The Potential for High-Value Agricultural Products Under the North American Free Trade Agreement: The Case of Beef in Mexico and Canada: Comment

Jong-Ying Lee and Mark G. Brown

In his recent article, Onianwa used the absolute price version of the Rotterdam demand model to estimate Mexican and Canadian import demands for U.S. beef products. The specification of the Rotterdam model allows straightforward testing of the basic theoretical properties of demand. Based on theory, a system of demand equations should obey adding up, homogeneity, symmetry, and negativity (Theil 1971, 1975; Deaton and Muellbauer). Adding up is guaranteed or automatically satisfied in the Rotterdam model and other similar demand models like the almost ideal demand system (AIDS). Hence, adding up cannot be tested. Negativity can be checked by calculating the eigenvalues of the Rotterdam Slutsky matrix (all eigenvalues should be nonpositive). The remaining two properties, homogeneity and symmetry, can be straightforwardly tested by using the likelihood ratio test, as in the present paper, or the Wald or Lagrange multiplier tests; homogeneity can also be tested separately for each demand equation using the F-test.

Unfortunately, in Onianwa's specification of the Rotterdam model, prices were incorrectly deflated [see equation (5), p. 379, in Onianwa's article] for testing homogeneity and symmetry. Below, we will demonstrate that deflating prices by a price or price index outside the Rotterdam demand system, such as the consumer price index (CPI), results in incorrect tests of the homogeneity and symmetry hypotheses, and is unnecessary when homogeneity restrictions are imposed.

First, we show how deflation of prices and income occurs in the Rotterdam model. The relative price version of the Rotterdam model (Theil 1975, p. 27) can be specified as

\[ w_i d \log q_i = \theta_i (d \log m - \sum_k w_k d \log p_k) \]

\[ + \sum_j v_{ij} (d \log p_j - \sum_i \theta_i d \log p_i), \]

where the subscripts \( i \) and \( j \) indicate products \((i, j = 1, \ldots, n)\); \( p, q, \) and \( w \) denote price, quantity, and budget share, respectively; \( m = \sum_i p_i q_i \) (i.e., total expenditure or income); \( \theta_i = p_i (\partial q_i / \partial m) \) (marginal propensity to consume); and \( v_{ij} = (\lambda / m) p_i w_j p_j \), where \( \lambda \) is the Lagrangian multiplier, and \( w \) is the \( i,j \)th element of the inverse of the Hessian. The time subscripts are omitted for simplicity.

Note that the income and price variables in the relative price version of the Rotterdam model are deflated by the Divisia price index \( (\sum_k w_k d \log p_k) \) and the Frisch price index \( (\sum_k \theta_k d \log p_k) \), respectively. In other words, real income and price variables are used in the relative price version of the Rotterdam model. In this specification of the Rotterdam model, homogeneity is imposed and not testable, provided the data are constructed so as to satisfy the adding-up condition as is commonly practiced; in this case, real income, \( d \log m - \sum_k w_k d \log p_k \), is replaced by \( d \log q = \sum_k w_k d \log q_i \), and \( \sum_k \theta_k = 1 \). It should also be noted that Rotterdam model (1) is not...
identified unless the \( v_{ij} \)s are restricted in some manner (Theil 1971, pp. 579–80).

A version of the Rotterdam model that allows for testing homogeneity is the absolute price version of the Rotterdam model which can be derived from (1) as follows:

\[
(2) \quad w_i \frac{d \log q_i}{d \log p_i} = \theta_i \frac{d \log Q}{d \log p_i} + \sum_j \pi_{ij} \frac{d \log p_j}{d \log p_i},
\]

where \( \pi_{ij} = v_{ij} - \phi \theta_i \phi \) (the Slutsky coefficient of the Rotterdam model), and \( \phi \) is a factor of proportionality (Theil 1975, pp. 47–48). Note that adding up requires \( \sum_i \theta_i = 1 \), and \( \sum_j \pi_{ij} = 0 \); homogeneity requires \( \sum_j \pi_{ij} = 0 \); and symmetry requires \( \pi_{ij} = \pi_{ji} \).

Homogeneity is also known as absence of money illusion. That is, if all prices and income double (or increase proportionally), demand for each product is unchanged; i.e., only relative prices matter. One way to impose homogeneity in Marshallian demand specifications is to divide or deflate all prices and income by one of the products’ price—say the price for product \( n \) (i.e., \( p_i/p_n \) and \( m/p_n \)). In Hicksian or compensated demand specifications, like the Rotterdam model, we only need to deflate prices (i.e., \( p_i/p_n \)); in the log differences of the Rotterdam model, homogeneity can thus be imposed by \( d \log p_i - d \log p_n \), where \( i = 1, \ldots, n - 1 \). Of course, this way of imposing homogeneity would be equivalent to imposing \( \sum_j \pi_{ij} = 0 \).

Is it necessary to deflate prices by the CPI in the paper in question? Presumably this deflation of prices was done to make prices real. However, this way of specifying real prices is not necessary, since the homogeneity restriction \( \sum_j \pi_{ij} = 0 \) already deflates prices. If deflation by the CPI is made and homogeneity is imposed, the CPI cancels out; i.e., \( d \log (p_i/\text{CPI}) - d \log (p_n/\text{CPI}) = d \log p_i - d \log p_n \). That is, deflation of prices by the CPI is redundant.

When price variables are deflated by the CPI in equation (2) above, as described in equation (5) of Onianwa’s study, the above model becomes

\[
(3) \quad w_i \frac{d \log q_i}{d \log \text{CPI}} = \theta_i \frac{d \log Q}{d \log \text{CPI}} + \sum_j \pi_{ij} \frac{d \log (p_j/\text{CPI})}{d \log \text{CPI}} + \beta_i \frac{d \log \text{CPI}}{d \log \text{CPI}},
\]

where \( d \log \text{CPI} = \log(\text{CPI}_{t+1}/\text{CPI}_t) \), and \( \beta_i = -\sum_j \pi_{ij} \).

Equation (3) shows that deflation of prices by the CPI in the Rotterdam model is equivalent to adding the term \( \beta_i d \log \text{CPI} \) to equation (2) under the restriction that \( \beta_i = -\sum_j \pi_{ij} \). For testing homogeneity, or homogeneity and symmetry, the additional term \( \beta_i d \log \text{CPI} \) in equation (3) creates a problem. Namely, homogeneity (homogeneity and symmetry) is a test of restrictions \( \sum_j \pi_{ij} = 0 \) (\( \sum_j \pi_{ij} = 0 \), and \( \pi_{ij} = \pi_{ji} \)) in Rotterdam model (2), not in the ad hoc specification defined as model (3). That is, the unrestricted model is (2), not (3); and the restricted model is (2), or redundantly (3), with \( \sum_j \pi_{ij} = 0 \) imposed. The addition of the term \( \beta_i \) dlog \text{CPI} can be expected to change the likelihood value of the unrestricted model, and perhaps may alter one’s conclusions regarding the tests for homogeneity and symmetry hypotheses.

References


