{pi} = 0.98 \qquad \forall\, p$$

$$(6) \qquad z_{pj} = \sum_{i=1}^{n} h_{ji} x_{pi} \qquad \forall\, j$$

$$(7) \qquad l_{pj} \leq z_{pj} \leq u_{pj} \qquad \forall\, p, j$$

$$(8) \qquad \sum_{i=1}^{n} x_{pi} e_{pi} \leq E \qquad \forall\, p$$

$$(9) \qquad x_{pi}, z_{pj}, t_p \geq 0 \qquad \forall\, i, j, p$$

$$(10) \qquad t_p \geq t_{p-1}$$

where P_L is the live weight price, $G_p(z_1, \ldots, z_p, t_1, \ldots t_{p-1}, t)$ is live weight as a function of protein and time in a phase feeding program, D and L are the discounts on live weight and discounts or premiums on percentage of lean respectively[4], P_F is the price of the feeder pig, r_i is the price of ingredient x_{pi}, P_O is the price of other feed (O), t is time, and P_V is the price of daily production costs (utilities, veterinary medicine, etc.).

The objective function states that net re-

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[3] The squared correlation between actual and predicted values is used as an objective model validation criterion. In the case of live weight, the squared correlations are all greater than .89, suggesting a high degree of accuracy for predicting in the out-of-sample data. Information from a survey of feed companies which currently provide ration recommendations to pork producers was used to determine industry averages for the composition and timing of phase feeding rations recommended by these companies. Their recommended average nutrient and switching time values are fixed and the model optimized live weight for those values. When the live weights are compared against the actual weights from the experimental data, 88% are within two days of each other for both sexes. The average number of rations recommended in a diet by survey respondents is 3.2, and consequently, t_p (p = 1, 2, 3, 4) rations are considered in this analysis.

[4] The mathematical formulation of these are discussed in Boland, Preckel, and Schinckel.

Table 4. Optimal Returns, Time, Live Weight, and Compound Levels for Gilts, by Ration and Diet

	Number of Rations in a Diet[a]									
	One Ration	Two Rations		Three Rations			Four Rations			
Item	1st[b]	1st	2nd	1st	2nd	3rd	1st	2nd	3rd	4th
Return, $/day	.136	.223		.241			.243			
Live weight, lbs	266	152	271	114	194	274	96	144	204	275
Time, days	117	54	107	34	71	105	24	51	81	104
Energy, kcals/lb	1343	1343	1321	1343	1321	1321	1343	1323	1321	1318

[a] For example, Two Rations denotes a diet that has two separate rations where the results for the first ration are reported in the 1st column and the results for the second ration are reported in the 2nd column.

[b] 1st refers to the ration fed in the first phase, 2nd is the second phase, 3rd is the third phase, and 4th is the fourth phase.

turns to management and operator labor are maximized. To simplify the notation, equations (3) and (4) are presented as constraints. Equation (3) states that live weight growth is equal to H_P while equation (4) is the sum of total costs (C). However, both constraints are substituted into the objective function when the model is solved to ensure that all explicit constraints are linear.

The constraint (5) states that the sum of the ingredients in the pth ration equals .98 (the remaining 2% is a constant for other ingredients such as vitamins and minerals) while (5) states that the sum of the amounts of nutrient j in the ingredients of the pth ration equals the proportion of the nutrient j contained in the pth ration. The next constraint (7) states that the jth nutrient usage must be within the range (l and u denote upper and lower bounds respectively) of the experimental data. Equation (8) is a constraint on energy and says that the sum across ingredient contributions of the feed energy level (e_{pt}) must be less than or equal to E (1375 kcals per pound) which is the upper limit on the range of the experimental data. Finally, the last two constraints, (9) and (10), state that the amounts of the ith ingredient and jth nutrient must be non-negative and the pth time period is positive. The optimization model is formulated in GAMS 2.25 (Brooke, Kendrick, and Meeraus) using the MINOS solver (Murtagh and Saunders).

Results

As noted previously, the two-ration diet as described by equation (2) has three possible cases. All three cases are optimized separately and the respective cases for the one-, three-, and four-ration diets are also optimized. The case with the highest returns is chosen and presented in Tables 4 and 5. For the case where only two rations are used, the optimal time for replacement ($t_2 = T$) is greater than the initial switching point plus the length of transitional growth for the gilts. This corresponds to the third equation in (2). For barrows, the optimal replacement occurs before the completion of transitional growth ($t_1 < t_2 \leq t_1 + \Delta$). Consequently, the second equation in (2) is used to approximate live weight for that case. Similar results can be noted for the third and fourth rations.

Optimal Returns, Nutrients, and Marketing Time

With regard to the optimal diets, there are substantial economic incentives for producers to feed more than one ration. The highest returns for both sexes are found for feeding diets containing four rations. For the barrows and gilts, feeding three rather than two rations results in an increase in return of 22% and 8%, respectively. These changes are smaller for feeding four rations (less than 3%). These results sug-

Table 5. Optimal Returns, Time, Live Weight, and Compound Levels for Barrows, by Ration and Diet

	Number of Rations in a Diet[a]									
	One Ration	Two Rations		Three Rations			Four Rations			
Item	1st[a]	1st	2nd	1st	2nd	3rd	1st	2nd	3rd	4th
Return, $/day	.074	.180		.222			.241			
Live weight, lbs	261	153	267	109	187	267	52	114	188	268
Time, days	105	46	102	27	60	100	3	30	64	99
Energy, kcals/lbs	1323	1343	1321	1343	1321	1322	1326	1323	1322	1318

[a] For example, Two Rations denotes a diet that has two separate rations where the results for the first ration are reported in the 1st column and the results for the second ration are reported in the 2nd column.

[b] 1st refers to the ration fed in the first phase, 2nd is the second phase, 3rd is the third phase, and 4th is the fourth phase.

gest that adding additional rations has diminishing returns to management and operator labor, and are consistent with actual practice because most producers are feeding two to three rations. For the barrows, the single ration has less lysine than for the gilts. This result is expected because gilts require more amino acids due to an increased efficiency in producing lean tissue which means an ingredient with high levels of protein, such as soybean meal, displaces corn in the ration. With respect to feeding more than a single ration, the percentage of ingredients in the optimal ration is approximately the same for both sexes, suggesting that it is more economical to adjust synthetic amino acid levels than to adjust individual crop ingredients.

The optimal diet for gilts (four rations) includes a high level of protein (20%) initially followed by a decrease in protein levels in subsequent rations (16 to 15.1 to 14.9%). However, the optimal protein level for the gilts is higher than for barrows in the second and third rations (16% and 15%, respectively).[5] This difference probably results from the greater lean growth efficiency in gilts. For the gilts, the bound on protein for the first ration in all diets is binding which suggests that increasing protein by an additional unit would have increased returns. Protein levels are not binding for any other rations in the diets.

Similarly, the optimal level of lysine in all diets is greater (1.10%) in the first ration than in the other rations for the multiple diets. This result is consistent with the greater growth efficiency of younger animals. Furthermore, it agrees with the National Research Council recommendations for feeding a high lysine diet in the early stages of growth followed by successively lower amounts of lysine. With the exception of the first ration, the lower bound on lysine is binding for the rations in all diets. This result suggests that reducing lysine requirement by an additional unit would have increased returns. Lysine and protein both exhibit diminishing marginal returns. The lower bound on the level of phosphorus is binding for all rations in the diets. The bounds on the other ingredients are not binding. For the synthetic amino acids, threonine and tryptophan are not in any of the optimal diets for either genotype.

The average net revenue curve is relatively flat near the optimal solution. These live weights are much higher than those reported by Glen (220 pounds) in 1983 and Chavas, Kliebenstein, and Crenshaw (212 pounds) in 1985. The marketing live weight in the U.S. was 255 pounds in 1994 (Foster, Hurt, and Hale). Due in part to demands for heavier primal cut weights and lower trimming costs, plants began using live-weight discounts in the

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[5] Although only a single genotype was reported here, differences in rations between genotypes were observed in Boland.

late 1980s to attract heavier animals. Such discounts were not employed by Glen or Chavas, Kliebenstein, and Crenshaw in those two previous studies. In addition, different input and output prices and live weight growth functions are used which also contributed to differences in optimal weights.[6]

Marginal Cost of Nutrients

The marginal cost of adding an additional unit of a nutrient for each ration in all diets is calculated. These marginal costs represent the additional increase in feed ingredient costs from adding an additional unit of the nutrient when evaluated at the optimal levels. For all diets, the marginal cost of adding an additional pound of protein is higher for the first ration ($0.097). The main source of protein in these rations is soybean meal (whose cost was $0.13 per pound). The marginal cost of an additional unit of protein decreased to $0.07 in the second, third, and fourth rations for all diets.

The marginal cost of an additional unit of lysine is $1.14 for the first ration and increased ($1.49 to $1.57 per unit) in succeeding rations for the other diets. Synthetic lysine and soybean meal are the main sources of lysine. The marginal cost of methionine is lower for the first ration ($1.54) and increased slightly for succeeding rations ($1.57). The marginal cost of an additional unit of phosphorus is $0.09. Because threonine and tryptophan are adequately supplied for the rations in all diets, the marginal cost of these nutrients is zero. In order for any of these ingredients to enter the optimal solution, the prices would have had to decrease (*ceteris paribus*) by $1.32 per pound (threonine) and $0.04 per pound (tryptophan).[7]

Sensitivity Analysis

A sensitivity analysis for the ingredient prices, other input prices, output (live weight) price, and the premiums and discounts for live weight and percentage of lean is conducted for a 1% change. The arc elasticities of slaughter weight, time, and net returns for a 1% change above and below the base value are presented in Table 6. The elasticities of optimal slaughter weight and marketing time with respect to the ingredient prices are relatively small for both sexes. An increase in ingredient prices results in an increase in optimal slaughter weight or marketing time. This result suggests that increased ingredient prices have a greater effect on average net revenue than on marginal net revenue. While increased ingredient (and feeder pig) prices lead to increased costs which would suggest immediate replacement, the foregone revenues from replacing that animal with a new feeder pig are greater and the producer will delay replacement to obtain those revenues and, hence, increase returns. These results are consistent with Chavas, Klieben-

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[6] The solution and results for the four-ration phase feeding program are compared to the survey recommendations. On average, the feed company recommendations for the percentage of protein in a ration is a half percentage higher for gilts than for barrows. The model's optimal solution is to feed slightly higher percentages of protein in the first and fourth ration relative to the survey recommendations. The returns to management and labor using the industry averages are smaller than the returns from the rations from the bioeconomic model. In general, the returns differ by about $1.00 (5%) per animal when comparing the rations from the bioeconomic model and the averages. It should be noted that the results of the optimization model are only valid for the genotype used to estimate the live weight and cumulative feed intake equations. Feed company recommendations are based on the results of pooling many genotypes rather than just one genotype and Cromwell's survey indicated that feed companies were recommending higher levels of nutrients. However, as Gahl, Crenshaw, and Benevenga noted, economic optimization usually yields different results than biological optimization.

[7] In general, the objective function for this problem is not concave. However, to test for concavity over the feasible region, the following heuristic test was constructed by identifying 1000 points in a 10 × 10 × 10 grid over time, protein, and lysine. Of these, 712 lay within the feasible region. In all cases, the Hessian with respect to time, protein, and lysine was found to be negative definite for the 712 points evaluated in the feasible region. This result suggests that the objective is probably strictly concave over the feasible region. If true, this implies that the optima computed and presented here are global optima. To double-check the solution for the ingredients, feed costs are minimized using the optimal levels of the nutrients. For all rations in the diets the optimal levels of ingredients are the same as those derived from the profit maximization model.

Table 6. Arc Elasticities for a One Percent Change in Input and Output Prices, by Sex

			Barrows			Gilts		
Variable	Mean	1% Change	Weight	Time	Return	Weight	Time	Return
P_{corn}[a]	.055	.0006	1.79	−.02	−1.81	2.16	.00	−.71
$P_{\text{soybean meal}}$	.13	.0013	1.79	.00	−1.55	2.16	.00	−.63
P_{lysine}	1.23	.0123	1.76	.00	−.02	2.14	.00	−.01
$P_{\text{methionine}}$	1.65	.0165	1.80	.00	−.13	2.18	.00	−.05
$P_{\text{threonine}}$	1.57	.0157	1.80	−.02	−1.13	2.18	−.01	−.42
$P_{\text{tryptophan}}$	10.00	.1000	1.80	.00	−.68	2.18	.00	−.25
P_{dical}	.18	.0018	1.80	.00	−.07	2.18	.00	−.03
P_{feeder}	55.00	.5500	1.92	.15	−4.79	2.29	.12	−2.20
P_{day}	.10	.0010	1.80	.00	−1.84	2.18	.00	−.86
P_{live}	.49	.0049	1.75	−.06	11.84	2.15	−.03	5.53
γ[b]	14.00	.490	1.50	.00	−.07	2.05	.00	−.03
Δ[c]	.55	.006	1.51	.00	−.01	2.07	.00	−.01
$D(H_p)$[d]	−.05	−.001	1.84	.00	−.17	2.22	.00	−.11
$L(H_p)$[e]	−.02	.002	1.84	.00	−5.89	2.24	.00	−2.51

[a] A "P" denotes price while the subscript denotes the ingredient, fixed cost, or variable cost from the objective function.
[b] Denotes rate of transitional growth.
[c] Denotes the duration of transitional growth.
[d] Denotes the live weight discount rate.
[e] Denotes the premium or discount on percentage of lean.

stein, and Crenshaw and Boland, Preckel, and Schinckel (i.e., the opportunity cost of replacement is greater than the marginal net revenue of replacement).

With respect to returns for barrows, the elasticities are greater than one for the price of corn, soybean meal, threonine, feeder pig price, daily production costs, output price, and the premium/discount on percentage of lean. This result implies that the percentage change in returns is greater than the percentage change in ingredient prices, suggesting that a large share of feed costs comprise total costs and low returns in hog production. In addition, the signs on all elasticities with respect to ingredient prices are negative (except on output price and the premium/discounts on percentage of lean), indicating that increasing these prices results in a decrease in returns. These same elasticities for gilts are less than one. The difference is most likely due to barrows having a higher feed intake and less leanness than the gilts. Consequently, a given level of an ingredient price leads to optimal diets that limit the performance of barrows more than that of gilts. The arc elasticity for the output price is higher than any of the other elasticities for both genotypes (5.53 for gilts to 11.84 for barrows) which is again consistent with low hog returns.

Of notable significance is the large elasticity on the premium/discount associated with percentage of lean in the carcass relative to the discount on live weight. This result suggests that producer returns are much more sensitive to changes in variables—such as backfat depth—that are correlated with leanness. Increasing (decreasing) the premiums (discounts) for carcasses with higher (lower) levels of leanness can yield greater changes in producer returns relative to changes in these premiums or discounts for delivering an animal within a certain weight range.[8] Arc elasticities were also computed for the duration and rate of transitional growth. These were relatively small for returns to management and

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[8] This has not gone unnoticed in the pork industry. Since 1988 the number of hogs sold on some form of price discrimination for leanness has increased from 12% to 36% in 1993 to almost 70% in 1995 (Jekanowski, Akridge, and Boland).

labor, optimal slaughter weight, and time, suggesting that small changes do not affect their values.

Implications

An economic evaluation system was designed that uses curvilinear relationships to estimate the optimal source of nutrient intake that maximizes economic returns rather than animal growth. Furthermore, the impact of diminishing returns has been included in the development of a modern diet formulation as advocated by Gahl, Crenshaw, and Benevenga. The results suggest that there are substantial incentives for producers to use multiple rations. Furthermore, the model found lower levels of protein in the rations than industry recommendations. This excess protein is most likely excreted as nitrogen in the manure which may be costly for many producers to manage. This research is of interest to nutritionists, veterinarians, animal scientists, and producers who are concerned with helping producers lower costs through prescription feeding. Future research should investigate the impact of excess nutrition on manure nutrient excretion by animals.

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