A Risk Rationing Model

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Introduction

In recent years a renewed policy interest in rural credit for generally poor and underrepresented farmers has given rise to a more concentrated interest in factors affecting credit demand. An offshoot to this effort has culminated in the refinement of credit rationing to include not only the notion of price and quantity rationing but also risk rationing (Boucher, Carter and Guirkinger (2008)). Risk rationing describes an individual that having the asset wealth to qualify for a credit, voluntarily refrains from it for fear of losing his collateral. Unlike borrowers, risk rationing individuals believe that by taking a loan, a positive and sufficiently large probability of default may occur. The concept of risk rationing has been observed by many but not formally analyzed. An early sentiment of risk rationing is by Binswanger and Siller (1983) who state that “If the disutility of the loan is sufficiently high, small farmers may stop borrowing altogether, i.e. the credit market for small farmers may disappear because of lack of demand, despite the fact that small farmers may still have available collateral in the form of unencumbered land” (page 17), concluding that “It is important to realize that it is not an innate deficiency in the willingness of small farmers to take risks that hold them back” (page 19). Eswaran and Kotwal (1990) examining the use of credit markets argue that the smoothing of consumption between individuals of different income classes but identical utility functions can be differentiated by scale of operations in terms of credit demand but also observe that “What seems like an inordinate degree of risk aversion may be a merely a reflection of their inability to sustain downswings in income” (Page 480). Bell, Srintvasan and Udry (1997) examining linked credit estimate a credit demand relationship in which demand increases with liquid assets but decreases with fixed assets, a result they state is “both puzzling and unsatisfactory” (page 575). That the sign on liquid assets is positive is at least suggestive of risk rationing behavior, since liquid assets would buffer the collateralized value of fixed assets in case of default. Swain (2002), investigating credit rationing in Puri, India, finds evidence of credit rationing in the conventional sense, but also notes that “the lower number of households demanding loans from the formal sector might be a choice decision of the household… Such households restrict their demand for production loans even if they have access to them” (Pages 4-5). Bhattacharyya (2005) reveals data from West Bengal showing that even in the presence of formal lending, 62.3% of farmers used informal credit with 86% of these willing to pay a substantial premium for not having to give up collateral.

Of those studies that have instrumented field research to identifying risk rationing there is substantial evidence that it is not a trivial matter. In the current paper we find that 35% of Mexican farmers surveyed are risk rationed. In related research we find 6.5% of Chinese farmers are risk rationed. Barham, Boucher and Carter (1996) report that 32% of Guatemalan farmers surveyed did not apply for credit and were fully constrained in their credit choice due to either transactions costs (transaction cost rationing) or fear of risk leading to self-insure (but without using the term ‘risk rationing’). Boucher, Guirkinger and Trivelli (2009) find 8.6% of surveyed Peruvian farmers in 1997 were risk rationed and (with) Fletschner, Guirkinger and Boucher (2012) find 21% to 25% of a resample of Peruvian farmers in 2003 to be risk rationed. Boucher, Carter and Guirkinger (2008) report results from a number of surveys that 19% of Peruvian farmers, 16% of Honduran farmers and 12% of Nicaraguan farmers were identified as risk rationed.

The only study to attempt to place risk rationing in a theoretical context is Boucher, Carter and Guirkinger (2008). Their model is based on asymmetric information that leads to loan contracts with high collateral contracts, whereupon default the farmers lose productive assets. Consequently, the farmer will self-ration
out of the market in order to preserve capital. If the farmer is to accept the risk of borrowing it is assumed that external risks can be controlled by some level of effort, but such effort also lowers utility in the good state. The lender offers a suite of contracts with the existence of an insurance contract available to reduce collateral requirements. Incentive compatibility is determined by a mix of high or low interest rates, insurance purchases and collateral. They then go on to show that there is some level of financial wealth (e.g. liquidity) that bifurcates two economic outcomes. The first is the decisions to not borrow and expend high effort on subsistence activity and the second is to borrow for commercial activities with the potential loss of collateral or costly mitigation through some combination of insurance and high interest rates. Whether a farmer operates risk rationed or under a credit contract depends upon utility in high and low effort states, risk aversion and prudence. On this latter point Boucher et al. (2008) argue that any agent with prudence being three times absolute risk aversion, and wealth greater its critical value will choose risky commercial activities with credit, while those with the same level of wealth but prudence less than three times absolute risk aversion will choose risk-rationed subsistence activities. Those with financial wealth below the threshold will do the opposite. In other words a farmer with low financial wealth but with prudence less than 3 times risk aversion will choose to borrow and undertake risky commercial activity.

Risk rationing affects the choices of risk coping mechanisms such as income and consumption smoothing. Morduch (1995) mentions that “Income smoothing is more likely to occur when households anticipate being unable to borrow or insure.” implying income smoothing practices are more likely to occur in risk rationed than price rationed. Conservative production decision is one way in which risk rationing smooths income by limiting their exposure to risk. Other income and consumption smoothing methods include alternative income generating activities, diversification, borrowing, savings and informal insurance agreements. Under imperfect markets, which are mostly the case in the agricultural sector in developing economies, production risk cannot be diversified away and thus it is not independent of consumption level. Under imperfect markets, or where “capital markets are inefficient” (Masson), jump disutility and its corresponding avoidance behavior is more likely to occur.

The existence of a jump disutility can affect risk taking behavior. Masson states that under disaster avoidance a risk-averse investor may choose an investment as its variance increases, as long as its lower bound is above the threshold level. Even though this behavior can be found in the absence of disutility jumps, the presence of these can induce this paradoxical behavior. This action relates to Roy’s (1952) safety first criterion. Robinson and Lev (1986), referring to Masson’s model, provide an example of a firm facing liquidation costs as the source of a disutility jump. They explain that the firm’s decisions are to avoid falling below the threshold that causes the disutility jump. An investor may refrain from borrowing, even if he wants to borrow at the market interest rate, if the disutility caused by liquidating assets in case of default is sufficiently large. This approach relaxes some of the more restrictive assumptions in Boucher et al (e.g. effort differentiation and insurance markets) but also confirms some of the more critical aspects dealing with asset wealth, risk aversion and prudence. In our view it is the exogenously determined probability of default, along with asset values (principally land), that creates a state of disutility to be avoided. A sure way to avoid falling into this state is by not taking a loan. This consideration of preserving wealth by means of minimizing the probability of falling below a threshold income level was first analyzed by Roy in his Safety First model.
Another characteristic of risk rationing is the discount factor of future consumption. Risk rationing refrain from engaging into the risky activity by not borrowing; this forgone expected increase in revenues due to the commercial activity is compensated by current consumption, implying a larger discount factor than price rationed under homogeneous production and individual characteristics. Taking the risky commercial activity means sacrificing current consumption for future one. According to Pender (1996), the discount factor, or inter temporal rate of substitution, can be affected by marginal utilities of consumption or by time preference. He continues to say that under binding credit constraint borrowers’ discount rate is higher than the market interest rate. In his paper, he looks at the ratio of marginal indirect utilities of initial wealth and future wealth as a measure of the discount factor. Alternatively, we can look at the utility of initial wealth for two identical individuals that have the same expected returns. That with the higher utility at initial wealth level prefers current consumption more. Risk rationed people are expected to have a higher initial utility, reflecting in part their time preference for consumption.

One objective of this study is to investigate the extent of risk rationing amongst Mexican farmers. However, we present a model with a somewhat different structure from Boucher et al. (2008) preferring to develop risk rationing around the exogenous risk conditions that give rise to a jump in utility functions. If a firm’s loss due to asset liquidation is sufficiently large, it may exhibit a jump discontinuity at a critical income level. Falling below that level would force the firm to liquidate assets in order to meet its obligations. Masson (1974) provides evidence of a utility function with a discontinuity in the form of a vertical jump, representing a large loss in utility due to income falling below a critical level. He makes analogies that the threshold is some situation that will lead us into an undesirable state of nature and thus cause a large disutility, like a divorce or being declared bankrupt. In this model, people get utility from both: income (or any variable of interest), and the state of nature. The different states are defined by being above or below the threshold level.

The second objective of this paper is to a model for risk rationing based on a jump disutility model. We show that a jump disutility requires the incorporation of third moments of the distribution of returns, or any variable of interest. The model is tested using data from Mexican grain farmers from a survey in 2011. With the responses on lowest and highest possible, and expected yield and price for their crops, we simulated the revenue mean, variance and skewness using a Pert distribution. Our results suggest that risk rationing can be modeled as having a jump disutility. Preference for risk rationing depends on the difference in the expected return distribution moments of the two states (debt and no debt). Our results suggest that risk rationed have a stronger preference for skewness than borrowers (price rationed), but not necessarily for the first two moments. Also, the utility at their initial level of wealth is higher for risk rationed, suggesting that their discount rate is larger than price rationed. Our results provide elements for policies to integrate risk rationed into the formal credit market.

**Risk rationing and jump discontinuity**

The concept of a utility function with a jump discontinuity and risk rationing are closely related. Risk rationing refers to an individual who, in spite of his willingness to borrow from financial institutions at the market interest rate, refrain from borrowing for fear of losing collateral. The probability of losing collateral creates a large disutility at the point where his revenues cannot cover the debt repayment. We
can think of this disutility as a large vertical drop in utility caused by the collateral loss. The collateral loss, or jump disutility, depends on the size of the loan and on the divisibility of the collateral.

Models that incorporate a jump disutility relate the jump to a variable of interest reaching certain threshold value; however, the disutility is not caused directly by income falling low, but by the resulting state created by it. Low income triggers a disutility caused by another variable. In Robinson and Lev (1986), this variable is liquidation costs; In Masson (1974), it is the state of being declared bankrupt (fig. 1).

The connection between income and loss of wealth resulting in a disutility jump can be illustrated as a firm that cannot meet its debt obligations and is forced into asset liquidation. This can occur if a firm’s total revenue is less than its total costs. A firm will be indifferent between producing or not if their total revenue equals its total costs, but it will shut down if its total costs are not covered by its total revenue in the long run. From this condition we can have a relationship between revenue and liquidation loss. If $TR < TC$, or consequently if the ratio $\frac{TR}{TC} < 1$, the firm will shut down and liquidate assets to pay its debt obligations. The amount of assets that the firm must liquidate, and all its associated costs, is the loss in total wealth of the firm and the source of jump disutility. The greater the firm’s financial obligations, the larger the wealth loss in case of default. The size of the utility discontinuity is directly related to the amount of fixed assets that a firm should liquidate in order to pay its obligations; if the liquidation of assets is large enough the firm may decide not to enter into the credit market, and thus become risk rationed. The firm will not borrow if the disutility associated to asset liquidation is deemed to be sufficiently large. Masson’s discontinuous function does not capture the interaction between income and wealth loss, it assumes that a given level of income the disutility occurs. To better understand the relationship of income (or revenue) and asset liquidation, we need to include a function of wealth over the ratio $\frac{TR}{TC}$, and a utility function of wealth given that ratio.
Assume the profit function of an individual is the following:

\[ \pi = \alpha + R(l,r,d) + py - C(w,y) - D \]  

where \( \alpha \) is the initial wealth, or savings, \( R(l,r,d) \) is revenue from labor, \( l \), remittances, \( r \), and amount borrowed, \( d \); \( py \) is revenue from farm operations, output price is \( p \) and output \( y \); \( C(w,y) \) is the cost function of the farm and \( D \) is the debt repayment, if any.

Total wealth at end of period, \( t = 1 \), becomes:

\[ W_h = \pi_1 \quad \text{if} \quad TR \geq TC \text{ at period } t = 0 \]  
\[ W_l = \pi_1 - L \quad \text{if} \quad TR < TC \text{ at period } t = 0 \]  

Where \( L \) is the value of the land or total fixed assets that must be liquidated to repay the loan. When the firm’s total revenue is less than its total costs, it will liquidate its assets to pay for any debt obligations it may have incurred.

\[ TR = TC \quad \rightarrow \quad R(l,r,d) + py = C(w,y) + D - \alpha \quad \rightarrow \quad R(l,r,d) + py / C(w,y) + D - \alpha = 1 \]
We call $\gamma$ the ratio $\frac{TR}{TC}$. The value of the variable $\gamma$ determines the different states of the world. As long as $\gamma \geq 1$ the firm will be able to repay its loan obligations and will not be forced to shut down operations. Liquidation of assets occurs if $\gamma < 1$. The amount of asset loss due to liquidation is proportional to the amount of loan, $D$. Assume $L > D$, $L$ is indivisible and is used fully as collateral for $D$. Let the liquidation cost in case of $\gamma < 1$ be $L$ for simplicity. Assuming $d < D$, borrowing decreases the value of $\gamma$; thus, increasing the chance of liquidating assets. As expected, revenues increase the value of $\gamma$ while costs decrease it.

Since a liquidation of assets represents a loss in total wealth for the producer, the relationship of wealth and $\gamma$ can be represented in figure 2. At $\gamma = 1$, a small change, $\varepsilon > 0$, to the left of it will trigger the wealth loss. Unlike our model, however, the jump disutility can also result from being in a different state of the world not necessarily dependent on wealth changes.

**Figure 2. Wealth with respect to $\gamma$, good and bad state at right and left side of $\gamma=1$**

Now that we have a relationship of wealth and $\gamma$ we can have a utility function of wealth dependent on $\gamma$. This utility function exhibits not only a vertical discontinuity at a given level of wealth, but also a change in wealth that separates the good and bad state of nature. Looking at figure 3, the point b is where $\gamma = 1$, the limit of the “good state”. Once $\gamma$ drops below 1, the function takes the value at point a, the upper limit.
of the “bad state”. The separation of the two states occurs if a person borrows and has to liquidate his assets to repay a loan. The vertical jump from b to a is the change in utility from being at either state. This disutility is caused by the asset loss needed to meet obligations, represented by the horizontal gap between points a and b. If this gap in wealth is related to the indivisibility of land, uncertainty of cash flows or current level of wealth; then, if a risk rationed individual can use other assets than his land as collateral, if he has a revenue insurance product, if he has other sources of income, or if there is a government policy that provides him with a steady cash transfer, then a risk rationed may engage into a higher expected yield enterprise, i.e. become price rationed.

There may also be some disutility caused by the stigma of being at the bad state, like Masson suggested, but for the purpose of this paper we exclude it from the analysis; however, this stigma could cause a downward shift of the segment of the utility function left of point a, and thus, increase the jump, assuming the marginal utilities are invariant in any state. Otherwise, the slope of the utility function may change as well.

For a risk rationed, it is assumed that the starting value of $\gamma$ is on the right of 1. When the starting value is located to the left side of 1, the individual could behave as risk seeker. His expected revenue distribution becomes positively skewed. At the “bad state” farmers may not have access to formal credit; nonetheless, they may seek a production technology that offers them the highest variance to have a positive probability of reaching $\gamma = 1$, and so be above the threshold level or in the “good state”.

**Figure 3. Utility function of wealth with a discontinuity given $\gamma$**
The relationship between the indirect variable, $\gamma$; the direct variable, wealth; and the utility function, is shown in figure 4. Quadrant 1 shows utility as a function of $\gamma$. The loss in utility, $J$, occurs once $\gamma$ is at the threshold level. This loss in utility is the result of the loss of assets, $L$, from liquidation when of $\gamma < 1$, fig. 2 and quadrant 4. Quadrant 2 shows the relationship of utility and wealth when facing liquidation costs. The discontinuity is derived from the loss of wealth, $L$, and utility, $J$.

Figure 4. Multi-Quadrant Derivation of a Utility Function with Jump Disutility and Wealth Loss.
The relationship of utility, wealth and the indirect variable, $\gamma$, can be expressed first as wealth as a function of $\gamma$, and then as the utility of wealth at each of the two states of nature. Looking at quadrant IV, the good state is at the left of $W_h$, and the bad state at the right of $W_i$. Summarizing [1] and [2], and following Lev and Robinson’s ad hoc decision rules, we have:

\[ W = \begin{cases} \pi & \text{for } \gamma \geq 1 \ (\text{good state}) \\ \pi - L & \text{for } \gamma < 1 \ (\text{bad state}) \end{cases} \]

\[ V(W) = \begin{cases} U(\pi) & \text{for } \gamma \geq 1 \ (\text{good state}) \\ U(\pi - L) & \text{for } \gamma < 1 \ (\text{bad state}) \end{cases} \]

Where the ad hoc decision rule to maximize is:

\[ \text{Max } V(W) = U(\pi)(1 - F(\gamma < 1)) + U(\pi - L)F(\gamma < 1) \quad [3] \]

Solution of [3] is equivalent to Roy’s safety first model, since we are minimizing $\text{Prob}(\gamma < 1)$.

Equation [3] refers to a price rationed individual; he is taking the loan and thus it incurs into a positive probability of losing $L$. This doesn’t mean that by not taking a loan the probability of liquidation, or
\( \gamma < 1 \), is not positive. This probability may in fact be positive, but the size of \( L \) may not be significant as to create a disutility jump. From this we can assume that for a small enough \( L \), the utility function of a risk rationed, or non-borrower, is continuous over wealth. However, we should consider the cases where the individual’s utility function changes by taking the loan or not, and also his expectation on the distribution of revenues in the case of taking the loan or not. We could also expect that taking a loan increases the probability of higher returns. If this is so, then we have four different scenarios:

1) Utilities don’t change between states (debt and no-debt), and neither does return distributions.

2) Utilities between states are the same, but return distributions differ by state.

3) Utilities are different between states, and return distributions are same.

4) Utilities are different between states, and return distributions are different too.

As an illustration of scenario (2), Figure 5 shows the same utility function of an individual in the two states, debt and no-debt, and only their revenue distribution changes between states. Since price rationed are those who borrow formally, we would expect that their upper bound on returns to be higher than under no debt. Also, we would expect that their benefits of increase revenue outweigh the cost of losing their collateral. That increase in expected revenue is reflected in Figure 5 as an increase in the upper bound of wealth from \( W_{nd,M}(\gamma) \) to \( W_{d,M}(\gamma) \), with a corresponding increase in utility at the upper bounds from \( U(W_{nd,M}|\gamma) \) to \( U(W_{d,M}|\gamma) \). Borrowers would face the wealth loss of \( W_{d,h}(\gamma) - W_{d,i}(\gamma) \) with the corresponding jump disutility of \( U(W|\gamma = 1) - U(W|\gamma = 1 - \varepsilon) \).

**Figure 5. Utility function of a risk and price rationed individual under different revenue distributions**
The expected utility of a borrower is the integral from 0 to the higher upper bound of wealth $W_{d,M}(\gamma)$ minus the expected disutility created by the jump.

$$E[U(W^d)] = \int_0^{W_{d,M}} U(W | \gamma) \varphi_d(\gamma) d\gamma$$

$$= \int_0^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma - \int_{W_{d,l}}^{W_{d,h}} U(W | \gamma) \varphi(\gamma) d\gamma + \int_{W_{nd,l}}^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma$$

The expected utility of a non-borrower is the integral from the origin to the maximum expected revenue under no debt.

$$E[U(W^{nd})] = \int_0^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma$$

An individual is risk rationed if $EU(W^{nd}) > EU(W^d)$.

$$\int_0^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma > \int_0^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma - \int_{W_{d,l}}^{W_{d,h}} U(W | \gamma) \varphi(\gamma) d\gamma + \int_{W_{nd,l}}^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma$$

$$EU(W^{nd}) > EU(W^d) = \int_{W_{d,l}}^{W_{d,h}} U(W | \gamma) \varphi(\gamma) d\gamma > \int_{W_{nd,l}}^{W_{nd,M}} U(W | \gamma) \varphi(\gamma) d\gamma$$

Similar analyses follow for the other three scenarios.

**Effect of Higher Moments in the Expected Utility Function of a Risk Rationed Individual.**
The justification for including the third moment in the utility function is provided next. The drop in utility caused by the possibility of collateral loss has the effect of reducing the skewness of the expected revenue distribution. The reason for this is that there is a probability shift from the value of $y$ that triggers the collateral loss to the values of $y$ with the loss incurred. These values are to the left side of the trigger values, thus, shifting probability weights toward the left tail of the distribution. Rothschild and Stiglitz (1970) mention that a density function created from another one by taking probability weight from the center and adding it to the tails becomes more variable. In our case, however, the probability function created by shifting weights to the left tail not only becomes more variable, but also more negatively skewed.

Figure 6. Probability Shift from $A'$ to $A''$ for Borrowers under jump disutility

Figure 6 shows a probability transfer towards the left tail of the expected wealth distribution as described in [2]. The area $A' = \int_{W_l}^{W_h} f(w|y)dw$ is the probability of wealth being in the gap between $W_h$ and $W_l$. This is the loss of wealth due to collateral liquidation. The borrower cannot attain the wealth values between $W_h$ and $W_l$ anymore, the probability of being between them is transferred, from $A'$, to the left of $W_l$ at $A''$. How spread is the new added probability at the left of $W_l$ is unknown. However, we can denote the spread as being a positive number, $\xi$. Regardless of the value of $\xi$, the new distribution would have more weights towards its left tail, thereby decreasing skewness.
Distribution of wealth might appear bimodal (or multimodal) amongst risk rationed, particularly if some of them used to be borrowers and lost collateral; while for borrowers it may appear unimodal.

Following the justification, we can characterize the expected utility model of an individual through a third-order Taylor expansion over $\gamma$. The expected utility function of wealth depends on the current level of wealth and on the distribution of $\gamma$. The incorporation of the third moment into the expected utility function is the result of the shape of the distribution of $\gamma$: the starting value of $\gamma$, $\bar{\gamma}$, plus the random term $\tilde{\gamma}$. The distribution of $\gamma$, $\psi$, is characterized by its first three moments: $\bar{\gamma} \sim \psi(\mu, \sigma^2, m^3)$, where $\mu$ is the distribution mean, $\sigma^2$ its variance, and $m^3$ its third moment or unstandardized skewness. The jump disutility of $\gamma$ creates negative skewness in its distribution. This jump process transfers risk from the right side to the left side of $E(\bar{\gamma})$. For the same reason that limiting downside risk increases skewness, increasing downside risk decreases it.

The inclusion of the third moment of a distribution in the utility function reflects the downside risk of a random variable. A positive skewness decreases downside risk, while a negative one increases it. Menezes et al. (1980) gives a general definition of increasing downside risk as the following: “one distribution has more downside risk than another if it can be obtained from the other by a sequence of probability transfers which unambiguously shift dispersion from right to the left without changing the mean and variance.” Cain and Peel (2004) studied the preference for gambles and state that a risk-averse person has a preference for skewness; moreover, a tradeoff exists among mean, variance and skewness. People are willing to trade a negative expected mean of returns for a positive skewness. According to Golec and Tamarkin (1998), the preference for skewness can be sufficiently large that even though people are faced with negative expected return and high variance on a gamble, they would still take the gamble as long as the skewness is sufficiently large. Peel (2012), on the other hand, provides examples where given different characteristics of lotteries and utility functions, risk-averse individuals do not necessarily prefer a more skewed distribution with equal mean or variance. However, investor’s preferences for skewness in returns are so common that there exist many mechanisms in the market to increase positive skewness of returns. Tsian (1972) cites limited liability, prearranged stop-loss sales on stocks, and put and call options as examples of market mechanism to increase positive skewness of returns. Diminishing the magnitude of a financial loss and increasing the magnitude of the gain are ways to increase the skewness of returns. Similarly, increasing the size of a loss and limiting gains decrease the skewness of the return distribution, which is avoided by investors. This can occur if the income level of a firm is sufficiently low that it may be forced into bankruptcy and into liquidation of assets. The size of the loss in assets due to liquidations is related to the amount owed to creditors.

The expansion of the utility function of wealth over $\bar{\gamma}$ becomes:

\[
E[U(W|\gamma)] = U(\bar{\gamma}) + U'(\bar{\gamma})\mu + \frac{U''(\bar{\gamma})\sigma^2}{2} + \frac{U'''(\bar{\gamma})m^3}{6} \tag{6}
\]

Where $U' > 0$, $U'' < 0$, $U''' > 0$, $E(\gamma - \bar{\gamma}) = \mu$, $E(\gamma - \bar{\gamma})^2 = \sigma^2$, $E(\gamma - \bar{\gamma})^3 = m^3$, and $E\left(\frac{\gamma - \bar{\gamma}}{\sigma}\right)^3 = Sk$ is the standardized skewness. Higher order terms are not considered in this analysis. After multiplying the second and third moments of [6] by $\frac{U'}{U''} \text{ and } \frac{U'''}{U'''} \text{ respectively, and standardizing the skewness, we transform [6] as a function of risk aversion and prudence.}
\[
E[U(W|\gamma)] = U(\bar{\gamma}) + U'(\bar{\gamma})\mu - \frac{1}{2}U'(\bar{\gamma})R\sigma^2 + \frac{1}{6}U'(\bar{\gamma})RPSk\sigma^3
\]

\[
= U(\bar{\gamma}) + U'(\bar{\gamma})[\mu - \frac{1}{2}R\sigma^2(1 - \frac{1}{3}PSk)]
\]  

[7]

Where \( R = -\frac{u'''}{u''} \) is the Arrow-Pratt absolute risk aversion coefficient, and \( P = -\frac{u'''}{u''} \) is the absolute prudence coefficient, Kimball (1990). The expected utility function characterized by the first three moments is derived in equation [6]. The effect of the change in expected revenue, or whatever \( y \) is, to expected utility is \( \frac{\partial E[U(W|\gamma)]}{\partial \mu} = U'(\bar{\gamma}) > 0 \), due to local non-satiation. The effect of revenue risk on expected utility is \( \frac{\partial E[U(W|\gamma)]}{\partial \sigma} = -U'(\bar{\gamma})R\sigma + \frac{1}{2}U'(\bar{\gamma})RPSk\sigma^2 \leq 0 \). When skewness is 0, the derivative is negative except under risk neutrality, that is, when \( R = 0 \). If \( R \) and \( P > 0 \), the derivative can be positive when skewness is positive and sufficiently large, i.e. when \( Sk > \frac{2}{P\sigma} \). This is consistent with a case stated by Masson where an individual may prefer a production technology with a larger variance as long as the mean return of the two technologies is the same and the riskier technology does not fall below a minimum threshold. If as a riskier production technology is left bounded, which increases its skewness, a person may in fact prefer it. Skewness has a positive effect on expected utility, \( \frac{\partial E[U(W|\gamma)]}{\partial Sk} = \frac{1}{6}U'(\bar{\gamma})RPSk\sigma^3 \geq 0 \), except again, under risk neutrality, or prudence neutrality (when \( U'''' = 0 \)). The derivative increases with variance when both \( R \) and \( P > 0 \). This is equivalent to say that the larger the downside risk aversion coefficient is, \( D = RP \) (Modica and Salvatore 2005), the larger the preference for skewness. As long as \( D = \frac{u''''}{u''} > 0 \), skewness increases expected utility under larger variance.

When prudence equals \( \frac{3}{Sk\sigma^3} \), the expected utility function is characterized by the first moment only. If it is greater than that, a larger risk aversion values would increase expected utility. In a similar way, when prudence is lower than \( \frac{3}{Sk\sigma^3} \), a larger risk aversion would decrease expected utility. Prudence would increase expected utility as long as skewness is positive.

The three-moment expected utility also affects risk premium through skewness. Risk premium is equivalent to \( E[U(W + e)] = U(W - \pi) \), where \( e \sim f(\mu, \sigma^2) \) is the distribution of the gamble’s outcome and \( \pi \) is the risk premium willing to pay in order to avoid the gamble \( e \). Under mean-variance analysis, the risk premium is approximated as \( \pi = \frac{1}{2}\sigma^2A - \mu \), where \( A \) is the Arrow-Pratt coefficient of absolute risk aversion. Similarly, for a three-moment expected utility, the risk premium is \( \pi = \frac{1}{2}\sigma^2A - \frac{1}{6}\sigma^3DSk - \mu \). When skewness increases, risk premium decreases, as long as \( D > 0 \).

The state of being risk rationed or price rationed is endogenous to each individual. We assume that they have complete information about their production technologies and risks, from which they base their decision to be in either state. So far we have not specified the form of the utility function since for the purpose of explaining the decision to be risk rationed, it is sufficient to have the general form. The utility functions, however, can differ at each state. That is, the utility function of a risk rationed may be different from that of a price rationed. The resulting expected utility for each state is derived from the interaction of the utilities functions along with the distribution of expected returns.
Farmer’s decision to be at each state can be summarized as follows. Suppose that each farmer starts with the decision to borrow or not at time 0. At this point he has the opportunity to borrow, and he indeed wants to borrow, in order to adopt a higher yield technology with higher expected returns; or he can choose not to borrow and produce with a lower yield technology if he deems that the probability of default is large enough as to force him to liquidate his assets in order to repay if default occurs. That doesn’t mean that farmers do not consider borrowing, they do want to borrow and have access to credit, but the disutility caused by the collateral loss risk is greater than the utility gained from the higher expected returns. The extra revenue from using the high yield technology does not compensate for the probability of losing collateral in case of a bad scenario. These people are risk rationed. If, on the other hand, the disutility from the probability of losing collateral due to default is lower than the utility gained from the higher expected returns generated by the adoption of the high yield technology, then this farmer would be price rationed. Therefore, the farmer has two options based on his utility function and on his risk assessment of returns.

**Random utility model for risk rationing under static time**

In this section we investigate the characteristics of risk and price rationing using a random utility model. A dichotomous variable indicating credit rationing status is used as dependent variable, and the first three moments of the expanded expected utility as independent variables.

To accomplish this, we specifically designed a survey to unambiguously determine credit status (see appendix for a schematic of the survey). This survey divide individuals into three mutually exclusive groups: risk, price and quantity rationed. Quantity rationed are those people who requested a loan but the lender did not offer any amount or a lower amount than requested. They are externally rationed. As previously defined, Risk rationed are those people who do not borrow, or borrow less than offered, for fear of losing collateral; while price rationed are those who borrow or refrained from borrowing for reasons other than fear of collateral loss. These two groups are internally determined by the borrower. The survey was meticulously created to eliminate any endogeneity. For instance, observing that a person has no formal debt cannot determine if he is price, quantity or risk rationed; and thus instrumental variables may be needed.

Under the random utility framework, a farmer would decide not to borrow if his expected utility from not borrowing is larger than that from borrowing; that is, if $E_R[U(w|γ)] > E_P[U(w|γ)]$. Subscript $R$ stands for risk rationed and $P$ for price rationed. The probability that risk rationing is chosen, $Pr(R = 1)$, is then:

$$Pr \left\{ U_R(γ) + U_R′(γ)μ_R + \frac{1}{2} U_R″(γ)σ_R^2 + \frac{1}{6} U_R‴(γ)m_R^3 + ε_R > U_P(γ) + U_P′(γ)μ_P + \frac{1}{2} U_P″(γ)σ_P^2 + \frac{1}{6} U_P‴(γ)m_P^3 + ε_P \right\}$$

$$Pr \left\{ ε_P - ε_R < U_R(γ) - U_P(γ) + U_R′(γ)μ_R - U_P′(γ)μ_P + \frac{1}{2} \left( U_R″(γ)σ_R^2 - U_P″(γ)σ_P^2 \right) + \frac{1}{6} \left( U_R‴(γ)m_R^3 - U_P‴(γ)m_P^3 \right) \right\} \quad [8]$$

The probability of choosing not to borrow is given by the cdf of $(ε_P - ε_R)$ to the point of the right hand side of [8]. That is,

$$Pr(R = 1) = \Phi \left( \frac{U_R(γ) - U_P(γ) + U_R′(γ)μ_R - U_P′(γ)μ_P + \frac{1}{2} \left( U_R″(γ)σ_R^2 - U_P″(γ)σ_P^2 \right) + \frac{1}{6} \left( U_R‴(γ)m_R^3 - U_P‴(γ)m_P^3 \right) }{σ} \right)$$

Equation [9] can be estimated using a linear probability, probit or logit model. The independent variables are the moments of the expected distributions of returns for each state. The dependent variable is a
dummy variable with a value of 1 if the individual is risk rationed, and 0 for price rationed. The estimated coefficients are the utilities and its first three derivatives for each state, however, the constant term would be the difference in utilities at the initial wealth level. Going back to the four scenarios previously mentioned, we can test for different utilities functions, or its derivatives, by solving [9] and testing for differences in coefficients. On the other hand, if we believe that the moments are invariant between states, then the only source of differences in expected utility would be different utility functions. The probability of being risk rationed under the assumption of invariant moments becomes:

\[ \Pr(R = 1) = F\left\{[U_R(\bar{\gamma}) - U_P(\bar{\gamma})] + [U'^R(\bar{\gamma}) - U'^P(\bar{\gamma})]\mu + [U''^R(\bar{\gamma}) - U''^P(\bar{\gamma})] \frac{\sigma^2}{2} + [U'''^R(\bar{\gamma}) - U'''^P(\bar{\gamma})] \frac{m^3}{6}\right\} \]  

Model [10] can be justified if there are no significant differences between the moment distributions at each state. The coefficients in [10] now become the differences in coefficients between states. A coefficient that is not significantly different from zero means that it cannot explain the state preference between risk rationed and price rationed. As in [9], the constant term is the difference in utilities at the initial wealth level. The rest of the coefficients are also measured at the initial wealth, \( \bar{\gamma} \). If the coefficients of [10] are different from zero, each state would have a different utility function. Our data supports the use of model [10].

If the moment distribution is different across states but with equal utility function and initial wealth, the equation to solve becomes:

\[ \Pr(R = 1) = F\{U(\bar{\gamma}) + U'(\bar{\gamma})(\mu_R - \mu_P) + U''(\bar{\gamma})\frac{1}{2}(\sigma^2_R - \sigma^2_P) + U'''(\bar{\gamma})\frac{1}{6}(m^3_R - m^3_P)\} \]  

Again, we can test for different utilities and its derivatives between states by testing the corresponding coefficients of [9].

Once we have estimated the coefficients we can proceed to analyze some characteristics for each group. For instance, if we use model [9] we can estimate the values of the utility at initial wealth, and expected marginal utilities for each group. By dividing the utility at initial wealth by the expected marginal utility of each group, we can get their discount factor (Pender, 1996). Also, by dividing the expected marginal utilities of each group we get the marginal rate of substitution between groups. This tells us the amount of expected income that can be given up in one state in exchange of one unit of expected income in the other while keeping utility constant. In other words, this is the value of expected income in one group measured in a unit of expected income in the other. Risk aversion, prudence and downside risk aversion coefficients can also be estimated by using model [9]. If model [10] is used, the analysis is more limited since the coefficients are the difference of utilities and its expected moments between groups. The constant coefficient can tell us which group has a stronger preference for current consumption, implying a larger discount rate. If the value of the coefficient is positive, it means that risk rationed get more utility than price rationed at the initial wealth level, or at the present. Similar to the marginal rate of substitution between expected incomes of the two states, the difference of expected marginal utilities can tell us which state requires values expected income more, but not the actual price ratio like the marginal rate of substitution. This difference in marginal utilities indicates the expected income preference under risk. A small value means that an individual can easily substitute expected income between the two states, and then the state preference will depend on the higher moments.
Figure 7. Marginal Rate of substitution between expected income for each state

In figure 7, the horizontal axis is the expected income under risks rationed, while the vertical is under price rationed. The slope of the indifference curve is the units of expected income that should be given up under price rationed for one unit under risk rationed.

Comparative statics of demand for credit under a jump utility model with intertemporal consumption.

Here we analyze the effects of key parameters on demand for credit for a farmer with a jump disutility using an intertemporal utility model. This section is to illustrate some comparative statics on demand for credit, and show how the discount rate is obtained. Similar to Pender (1996), we created a credit constraint where a farmer faces liquidation costs under a probability of default. We estimate the effect of
assets, non-farm revenue, and expected farm revenue on credit demand. Assume the farmer has initial assets $\alpha$, which includes land value. He complements his income by non-farming activities, $R$. He decides how much to borrow, $d$, before the planting decision. After harvest, he receives farm income $\tilde{y}$ with distribution $f(\tilde{y})$. He also repays his formal loan at the market interest rate, $(1 + r)d$, but if his total income cannot meet the debt obligation, he is forced into liquidation of assets $L$. The probability of default is $F(y)$. The farmer has savings rate of $K$ for the assets at first period. His formal credit line is limited by $D$. The farmer’s problem becomes:

$$\text{Max}_{d < D} U_1(\alpha + R + d) + EU_2[\tilde{y} - (1 + r)d - LF(y) + K(\alpha + R + d)] + \lambda(D - d) \quad [12]$$

Applying the envelope theorem to [12] we obtain the effects of the different parameters on credit demand. The effect of non-farm income on credit demand is negative if the marginal expected utility of non-farm income is greater than the expected marginal utility of debt, as long as $\frac{\partial y}{\partial d} + K > (1 + r) + L\frac{\partial F(y)}{\partial d}$, that is, if the marginal farm income from investing the loan plus savings rate is greater than marginal probability of default from an increase in debt times liquidation cost plus the market interest rate. In other words, if the farmer’s expected marginal utility of non-farm income is greater than his expected marginal utility of debt, then his demand for formal credit would increase as his non-farm income increases if the liquidation costs are large, or the probability of default is high. This reflects the effect of a buffer cash to cope with bad outcome. This interaction is complex, but one of the main factors is increase in expected farm income, liquidation cost and probability of default. The effect of $LF(y)$ is the dominant force in determining credit demand for risk rationing; however, the interaction with the rest of the parameters will ultimately decide the credit demand. Demand for formal credit given non-farm income is:

$$\frac{\partial d}{\partial R} = \frac{EU'_R(C_2)(L \frac{\partial F(y)}{\partial R} - K) - U'_R(C_1)}{(EU'_d(C_2) - EU'_d(C_2)) \left( \frac{\partial y}{\partial d} + K - (1 + r) - L \frac{\partial F(y)}{\partial d} \right)} \quad [13]$$

The Lagrange multiplier is $\lambda^* = U'_d(C_1) + EU'_d(C_2)\left( \frac{\partial y}{\partial d} + K - (1 + r) - L \frac{\partial F(y)}{\partial d} \right)$, and $C_1, C_2$ are consumption at each period.

$\lambda^*$ is positive if the marginal farm income from investing the loan is greater than marginal probability of default times liquidation cost. Normally this is the case, unless the probability of default is large.

How expected farm income affects credit demand depends also on the probability of default and liquidation cost, but also on the difference in expected marginal utilities of farm income and debt. If this difference is large enough, it would require a high savings rate for expected farm income to have a negative effect on credit demand. If both expected marginal utilities are close in value, credit demand would increase as $\frac{\partial y}{\partial d}$ increases.

Similar results hold for the effect of assets on credit demand. Asset wealth creates a safety net for borrowing that even with high probability of default credit demand is positive.

One characteristic of risk rationed is that they forego the chance of higher future income, by not taking the risky activity. This may be also caused by time preference of consumption. Although risk rationing
may exhibit larger discount rate than price rationing, we believe this is a sufficient but not a necessary condition for risk rationing. Large discount rate can explain risk rationing behavior, but as mentioned before, many other factors affect that decision. Pender defined the discount rate \((1+d)\) as the ratio of the marginal utilities between state 1 and 2, the smaller \((1+d)\), the larger future consumption is discounted. His discount rate is obtained from the Taylor expansion of \(V(w, \bar{y})\), the function that solves the UMP. The expansion becomes: \((w, \bar{y}) + x \frac{\partial V(w, \bar{y})}{\partial w} \equiv V(w, \bar{y}) + (1 + d)x \frac{\partial V(w, \bar{y})}{\partial \bar{y}}\). Pender defined \(x\) as the present reward and \((1 + d)x\) as the future reward. For a small \(x\), the Taylor expansion provides an approximated solution.

In our model, the discount rate is \((1 + d) = \frac{\frac{\partial V(w, \bar{y})}{\partial w}}{\frac{\partial V(w, \bar{y})}{\partial \bar{y}}} = \frac{U'_w(C_1) + EU'_w(C_2)[\frac{\partial y}{\partial w} + K]}{EU'_y(C_2)}.\) The time preference depends on the expected utility from farm income, as well on the expected utility of certain wealth, \(w\), and the savings rate. Unless \(EU'_y(C_2)\) is very large, risk rationing would prefer current consumption.

These models show that the demand for credit is the result of complex interactions. When demand for credit decreases for a given set of parameters, we can say that the individual is approaching risk rationing status, or is currently quasi-risk rationed. The advantage of our study is that we are not looking at those people who have diminishing demand for credit, but instead look at the determinants of risk rationing. This analysis is possible because we used a specific survey to unambiguously determine credit rationing status.

### Data and Empirical Results

The data we use in this paper was collected from a survey of Mexican grain farmers that we conducted in September 2011. The survey took place in a grain producing region in eastern San Luis Potosi. We collaborated with a local grain marketing cooperative for logistics and survey sampling support. In total we interviewed 372 grain farmers that produce among corn, sorghum and soybeans, or a combination of them. Survey participants were compensated with $100 pesos for participating, which is about a day’s wage.

The survey included demographic as well as production questions. We asked them about their current production decisions, land size, total assets and expectations about the next season. Specifically, we asked farmers to give an estimate on the lowest possible, most likely and highest possible crop price they expect to sell. They provided the same estimation for their crop yields: lowest possible, most likely and highest possible yields per hectare of the crops intended to plant, and in some cases for the rest of the crops. Once we gathered the price and yield data, we estimated their expected revenue for next season. That is, we estimated their minimum, most likely and highest possible revenue based on their price and yield estimations.

This same survey asked questions that define their credit rationing group. As mentioned before, this survey unambiguously categorizes each farmer, avoiding the use of instrumental variables to correct endogeneity.
With the estimated revenue data, we calculated the first three moments of the distribution through simulation. Using the software atRisk, we inputted the minimum, most likely and maximum revenue values into a Pert distribution simulator. We ran 5,000 iterations for each farmer and from the resulting Pert distribution, we obtained the first three moments for each farmer. Those values were used in our analysis. Parameterizing a Pert distribution when data is limited to expert opinion is a common practice. The parameters needed to fit a Pert distribution are minimum, most likely and maximum. Another advantage of this distribution is that it allows for skewness.

A dichotomous variable was used as the dependent variable: 1 for risk rationed and 0 for price rationed. As regressors we used the estimated revenue variables for each of the three crops. If somebody planned to grow more than one crop, only the main crop in acreage was used for the analysis. For instance, if his main crop was corn, then the first three moments of corn would be used. We tested for differences in mean between groups for the estimated values of revenue and found them to be not significantly different. Because of this similarity, we used model [10].

Most of the farmers in this analysis have the intention of planting sorghum as their main crop for the next season, 53%; followed by soybeans, 24%; and corn, 23%.

The results of the regressions are shown in tables 1 and 2 for each crop.

The coefficients in the regressions represent the difference between the utility at initial wealth level, marginal utility of expected revenues, change in the marginal utility of expected revenues, and the third derivative of the utility function of expected revenues. Thus, a positive coefficient indicates that its value under risk rationing is larger than that under the price rationing state.

Table 1. Probit Regression of Revenue Moments on Risk Rationed

<table>
<thead>
<tr>
<th>Revenue</th>
<th>Sorghum</th>
<th>Corn</th>
<th>Soybeans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Ration = 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0000303</td>
<td>0.000217*</td>
<td>0.0000806</td>
</tr>
<tr>
<td></td>
<td>(0.669)</td>
<td>(0.019)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>Variance</td>
<td>-7.38e-08</td>
<td>-0.000000143*</td>
<td>-0.000000166</td>
</tr>
<tr>
<td></td>
<td>(0.211)</td>
<td>(0.014)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.531</td>
<td>0.523</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>(0.279)</td>
<td>(0.406)</td>
<td>(0.563)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.246</td>
<td>-1.011</td>
<td>-0.865</td>
</tr>
<tr>
<td></td>
<td>(0.474)</td>
<td>(0.128)</td>
<td>(0.303)</td>
</tr>
<tr>
<td>N</td>
<td>112</td>
<td>49</td>
<td>51</td>
</tr>
<tr>
<td>p-values in parentheses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* p&lt;0.05, ** p&lt;0.01, *** p&lt;0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Linear Probability Regression of Revenue Moments on Risk Rationed

---
<table>
<thead>
<tr>
<th>Revenue</th>
<th>Sorghum</th>
<th>Corn</th>
<th>Soybeans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0000173 (0.451)</td>
<td>0.0000743* (0.014)</td>
<td>0.0000227 (0.416)</td>
</tr>
<tr>
<td>Variance</td>
<td>-1.58e-08( \times 10^{-8} ) (0.163)</td>
<td>-4.85e-08** (0.007)</td>
<td>-3.68e-08* (0.037)</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.170 (0.342)</td>
<td>0.159 (0.455)</td>
<td>0.101 (0.596)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.604*** (0.0)</td>
<td>0.155 (0.518)</td>
<td>0.178 (0.467)</td>
</tr>
</tbody>
</table>

| N | 112 | 49 | 51 |

p-values in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Equation [10] provides the interpretation of the coefficients. They measure the difference in expected utility and its derivatives between risk rationed and price rationed. The constant term measures the difference in utility at initial wealth level between a risk rationed and a price rationed. This random utility model measures the likelihood that a person would be risk rationed given changes in the moments of the expected revenue distribution.

Using the coefficients for sorghum under the linear probability model we have the following equation:

\[
Prob(R = 1) = 0.604 - 0.0000173\mu - 0.000000158\sigma^2 + 0.17 Sk + e \quad [14]
\]

We selected sorghum as an illustration since it is the crop that is most planted in our survey. Also, the coefficients of the linear probability model are used since they measure the marginal effects of the moments of the expected revenue distribution.

Looking at the results from equation [14], we find that the utility of risk rationed is larger than that of price rationed at the initial wealth level. This is given by the positive value of the constant coefficient. However, the negative coefficient of \( \mu \) shows that the expected marginal utility of revenue for price rationed is larger than the expected marginal utility of revenue for risk rationed. The larger the expected revenue for sorghum, the more likely someone would be price rationed. The sign of the coefficient for \( \sigma^2 \) is negative also, this means that the change in marginal utility of expected revenue for price rationed is lower than that of risk rationed. The larger the variance in expected returns, the more likely someone is price rationed, that is, risk rationed are more adverse to expected variance in returns than price rationed. The positive value of the coefficient of skewness indicates that the third derivative of the expected utility of revenue for risk rationed is larger than that of price rationed. This means that risk rationed people prefer more skewness of the expected revenue distribution than price rationed. Recall that by staying risk rationed, the skewness of their distribution becomes less negatively skewed relative to price rationed, indicating that price rationed respond less to skewness.

Similarly, corn and soybeans farmers have the same sign in the coefficients of variance, skewness and constant. They differ from sorghum growers in the sign of the expected revenue. Unlike sorghum, they
have positive signs. This means that the larger the expected revenue for these two crops, the more likely a farmer would become risk rationed.

The constant term has different signs in model 1 and 2 for corn and soybeans growers. In the probit model (table 1) corn and sorghum have negative coefficients, while in the linear probability model (table 2) all crops have positive coefficients. From equation [10], the coefficient of the constant term can be thought of as the difference in utilities at the initial wealth level between risk rationed and price rationed. A positive coefficient indicates that at the current wealth level, the utility of being risk rationed is larger than that of a price rationed. The signs of the coefficients for sorghum (the most common crop) in both models, and for the rest of the crops in the linear probability model, are all positive. This suggests that at the initial period, risk rationed have more utility than price rationed, and this may also indicate a preference for current consumption. When current utility is higher in risk rationed state, those farmers may not want to engage in the risky activity unless the utility derived from future revenues outweighs the utility at the present wealth level.

The results for both table 1 and table 2 show similar results regarding coefficient signs except in the constant terms for corn and soybeans. The results for corn and soybeans show that risk rationed people have a lower second derivative of expected utility of revenue than price rationed. This, along with the positive sign for the mean coefficient, makes risk rationed farmers of corn and soybeans more risk avert than price rationed. The coefficient of the difference in expected revenue is positive, meaning that the marginal utility of expected revenue for risk rationed is larger than that of price rationed. At the same time, the difference in expected variance coefficients is negative, implying that the change in marginal utility of expected revenue is larger for the risk rationed. Risk rationed farmers who grow sorghum are also more risk averse than price rationed. This can be obtained by looking at the signs of the coefficients of mean and variance. Since they are both negative this means that the values of the first and second derivatives of the expected utility function of a price rationed are larger than those of a risk rationed. Simple rearranging shows that for a risk rationed to be more risk averse than price rationed, the ratio $\frac{U_R'''}{U''}$ has to be smaller than $\frac{\delta}{\varepsilon}$, where $\delta = U_p'' - U_R''$, and $\varepsilon = U_p' - U_R'$. This is true since both $\delta$ and $\varepsilon$ are both positive and the $U_R''' < 0$.

The negative signs in all variance coefficients and the positive signs in all skewness coefficients indicate that all price rationed farmers have a larger prudence coefficient than risk rationed farmers. The inequalities obtained from the coefficients signs for all crops are $U_p'' > U_R''$ and $U_p'''' > U_R''''$. Based on these inequalities, the prudence coefficients (Kimball, 1990) are then obtained. From our results we find that $-\frac{U_R'''}{U_R''} < -\frac{U_p''''}{U_p''}$. This coefficient measures the importance of precautionary savings when there is uncertainty in the future.

Another coefficient obtained from the third derivative of the utility function is the downside risk aversion coefficient. This value is given by $\frac{U'''}{U''}$ (Modica and Scarsini, 2005). Modica and Scarsini use the definition of an increase in downside risk provided by Menezes, Geiss and Tressler (1980) as a “mean-variance preserving transformation that shift probability from the right to the left of a distribution”. From the illustration presented earlier in this paper of a jump disutility of borrowers is that we believe risk rationed are more risk adverse than price rationed. Our data suggests this hypothesis. In the case of
sorghum growers, they have a negative coefficient of the expected revenue (mean), and positive coefficient in the skewness of the expected revenue distribution. From these coefficients signs it can be shown that risk rationed farmers are also more risk adverse than price rationed. The positive sign in the skewness coefficient implies that $U''_R > U''_P$ for all crops. In the case of sorghum, however, the negative sign in the mean coefficient implies that $U'_R < U'_P$. Therefore, for sorghum the value of the risk aversion coefficient for risk rationed is larger than for price rationed, $\frac{U''_R}{U'_R} > \frac{U''_P}{U'_P}$. For the other crops, corn and soybeans, the positive sign of the mean coefficient means that there is a condition for which risk rationed are more downside risk averse than price rationed. This condition is reached in the following way. For both crops the positive sign of the mean and skewness coefficients indicates that $U'_R > U'_P$ and $U''_R > U''_P$. This is equivalent to $U'_R > U'_R - \epsilon$, where $\epsilon = U'_R - U'_P$. Similarly, $U''_R > U''_R - \Delta$, where $\Delta$ is $U''_R - U''_P$. Then, a risk rationed farmer has a larger downside risk aversion coefficient if $\frac{U''_R}{U'_R} > \frac{U''_R - \Delta}{U'_R - \epsilon}$. The condition is simplified to $\frac{U''_R}{U'_R} < \frac{\Delta}{\epsilon}$. The differences in marginal utilities and in the third derivative of the expected revenue between a risk rationed and a price rationed are given in the coefficients of mean and skewness. These values are $\epsilon$ and $\Delta$ respectively. Looking at the values from Table 2, the ratio $\frac{\Delta}{\epsilon}$ for corn is 2.162; while for soybeans is 4.545. Clearly, these values are much higher than the ratio $\frac{U''_R}{U'_R}$ for any type of utility function; therefore, the condition for risk rationed to have a larger coefficient holds.

These results test the hypothesis that risk rationing is the result of a jump disutility of a borrower. This jump creates a probability shift towards the left of the distribution which in turn increases downside risk. When the jump disutility is large enough, farmers would restrain from borrowing in order to avoid falling below the threshold that triggers the jump.

The positive coefficient for skewness is an indication that risk rationed prefer a revenue distribution that is more positively skewed, this includes the probability of large gains as well as the minimization of losses. In our data, we find evidence that risk rationed are more downside risk averse than price rationed since most farmers who do not use fertilizers are risk rationed. This occurs despite the higher increase in yields and revenue per hectare from utilizing fertilizers. These farmers are giving up the possibility of higher income by limiting the loss in case of disaster. Limiting downside risk becomes a dominant factor for input use.

Finally, following our model, the coefficient captures the effect of the differences of utility at the initial wealth. The sign of the coefficients tell us the difference in utility of initial wealth for the two credit rationed groups. All of the three crops have positive coefficient. This means that the utility at the initial state of wealth is larger for the risk rationed group for all crops. This may also be a reflection of being risk rationed, since they are better off at the initial state than price rationed, it is more difficult for them to want to move from that initial state. By preferring the status quo, they have more at stake when borrowing formally because that status quo is jeopardized.

The difference in utility at the initial wealth level, given by the constant term, measures the preference for current consumption. In this case, risk rationed farmers have a stronger preference for current
consumption, suggesting that their discount rate for future consumption is higher than that of price rationed.

Since risk rationing farmers do not borrow formally, by definition, they rely on informal borrowing or own saving to smooth consumption. Following Morduch’s empirical tests for complete markets we regress household expenses on informal loan as a proxy for transitory income and savings. Informal loan is negative and marginally significant (p= 0.18), and saving positive and significant (p= 0.0). These results suggest that farmers use these mechanisms to smooth consumption, and thus implying that credit markets are not complete.

Income smoothing mechanisms for risk rationed are also observed in our study. Our data shows that risk rationing farmers are much less likely to grow a high revenue- high risk crop in favor of low revenue-low risk ones. For instance, the percentage of risk rationed farmers who grow corn or sorghum, deemed as less risky crops, are about the same as price rationed farmers; however, the ration of price rationed farmers who grow soybeans, the high revenue crop, is more than double than that of risk rationed, 67 and 33%.

### Conclusion

This paper explores an alternative view to understand risk rationing among farmers in the context of a developing country. This new approach is formulated from Mason’s disutility jump and the implications of the jump in the preference for skewness, which imply a large downside risk aversion. In this paper we estimated farmer’s expected revenue distribution through means of simulation a Pert distribution with three parameters that we observed from a previous survey in Mexico. Using the simulated distributions’ moments we performed a random utility analysis between people classified as risk rationed and price rationed. Risk rationing preference depends on expected distribution, skewness preference and on discount rate.

Following the jump disutility theory proposed by Mason, we created a model where this jump disutility refers to the collateral loss that risks rationed avoid by not entering into the formal credit market. From there we have two possible distributions of future revenue, one under debt and one without debt. A formal debt would create, or increase, a disutility jump at a given revenue threshold, which if reached, triggers the jump disutility in the form of collateral loss. Reaching the threshold level of revenue is avoided at all costs under risk rationing. On the other hand, the distribution of future revenues without debt does not have this significant disutility jump. Thereby, making that distribution less negatively skewed compared to under risk rationed. Risk rationed have a stronger preference for skewness, and their expected revenue distribution is less positively skewed, or more negatively skewed, than that of price rationed.

Through a random utility model analysis, we estimated the preferences for expected revenue moments, and for present consumption. By looking at the differences in utilities and their derivatives between credit groups, given by the coefficients of the random utility model, we estimated that risk rationed have a higher discount factor that price rationed. This suggests that risk rationing can also be explained by the preference of current consumption. Price rationed, on the other hand, sacrifices current consumption in hopes to have a larger future consumption. This is measured by the difference between utility at initial wealth level. This difference is the constant term in the regressions.
The expected revenue moments were estimated by Monte Carlo analysis. From the results of a 2011 survey of Mexican grain farmers we obtained their minimum possible, maximum possible and most likely revenue for their next season. With these three parameters we simulated a Pert distribution for each farmer. Pert distribution is used because it is less sensitive to extreme values unlike the triangular distribution. From these distributions we obtained their first three moments. These moments were used as independent variables for our random utility analysis. This approach of simulating expected distributions based on farmers’ responses have not been fully exploited in the literature.

A novel approach to classifying credit rationing is also done in this study. Unlike previous research that establish risk rationing status a posteriori, here we use a survey specifically design to unambiguously classify credit rationing status (figure 8). Using this method, we eliminate potential problems of endogeneity. For instance, observing no credit demand does not imply risk rationing, it can be a price rationed that does not demand credit at current interest rates, or it may be a quantity rationed person, someone who was rejected from a formal loan.

The results in this paper test the hypothesis that risk rationed farmers face a jump disutility in case they borrow formally. This jump is trigger by revenues falling below a threshold level. The consequence of the jump disutility model is that risk rationed farmers are more downside risk averse than price rationed. This is tested empirically for all crops. Risk rationed farmers also have a larger risk aversion coefficient than price rationed; however, price rationed farmers have larger prudence coefficients than risk rationed.

Although we believe that the analysis of risk rationing through the use of a random utility model shed lights on time preference of consumption, skewness preference, and degree of risk aversion and downside risk aversion, we believe that there are other approaches to understanding risk rationing. One approach that is the topic of future inquiry is in the application of the Dual Process theory. Farmers’ response to the risk of losing collateral by not borrowing needs to be examined through the interaction of emotions and calculative probabilities of default. This is a natural extension of this topic that can provide further insights on the relationship between risk preferences and emotions.

Appendix

Figure 8. Schematic of the survey to classify according to credit rationing
References

Astrebo, T., Mata, J., & Santos-Pinto, L. (2009). Preference for skew in gambling, lotteries and entrepreneurship, mimeo


