Marketing Margins and Input Price Uncertainty

Josh Maples
Maples.msu@gmail.com

Ardian Harri
(662) 325-5179
Harri@agecon.msstate.edu

John Michael Riley
(662) 325-7986
Riley@agecon.msstate.edu

Jesse B. Tack
(662) 325-7999
Tack@agecon.msstate.edu


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Introduction

Increased volatility of agricultural commodity prices as well as market linkages between the agricultural and energy markets expose producers to different types of systematic price risk (Schweikhardt, 2009 and Harri, Nalley, and Hudson, 2009). Increased market volatility in recent years can be attributed to various factors including reduced government stockholding, adverse weather events, and government policies that have led to greater demand for agricultural products. This has led to increased variability in prices (Schweikhardt, 2009). As an example, the Renewable Fuel Standard has been shown to have created strong linkages between agricultural commodity prices and energy prices (Harri, Nalley, and Hudson, 2009).

Specifically, commodity prices have been shown to be linked to biofuels production (Trostl, 2008). This interdependence among markets has increased uncertainty and volatility in the agricultural cash and futures markets as commodity markets, especially biofuel feedstocks such as corn, seem to be importing price volatility from the biofuels sector (Hertel and Beckman, 2011).

Producers that operate on margins involving both input and output price uncertainty are perhaps the most adversely affected by these volatility changes. One such industry is the beef cattle finishing industry, which is the focus of this research. Beef cattle finishers purchase 700 to 850 lbs. cattle referred to as feeder cattle and use predominantly grain based-feeds to add the weight needed to sell the cattle to meat processors as fed cattle (Anderson and Trapp, 2000). Thus, the margin for finishers is the difference between the price they receive for fed cattle and the price at which they purchase those same animals as feeder cattle. Due to the several month feeding periods, the margin for finishers is complicated by not only the difference in form
between input and output, but also the dynamic behavior of prices throughout the feeding process.

The margins are typically very thin as finishers compete in both the input and output markets (Lawrence, Wang and Loy 1999). The aforementioned long feeding (production) periods increase risk exposure and margin uncertainty. Decision makers form expectations over the output price at the time they choose to purchase the cattle entering their production process and depend on these prices to hold steady or increase over the time they feed the cattle in order to make a profit. This output price uncertainty implies margin uncertainty for finishers.

However, output price risk is not the only variable contributing to this margin uncertainty. The cost of feed has a significant effect on margins for finishers. Again due to the long feeding periods, finishers also must form expectations over the amount it will cost to feed the cattle they purchase. Under the assumption that finishers do not perfectly hedge the cost of feed at the time they purchase cattle, feed price uncertainty also influences margin uncertainty. More specifically, finishers face input price uncertainty throughout their feeding periods. The cost of feed is directly linked to the price of corn as most feed used is corn based (Anderson and Trapp 2000, Dhuyvetter, Schroeder, and Prevatt 2001). Further, Schroeder et al. (1993) showed that between 60 and 72 percent of the variability of feeding cost of adding weight to cattle in feedlots can be attributed to the variability of the price of corn. This implies that the previously mentioned increased volatility of the price of corn is causing increased uncertainty of the price of feed, and thus, increased margin uncertainty for finishers.

Therefore, cattle finishers operate on margins that are complicated by not only the dynamic behavior of output prices, but also that of input prices. A better understanding of these dynamics would be beneficial to cattle finishers concerning effective production decisions,
reduction of costs, and increased profitability especially in times of increased commodity price volatility. The purpose of this research is to analyze the effect of input price uncertainty on expected margins for firms operating under output and input price uncertainty. A theoretical and empirical model will be developed to analyze these effects. Such research would provide a more accurate representation of the risk associated with beef cattle finishing operations.

The contribution of this research relates to both the conceptual model and empirical model decision-making under both output and input price uncertainty. Conceptually, a general model for firms operating on a margin with risk present in both the input and output pricing channels will be introduced. Empirically, hypotheses from the conceptual model will be tested. The combination of these two will allow for new theoretical framework in the study of decision-making under both output and input price uncertainty. Further, the methods and results of this research could possibly be used to develop an instrument for use by cattle finishers to better estimate their expected margins.

**Literature Review**

This section starts with a presentation of the literature concerning the development of a theoretical framework for firms operating on a margin under output price uncertainty. Then, the literature focusing on the empirical analysis of beef cattle production and the analysis of firms operating on margins under price uncertainty is discussed.

*Theory of the competitive firm operating on a margin under output price uncertainty*
The literature pertinent to the development of a theoretical model for the competitive firm operating on a margin under output price uncertainty begins with Sandmo’s (1971) seminal paper concerning theory of the competitive firm under output price uncertainty. Sandmo (1971) showed that under price uncertainty, a risk averse firm produces less than they would in the case of output price certainty. Ishii (1977) extended Sandmo’s (1971) findings by showing that if the firm exhibits decreasing absolute risk aversion (DARA), an increase in output price uncertainty is associated with a lower quantity of output. Batra and Ullah (1974) showed that the risk averse firm uses less input, and thus produces less output than it would in the case of output price certainty. Further, Batra and Ullah (1974) showed that under DARA preferences and when inputs have a complementary relationship, an increase in input price is associated with a decrease in quantity of output produced for a competitive firm facing output price uncertainty.

Brorsen et al. (1985) established the link between theory of the competitive firm under output price uncertainty and theory of the competitive firm operating on a margin under output price uncertainty. They utilized findings from Sandmo (1971), Ishii (1977) and Batra and Ullah (1974) to show that an increase in output implies an increase in expected margin given DARA preferences and that an increase in output price uncertainty implies an increase in expected margin. Further, they showed that under Batra and Ullah’s (1974) assumptions, an increase in input price implies an increase in expected margin.

Empirical analyses of beef production and marketing margins

Empirical studies on the beef cattle finishing industry have generally focused on factors influencing actual profits or observed margins. Studies by Mark, Schroeder, and Jones (2000) and Lawrence, Wang, and Loy (1999) each used a single equation model to estimate variables
influencing actual profits for cattle finishers. Both studies found that the output price is positively related with profit and the prices of inputs feeder cattle and corn have a negative relationship with profit. Similarly, Langemeier, Schroeder, and Mintert (1992) showed that changes in fed cattle prices and corn prices contributed about 50 percent and 22 percent to the variability in profits for finishers, respectively.

Dhuyvetter, Schroeder, and Prevatt (2001) and Brorsen et al. (2001) analyzed price relationships in the finishing stage of beef production by studying the relationships of prices of cattle in different weight classes. This relationship is commonly referred to as the price slide in the agricultural economics literature and is defined as the difference in prices between cattle in different weight classes at a single moment in time. In general, cattle are worth less per pound as their weight increases due to the decreasing cost of feeding needed for that particular animal. The potential profit from adding weight to cattle is, thus, bid into their price. Dhuyvetter, Schroeder, and Prevatt (2001) showed that a decrease in the price of corn increases the price slide due to a decreased cost of feeding cattle.

The bulk of the literature focusing on margins in the beef marketing channel has analyzed observed margins around the processing stage of beef production. Azzam and Anderson (1996) and Marsh and Brester (2004) analyzed observed farm to wholesale and wholesale to retail beef margins, respectively. Wohlgenant and Mullen (1987) analyzed the farm to retail margin which is defined as the difference between retail price of beef and fed cattle price. They showed that this margin can be simultaneously influenced by both supply and demand factors. Further, Wohlgenant (2001) states in his summary of marketing margins that “in general, the retail and farm prices and quantities, as well as the marketing margin, are jointly determined by the
exogenous shifters of the underlying demand and supply functions” (page 939). This implies that the use of a system including a supply and demand equation is more appropriate when modeling margins in the beef industry as opposed to a single margin equation. The movement of beef from processors to retail occurs over a rather short period of time and is, therefore, not exposed to the long production periods inherent to finishing operations.

The empirical studies of margins discussed thus far have each analyzed observed margins instead of expected margins. Brorsen et al. (1985) specified a single equation model for expected margins in the U.S. wheat industry. They found that an increase in output price uncertainty implies an increase in expected margin and that quantity marketed is positively related to expected margin. Holt (1993) applied Brorsen et al.’s (1985) work on margins under output price uncertainty to the beef production industry and analyzed expected farm to retail margins. Similar to Brorsen et al. (1985), Holt (1993) assumes firms form rational expectations of output price risk at the time when the raw input is purchased. Holt (1993) differs from Brorsen et al. (1985) in the way he calculates the expected margin to be used. Whereas Brorsen et al. (1985) used the annual average of the previous 12 monthly observed margins, Holt (1993) uses a time series model to forecast retail beef price which is the output price in his model. The expected margin is then calculated as the difference between the known farm beef (fed steer) price and expected retail beef price. Holt (1993) was the first study to model marketing margins in a system framework that includes a demand and supply equation. Like Brorsen et al. (1985), Holt (1993) found that expected margins have a positive relationship with output price uncertainty.
Wohlgenant (2001) provides insight for model specification in this study of marketing margins. Wohlgenant (2001) states that margins are most influenced by retail demand shifters, farm supply shifters, and changes in marketing costs. Marsh and Brester (2004) found similar results. Specific demand shifters include prices of related goods, income changes and population changes. Wohlgenant (2001) specifically notes the absence of adequate demand shifters in the studies by Holt (1993) and Brorsen et al (1985), and warns that without the proper specifications of demand, the influence of output price risk could be falsely signaling the influence of demand shifters.

Harri, Anderson, and Riley (2010) were the first to capture the dynamics of beef cattle finishing margins that include the long production periods previously stated. They developed a structural model for the finishing stage of beef cattle production which includes an equation for expected margin, demand, and supply. Expected margins for the finishing stage are calculated using monthly data from cash input prices and prices of futures contracts expiring five months from the current time period. Harri, Anderson, and Riley’s (2010) empirical results showed that the price of corn is positively related with expected margin for finishers and output price uncertainty is positively related with expected margin.

**Conceptual Model**

We assume that firms operate in competitive markets and that the decision makers form rational expectations of price and price risk as in Holt (1993). To determine the effects of input price and input price uncertainty on expected margins we follow the approach of Batra and Ullah (1974) and Brorsen et al. (1985). We assume that firms are price takers. The firm’s production function is given by

7
(1) \[ y = f(x, z) \]

where \( y \) is output, \( x \) is a raw material input and \( z \) is a vector of other inputs. We further assume that the marginal products of the inputs, \( f_x \) and \( f_z \), respectively, are positive. However, different from Batra and Ullah (1974) and Brorsen (1985) we assume that firms face an uncertain output price \( p \) and an uncertain price \( l \) for one input, \( z_1 \in z \). Decision makers seek to maximize the expected utility of the firm’s wealth. The objective function of the decision maker is then

(2) \[ \text{Max}_{x,z} EU[w + py - rx - lz_1 - m'z_- | y = f(x, z)] \]

where \( w \) is initial wealth, \( r \) is the price of the raw input, \( z_- \) is a vector of remaining inputs excluding input \( z_1 \) and \( m \) is a vector of prices for the inputs in \( z_- \), \( U_w = \partial U / \partial w > 0 \) and \( U_{ww} = \partial^2 U / \partial w^2 \geq 0 \) depending whether the decision maker is risk averse, risk neutral or risk preferrer. We assume that \( r \) and \( m \) are known at the time of the production decision while \( p \) and \( l \) are random variables with respective density functions \( f(p) \) and \( f(l) \) and means \( E(p) = \mu_p \) and \( E(l) = \mu_l \). Further, \( p \) and \( l \) are assumed to be nonnegative.

We can denote the optimal solutions as \( x^*, z_1^*, z_-^* \), and \( y^* \) corresponding to the optimization problem in (2). These are the risk-responsive input demand and output supply functions with each being a function of \( w, m, r \) and also the probability distributions of the output price \( p \) and the input price \( l \). Thus the optimal solutions above can be expressed as

\[
x^* = (w, r, m, \mu_p, \mu_l, \sigma_p, \sigma_l)
\]

\[
z_1^* = (w, r, m, \mu_p, \mu_l, \sigma_p, \sigma_l)
\]

\[
z_-^* = (w, r, m, \mu_p, \mu_l, \sigma_p, \sigma_l)
\]
\[ y^* = (w, r, m, \mu_p, \mu_l, \sigma_p, \sigma_l) \]

where \( \sigma_p \) and \( \sigma_l \) represent the second (and possibly higher) moments of the subjective probability distributions of \( p \) and \( l \).

Following Brorsen’s (1985) approach on specializing model (2) in the context of a price-taking firm operating in a vertical channel, we will make two restrictive assumptions. The first is we assume the production function (1) is weakly separable and thus can be written as \( y = g[x, h(z)] \). The second assumption strengthens the above separability by assuming that function \( g \) has a Leontief production function property that \( y \) is produced in fixed proportions from the raw input \( x \) (the other inputs \( z \) are used in variable proportions)

\[
(3) \quad y = \min\{x/k, h(z)\}.
\]

Modifying (2) to obtain

\[
(4) \quad \max_{x,z} \{EU[w + py - \min(rx + lz_1 + m'z_-)]\} \quad \text{s.t. (3)}
\]

will allow us to decompose the maximization into two stages: a cost minimization problem under uncertainty and an expected utility maximization problem.

The general cost minimizing input demand functions are denoted as \( x^+(r, l, m, y) \), \( z_1^+(r, l, m, y) \), and \( z_-^+(r, l, m, y) \). The indirect cost function for positive prices has the form

\[
(5) \quad C(r, l, m, y) = rx^+ + lz_1^+ + m'z_-^+ = rky + c_1(l, y) + c_-(m, y).
\]

\( C \) is linear homogeneous and increasing and concave in prices \( (r, l, m) \). It is convex in output \( y \). From the envelope theorem, we can derive
Using the results from (6a), (6b), and (6c), the second stage of the expected utility maximization problem in (4) becomes

\[(6a) \quad \frac{\partial C}{\partial r} = x^+ \]

\[(6b) \quad \frac{\partial C}{\partial l} = \frac{\partial c_1}{\partial l} = z_1^+ \]

\[(6c) \quad \frac{\partial C}{\partial m} = \frac{\partial c_-}{\partial m} = z_-^+. \]

Using the results from (6a), (6b), and (6c), the second stage of the expected utility maximization problem in (4) becomes

\[(7) \quad Max_y EU \ [w + py - rx^+ - lz_1^+ - m'z_-^+]

= Max_y EU \ [w + (p - kr)y - lz_1^+ - m'z_-^+]. \]

Let us now define the effective margin as \(M = p - kr\). Substituting this into (7) gives

\[(8) \quad Max_y EU \ [w + My - lz_1^+ - m'z_-^+] \]

which is an expected utility maximization problem under margin uncertainty. The solution to (8) will be the risk responsive supply function \(y^* = (w, r, m, \mu_p, \mu_l, \sigma_p, \sigma_l)\) or, using our definition of effective margin, \(y^* = (w, M, \tilde{\mu}, \mu_l, \sigma_p, \sigma_l)\) where \(\tilde{M} = E(M)\). Russell et al. (1998) show that no restrictions, and in particular the convexity of technology sets, are required in the aggregation of individual supplies of price-taking producers. In other words, “there is no loss of generality in positing the existence of a “representative producer”, which generates aggregate net-supply functions by maximizing aggregate profit subject to the constraint that the aggregate net-supply vector be contained in the aggregate technology set.” (p. 178). Russell et al. (1998) further state that “As a result, the Jacobian of the system of aggregate net supply functions has the same
properties as those of individual producers.” (p. 178). Therefore, the aggregate supply for the industry defined as \( Y^* = Y(w, m, r, \bar{M}, \mu, \sigma_p, \sigma_l) \) has the same properties as the firm supply \( y^* \).

Inverting the aggregate supply function gives the expected margin as the dependent variable

\[
(9) \quad \bar{M} = \bar{p} - r k = M(w, m, \mu, \sigma_p, \sigma_l, Y).
\]

This will allow us to better analyze the behavior of the expected margin under risk aversion.

We use methods similar to those in Sandmo (1971), Batra and Ullah (1974), Ishii (1977), and Brorsen et al. (1985) to derive the theoretical implications of model (9) in which we are interested. Brorsen et al. show that under DARA \( \partial \bar{M}/\partial l > 0 \). In other words, as input price increases the expected margin will also increase. Determining the sign of the partial derivative (10) below allows us to identify the impact input price uncertainty has on expected margin.

\[
(10) \quad \frac{\partial \bar{M}}{\partial \sigma_l} \leq 0
\]

The analytical solution to this partial derivative is provided in Appendix A.

**Determining the effect of input price uncertainty on expected margin**

Following Sandmo (1971) and Batra and Ullah (1974) we identify first the effect of the “overall uncertainty” or letting the input price \( l \) be stochastic rather than constant. Next we identify the effect of an “increase in uncertainty” in the form of a mean preserving change in the probability distribution of \( l \) as in Rothschild and Stiglitz (1970).
In Appendix A we show that the optimal quantity demanded of the input whose price is uncertain is lower than in the certainty case. It follows from this that the optimal output produced under uncertainty will also be lower than in the certainty case.

An important observation related to the “overall uncertainty” of the input price is as follows. The previous result was derived under the condition $p = \mu_p$. Batra and Ullah (1974) identify the effect of “overall uncertainty” with respect to the output price, $p$, to the optimal input quantity demanded and output level. They also found that under output price uncertainty both the optimal input quantity demanded and the optimal output produced are lower than in the certainty case. Thus the direction of the effect of both input and output price uncertainty on the optimal input quantity demanded and the optimal output produced is the same. This implies that under both input and output price uncertainty, the optimal input quantity demanded and the optimal output produced will be lower than under only input or output price uncertainty.

To derive the effect of the “increase in uncertainty” of input price on expected margin we first need to derive the effect of the “increase in uncertainty” of input price on the optimal quantity demanded of the input. Appendix A derivations show that $\frac{\partial z_1}{\partial \sigma_l} < 0$.

Thus, an “increase in the uncertainty” with respect to the input price has the same effect as the introduction of the “overall uncertainty” with respect to the input price, a reduction in the optimal quantity demanded of the input. It follows that the optimal output produced under increased input price uncertainty will also be decreased. This implies that

\[
(11) \quad \frac{\partial y}{\partial \sigma_l} < 0 \text{ and } \frac{\partial y^*}{\partial \sigma_l} < 0.
\]
To determine the effect of input price uncertainty on the margin, from (A9) and using the result in (11) we have

\[
\frac{\partial \bar{M}}{\partial \sigma_t} = - \left[ \frac{\partial Y^*}{\partial p} \right]^{-1} \frac{\partial Y^*}{\partial \sigma_t} > 0
\]

which shows that an increase in input price uncertainty will lead to an increase in expected margin.

**Data Description**

The data used to estimate the model are monthly time-series data from January 1990 through June 2012. Cash prices for feeder cattle are obtained from the *Oklahoma National Stockyards, Feeder Cattle Weighted Average* report of cash prices reported by United States Department of Agriculture’s (USDA) Agricultural Marketing Service (AMS) for Oklahoma City\(^1\) (USDA, AMS). Cash fed cattle prices are obtained from AMS’s *Five Area Daily Weighted Average Direct Slaughter Cattle* report (USDA, AMS). The Chicago cash price for corn is from AMS’s *Weekly Feedstuff Wholesale Prices* report (USDA, AMS). Wholesale beef prices are from AMS’s *National Weekly Boxed Beef Cutout and Boxed Beef Cuts* report\(^2\) (USDA, AMS). Wholesale price of pork is a monthly average of daily data from the *National Daily Direct Hog Prior Day Report - Slaughtered Swine* reported by AMS (USDA, AMS).

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\(^1\)Livestock Marketing Information Center (LMIC) calculates monthly weighted average cash prices for 100 lb. categories. The category used is the 700-800 lb. category.

\(^2\)For this analysis, monthly observations are calculated from weekly data from January 1990 through January 1999 while daily data is used from February 1999 through June 2012.
Quantity of beef production is federally inspected beef production in pounds from the National Agricultural Statistics Service’s (NASS) *Livestock Slaughter* monthly report (USDA, NASS). The quantity of cattle placed on feed (placements) is from NASS’s monthly *Cattle on Feed* report\(^3\) (USDA, NASS). The farm wage rate and fuel price index are each obtained from NASS. The consumer price index (CPI) is obtained from the Bureau of Economic Analysis’s (BEA) *Price Indexes for Personal Consumption Expenditures by Major Type of Product, Monthly* report (BEA). Personal disposable income is obtained from BEA’s *Personal Income and Its Disposition, Monthly* report (BEA).

The expected margin for the finishing stage is calculated as the difference between the live cattle futures contract maturing five months from the current month and the cash price in the current month for 700-800 lb. feeder cattle. Implied price volatilities for live cattle and corn are obtained from the Commodity Research Bureau (CRB). These implied volatilities are calculated using the options and futures markets and capture expected price volatility. Finishing basis is defined as cash price of fed steers minus nearby live cattle futures price. The historical volatility of finishing basis is calculated using the method described in the appendix. Unlike implied volatilities, historical volatilities are calculated using observed volatilities of basis for a number of days. Futures prices for live cattle and feeder cattle are obtained from CRB. All prices are deflated by the CPI.

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\(^3\) For the analysis of placements, the quantity has been converted from number of head placed on feed to number of pounds placed on feed. This was achieved by multiplying the number of head placed on feed for each weight group by the midpoint for each weight category and also adjusting for seasonality.
Empirical Model

Following Harri, Anderson, and Riley (2010) and Holt (1993), the empirical model consists of a system of three equations to be estimated simultaneously. The system includes an equation for demand, supply, and the expected margin for the finishing stage of beef cattle production. Further, the conditional variance dynamics for each equation are specified using GARCH processes.

The demand equation for the finishing stage can be considered fed cattle demand and is specified in price dependent form as

\[
\Delta p_F^t = a_0 + a_1 \Delta p_{BB}^t + a_2 \Delta p_{BB}^{t-1} + a_3 \Delta p_{BB}^{t-2} + a_4 \Delta p_{BB}^{t-3} + a_5 \Delta p_{BB}^t + a_6 \Delta PDI_t + a_7 \Delta Q_F^t + a_8 \Delta Q_{t-1}^F + a_9 \Delta Q_{t-2}^F + a_{10} \Delta Q_{t-3}^F + a_{11} ZP_F^t + a_{12} herf + \sum_{i=1}^{11} a_{12+i} month_i + \sum_{j=1}^{12} a_{23+j} \Delta P_{t-j}^F + \epsilon_{1t}
\]

where \( \Delta \) is the first difference operator. Dickey-Fuller (1979) and Phillips-Perron (1988) tests indicate the presence of unit roots for \( P^F, P_{BB}, P^p \) and \( PDI \). Thus, first differences are used and the error correction term \( ZP^F \) is included to capture the cointegrating relationship between the price of fed steers, price of boxed beef, price of pork and personal disposable income. \( P^F \) is the five-area weighted average live weight price for fed steers in dollars per pound and \( Q^F \) is the quantity of beef slaughtered in hundreds of millions of pounds. \( P_{BB} \) is the wholesale value of boxed beef in dollars per pound. Lagged values of \( Q^F \) and \( P_{BB} \) are included to capture responses to price and quantity changes over time. The cash price of pork is denoted as \( P^p \) and is measured in dollars per pound. \( PDI \) is personal disposable income in dollars. The variable \( herf \)
is the Herfindahl index for the finishing industry and our calculation of this variable is explained in Appendix B. Dummy variables for months January through November and are included as \( month_i \), where \( i = 1, \ldots, 11 \) to capture seasonality effects. Parameters to be estimated are \( a_0, \ldots, a_{35} \).

The supply equation for the finishing stage is fed cattle supply and is specified as

\[
Q_t^F = \beta_0 + t_2 \text{trend} + \beta_1 P_{t-5}^B + \beta_2 Q_t^B + \beta_3 P_t^C + \beta_4 P_{t-5}^C + \beta_5 FWR_{t-5} + \\
\beta_6 FUEL_{t-5} + \beta_7 \sigma_{t-5}^F + \beta_8 \sigma_{t-5}^C + \beta_9 D \sigma_{t-5}^C + \sum_{i=1}^{11} \beta_{10+i} month_i + \\
\sum_{j=1}^{12} \beta_{22+j} Q_{t-j}^F + \varepsilon_{2t}
\]

where \( P^B \) is the Oklahoma City cash price in dollars per pound for feeder steers in the 700-800 lb. weight range. \( Q^B \) is quantity of cattle placed on feed in hundreds of millions of pounds. The Chicago cash price for corn in dollars per bushel is denoted as \( P^C \) and captures supply responses near the end of the production period and at the time when the supply decision is made. \( FWR \) is the farm wage rate and \( FUEL \) is an index for fuel prices paid. The implied volatility of fed cattle price is denoted as \( \sigma^F \) and is used to capture fed cattle price uncertainty. The implied volatility of the price of corn used to capture feed price uncertainty is denoted as \( \sigma^C \) and \( D \) is an indicator variable which takes on a value of 1 starting in January of 2006 and a value of 0 otherwise. This indicator variable is included to capture the changing dynamics of corn price implied volatility after 2006. Other variables are as previously defined. A trend variable, \( \text{trend} \), is used to capture effects of variables like wealth and technology and \( \beta_0, \ldots, \beta_{33} \) are the parameters to be estimated.
The third equation for the finishing stage of beef cattle production is the expected margin equation and is specified as

\[
\bar{M}_{t}^{E} = \gamma_{0} + t_{3} trend + \gamma_{1} Q_{t}^{E} + \gamma_{2} P_{t}^{C} + \gamma_{3}(P_{t}^{C})_{t}^{2} + \gamma_{4} FWR_{t} + \gamma_{5} FUEL_{t} + \gamma_{6} \sigma_{t}^{E} + \\
\gamma_{7} \sigma_{t}^{C} + \gamma_{8} D \sigma_{t}^{C} + \gamma_{9} \sigma_{t}^{FB} + \sum_{i=1}^{11} \gamma_{9+i} month_{i} + \sum_{j=1}^{12} \gamma_{20+j} \bar{M}_{t-j}^{E} + \varepsilon_{3t}
\]

where \( \bar{M}_{t}^{E} = w_{1} E_{t}[P_{t+5}^{F}] - w_{2} P_{t}^{B} \). The average weights we use for fed cattle and feeder cattle are denoted as \( w_{1} \) and \( w_{2} \), respectively and are defined as \( w_{1} = 1200 \) lbs. and \( w_{2} = 750 \) lbs. \( E_{t}[P_{t+5}^{F}] \) is the price of the futures contract for live cattle with a maturity date five months or longer from the current time period. The squared term of the price of corn denoted as \((P_{t}^{C})_{t}^{2}\) is included to account for the presence of non-linearity in the relationship between expected finishing margin and the price of corn. The implied volatility of the price of corn is denoted as \( \sigma^{C} \). The historical volatility of finishing basis, \( \sigma^{FB} \), is included to account for the finishing operations choosing to manage the output price risk of their cattle (Gardner, 2001). Finishing basis is defined as the nearby live cattle futures price subtracted from fed cattle cash price and the calculation of the historical volatility for the finishing basis are similar to those used by CRB. Other variables are as previously defined. Parameters to be estimated are \( \gamma_{0}, \ldots, \gamma_{32} \).

GARCH(1,1) processes were used to specify the conditional variance dynamics for each of the equations above. Autocorrelations tests of the residuals for each equation indicate no signs of autocorrelation. The conditional variances are specified as

\[
h_{iit} = \kappa_{i0} + \eta_{i1} \varepsilon_{iit-1}^{2} + \psi_{i1} h_{iit-1},
\]

\[
h_{ijt} = \rho_{ij}(h_{iit} h_{jyt})^{1/2}
\]
\( i, j = 1 \left( P^F \right), 2 \left( Q^F \right), 3 \left( \bar{M}^F \right), i \neq j. \)

We use the full information maximum likelihood (FIML) estimation to estimate the six equation system. The log likelihood for observations \( t = 1, ..., T \) and equations \( n = 1, ..., N \) is specified as

\[
(17) \quad \ln L(\Theta) = -\frac{T N}{2} \ln(2\pi) + \sum_{t=1}^{T} \ln \left( \left| \frac{\partial \boldsymbol{e}_t}{\partial \boldsymbol{y}'_t} \right| \right) - \frac{T}{2} \ln(|\boldsymbol{H}|) - \frac{1}{2} \text{tr} \left( \boldsymbol{H}^{-1} \sum_{t=1}^{T} \boldsymbol{e}_t \boldsymbol{e}_t' \right)
\]

where \( \frac{\partial \boldsymbol{e}_t}{\partial \boldsymbol{y}'_t} \) is the Jacobian of the system, \( \boldsymbol{H} \) is the matrix of the parameters for the GARCH system specified in (16), and \( \Theta \) is the matrix of the parameters in equations (13) through (16).

One key assumption of maximum likelihood estimation is that residuals are normally distributed. This assumption is expressed as \( \epsilon_{it} \sim N(0, h_{it}^2) \). We use Shapiro Wilk’s (1965) method to test whether the residuals from each equation are normally distributed.

**Results**

Parameter estimates, standard errors and \( t \) values for the fed cattle demand, fed cattle supply and expected finishing margin equations are reported in Table 1. Parameter estimates have the expected signs for the fed cattle demand equation. Parameter estimates also have the expected signs in the fed cattle supply equation except for farm wage rate which is positive although not significant at conventional levels. It is interesting to note the relationships between

\[\text{Note the absence of the } \text{herf} \text{ variable to capture market concentration as specified in the previous section. This variable was only available from 1997 through 2012. We found it to be insignificant in our model and was, thus, omitted so not to lose the observations from 1990 to 1997.}\]
quantity of fed cattle slaughtered and the price of corn. The price of corn lagged five months has a negative relationship which implies that finishers purchase fewer cattle when corn prices are high. The current price of corn has a positive relationship with quantity of fed cattle slaughtered. This implies that when corn price increases, cattle near the end of the feeding process are slaughtered sooner (i.e. pulled forward) due to an increase in feeding cost. The volatility of the price of corn is negatively related with the quantity of cattle slaughtered. This relationship is consistent with theory derived in the conceptual model. Further, the volatility of the price of live cattle is negatively related with quantity of fed cattle slaughtered. This relationship is consistent with theory (Sandmo 1971, Ishii 1977 and Brorsen et al., 1985) and also results from previous empirical work (Harri, Anderson and Riley (2010).

Quantity of fed cattle slaughtered has a positive relationship with expected finishing margin and is significant at conventional levels. This is consistent with theory (Sandmo 1971, Batra and Ullah 1977, and Brorsen 1985) and with results from previous empirical work (Brorsen 1985, and Holt 1993). The price of corn has a positive relationship with expected finishing margin and is also significant. This is consistent with theory (Brorsen et al. 1985) and results from previous empirical work (Brorsen et al. 1985 and Harri, Anderson and Riley 2010). We find that wage rates and the fuel index have positive relationships with expected finishing margin although neither is significant at conventional levels. Although not significant at conventional levels, the volatility of the price of fed cattle is positively related to expected finishing margin. This is consistent with theory (Brorsen et al. 1985) and previous empirical work (Brorsen et al. 1985, Holt 1993, and Harri, Anderson, and Riley 2010). The effect of the volatility for the price of corn prior to 2006 is very small and not significant at conventional
levels. The parameter estimate for the volatility of the price of corn after January 2006 is positive and significant. This is consistent with theory developed in this paper. The volatility of finishing basis is negatively related to expected finishing margin and significant at conventional levels.

Table 2 reports the short run elasticities associated with the key variables in each equation for the finishing stage. Specifically, a one percent increase in the volatility of the price of fed cattle decreases the quantity of fed cattle slaughtered by 0.042 percent or 4.2 million lbs. A one percent increase in quantity of fed cattle slaughtered increases the expected margin for feeders by 0.257 percent while a one percent increase in the price of corn increases expected finishing margin by 0.271 percent. The volatility of the price of fed cattle positively influences expected finishing margin but this influence is small. We find a one percent increase in the volatility of the price of corn after January 2006 increases the expected margin for finishers by 0.058 percent.

Tests of the assumptions of the maximum likelihood estimation

One key assumption of maximum likelihood estimation is that residuals are normally distributed. Table 3 reports the results from a Shapiro-Wilk normality test of the residuals of each equation. Based on the results of table 3 we fail to reject the hypothesis that the residuals are normally distributed. Table 4 reports the results of Mardia Skewness, Mardia Kurtosis and Henze-Zirkler T tests of a multivariate normal distribution. The Mardia Kurtosis and Henze-Zirkler T tests each fail to reject the null hypothesis of a multivariate normal distribution at the one percent level. Results from the Mardia Skewness test imply that the joint distribution of the
residuals expresses higher than normal levels of skewness. This relationship is significant at the five percent level but not at the one percent level. FIML also accounts for the presence of (if any) cross correlation between equations. Table 5 shows the estimated correlations between equations are very small.

Conclusions

This research develops a conceptual model and empirically tests the hypotheses derived from the conceptual model to assess the influence of input price uncertainty on firms operating on an expected margin under output and input price uncertainty. Previous research focusing on marketing margins under price uncertainty has only examined output price uncertainty. We use the study of beef cattle finishing operations as one such example of firms operating on a margin under output and input price uncertainty. Long feeding periods in this stage causes uncertainty around the price of feed, a key input. Conceptually, we develop a theoretical model showing the expected relationships between input price uncertainty and expected margin under certain risk preferences. Empirically, we account for these long feeding periods and provide estimates for the effects of factors influencing expected margins for cattle finishers. Further, we empirically test the hypothesis derived from the theoretical model using a three equation system estimated by full information maximum likelihood.

Conceptually, we show that a firm operating on a margin produces less output when faced with input price uncertainty than in the case of input price certainty. Further, we show that an increase in input price uncertainty implies an increase in expected margin under DARA preferences. Results from the empirical analysis of finishers support a decrease in quantity
produced as input price uncertainty increases. This implies that finishers are less willing to purchase cattle during times of increased input price uncertainty. Results are also consistent with previous literature in showing that an increase in output price uncertainty implies a decrease in quantity produced. This implies that finishers purchase fewer cattle when output price uncertainty increases. Empirically, we find the expected margin for the finishing stage is positively influenced by an increase in input price risk.

The first implication of this research is the addition to the literature pertaining to the effect input price uncertainty has on firms operating on expected margins. This conceptual framework can be applied to other industries where firms operating on expected margins are exposed to input price uncertainty. Second, cattle finishers can use results from this research to better understand the dynamics of the price uncertainty to which they are exposed. An instrument could be constructed to predict expected margins for finishers depending on the level of output and input price uncertainty.

Future research on this topic could utilize different data sources and also analyze the different business structures for finishers. We used national level data to show the effect of input price uncertainty on the finishing industry. Researchers with access to firm level data could apply this research to different operation sizes. Such research would be a more accurate representation of a finisher based on the capacity of their operation.
<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameter</th>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fed Cattle</td>
<td>$\alpha_0$</td>
<td>Constant</td>
<td>-0.0132***</td>
<td>0.0019</td>
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<td>Demand</td>
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<td>0.0013</td>
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<td>Demand</td>
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<tr>
<td>Demand</td>
<td>$\alpha_8$</td>
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<td>0.0012</td>
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<td>0.0012</td>
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<td>Demand</td>
<td>$\alpha_{24-35}$</td>
<td>$\Sigma \Delta P_{F,t-i}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Fed Cattle</td>
<td>$\kappa_{10}$</td>
<td>Constant</td>
<td>0.0001**</td>
<td>0.0000</td>
<td>2.50</td>
</tr>
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<td>Conditional</td>
<td>$\eta_{II}$</td>
<td>$\varepsilon_{lt-1}$</td>
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<td>0.0887</td>
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<td>Supply</td>
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<tr>
<td>Supply</td>
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<td>Supply</td>
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<td>Supply</td>
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<td>$\sigma_{t-5}^C$</td>
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</tr>
<tr>
<td>Expected</td>
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<td>Margin</td>
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<td>Log Likelihood</td>
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<td>0.3404</td>
<td>0.41</td>
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</table>

Note: ***, **, * denote significance at the 1%, 5%, and 10% levels respectively.
Table 2. Estimates of Short-Run Elasticities for Finishing Operations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Variable</th>
<th>Elasticity</th>
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<tr>
<td>Fed Cattle</td>
<td>$p^{BB}_t$</td>
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<tr>
<td>Demand</td>
<td>$Q^F_t$</td>
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<tr>
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<td>$p^P_t$</td>
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<td>$PDI_t$</td>
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<tr>
<td>Fed Cattle</td>
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<tr>
<td>Supply</td>
<td>$p^C_t$</td>
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<td>Equation</td>
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<td>Probability</td>
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<tr>
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<tr>
<td>Fed Cattle Demand</td>
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<tr>
<td>Fed Cattle Supply</td>
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<td>Expected Finishing Margin</td>
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<table>
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<tr>
<th>Equation</th>
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<tr>
<td>Mardia Skewness</td>
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<td>Mardia Kurtosis</td>
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<td>Henze-Zirkler T</td>
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<th>Fed Cattle Supply</th>
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<td>Finishing Margin</td>
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<td>1.000</td>
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Appendix A

Determining the effect of input price uncertainty on expected margin

From (9) we have

\[ \frac{\partial \tilde{M}}{\partial l} = - \left[ \frac{\partial Y^*}{\partial \bar{p}} \right]^{-1} \frac{\partial Y^*}{\partial l}. \]

Following Sandmo (1971) and Batra and Ullah (1974) we distinguish between the “overall uncertainty” or letting the input price \( l \) be stochastic rather than constant, and an “increase in uncertainty” in the form of a mean preserving change in the probability distribution of \( l \) as in Rothschild and Stiglitz (1970).

To determine the effect of “overall uncertainty” we identify the change in the optimal quantity demanded of input \( z_1 \) and the optimal level of output, \( y \), when input price \( l \) is stochastic compared to the case when input price \( l \) is known with certainty. We start with the profit function, \( \pi \), defined as:

\[ \pi = py - l z_1 - rx - m'z_. \]

Taking expectations on both sides of (12) gives \( E[\pi] = \mu_p y - \mu_l z_1 - (rx + m'z_) \) or \( E[\pi] = \mu_p y + \mu_l z_1 = -(rx + m'z_) \). Substituting for the expression \( -(rx + m'z_) \) in (A2) we get \( \pi = py - l z_1 + E[\pi] - \mu_p y + \mu_l z_1 \) or \( \pi = E[\pi] + (p - \mu_p) y - (l - \mu_l) z_1 \). Batra and Ullah (1974) identify the effect of effect of “overall uncertainty” with respect to output price \( p \) on \( z_1 \) and \( y \). Given our focus on the effect of “overall uncertainty” with respect to input price \( l \), we set \( p = \mu_p \). This simplifies the previous equation to \( \pi = E[\pi] - (l - \mu_l) z_1 \). For a strictly concave
utility function ($U''(\pi) < 0$), then $U'(\pi) \leq U'(E[\pi])$ for $l \geq \mu_i$. Multiplying through by $l - \mu_i$,
$U'(\pi)(l - \mu_i) \leq U'(E[\pi])(l - \mu_i)$. This inequality holds for all $l$. Taking expectations on both
sides, we obtain

\[ (A3) \quad E[U'(\pi)(l - \mu_i)] \geq 0 \]

given that $U'(E[\pi])$ is a given number. On the other hand, the first-order condition of (4) with
respect to $z_1$, is

\[ (A4) \quad \frac{\partial E[U]}{\partial z_1} = E[U'(\pi)pf_{z_1}] - E[U'(\pi)l] = 0. \]

We can also write (A4) as $E[U'(\pi)pf_{z_1}] = E[U'(\pi)l]$. Subtracting $E[U'(\pi)\mu_i]$ from both sides,
$E[U'(\pi)pf_{z_1}] - E[U'(\pi)\mu_i] = E[U'(\pi)l] - E[U'(\pi)\mu_i]$ or $E[U'(\pi)(pf_{z_1} - \mu_i)] =
E[U'(\pi)(l - \mu_i)]$. Given (A3), this implies that

\[ (A5) \quad E[U'(\pi)(pf_{z_1} - \mu_i)] \geq 0. \]

Further, given that marginal utility is always positive, then $pf_{z_1} \geq \mu_i$. This means that under
input price uncertainty, the (expected) marginal value productivity of the respective input
exceeds its expected price, or that the (expected) output price exceeds the expected marginal cost
of the input. Under price (input and output) certainty, the marginal value productivity of each
input will equal its price. Further if $f_{z_1} < 0$ (a necessary condition to satisfy the second order
conditions of utility maximization under price certainty), then the optimal quantity demanded of
the input whose price is uncertain is lower than in the certainty case. It follows from this that the
optimal output produced under uncertainty will also be lower than in the certainty case.
We define the “increase in uncertainty” as a mean preserving shift in the probability distribution of $l$ as in Rothschild and Stiglitz (1970), Sandmo (1971). Define $l' = \sigma_l l + \theta$ where $\sigma_l$ is a multiplicative shift parameter and $\theta$ is an additive one with initial values of 1 and 0 respectively. An increase in $\sigma_l$ increases both the mean and the variance of the distribution of $l$. Therefore, to preserve the mean of $l$, requires that $dE[\sigma_l l + \theta] = 0$, or $\mu_l d\sigma_l + d\theta = 0$. This implies that as $\sigma_l$ increases, $\theta$ needs to be reduced by

\[
\frac{d\theta}{d\sigma_l} = -\mu_l.
\]

(A6)

Using the new definition for the input price, the profit function is now $\pi = py - (\sigma_l l + \theta)z_1 - rx - m'z_\pi$ and the first-order condition in (14) becomes

(A7) \[ \frac{\partial E[U]}{\partial z_1} = E[U'(\pi)(pf_{z_1} - (\sigma_l l + \theta))] = 0. \]

Differentiating (A7) with respect to $\sigma_l$ gives

(A8) \[ \frac{\partial z_1}{\partial \sigma_l} = \frac{z_1 E[U''(\pi)(pf_{z_1} - l)(l - \mu_l)] + E[U'(\pi)(l - \mu_l)]}{E[U''(\pi)(pf_{z_1} - l)^2] + E[U'(\pi)pf_{z_1}]}.
\]

The purpose here is to determine the sign of (A8). Starting with the first term in the numerator, $E[U''(\pi)(pf_{z_1} - l)(l - \mu_l)] = E[U''(\pi)(pf_{z_1} - l)((l - pf_{z_1}) + (pf_{z_1} - \mu_l))] = -E[U''(\pi)(pf_{z_1} - l)^2] + E[U''(\pi)(pf_{z_1} - l)](pf_{z_1} - \mu_l)$ given that $(pf_{z_1} - \mu_l)$ is a number. $U''(\pi) < 0$ implies that the first expectation is negative. Earlier it was shown that $(pf_{z_1} - \mu_l) \geq 0$ while Batra and Ullah (1974) show that under decreasing absolute risk aversion $E[U''(\pi)(pf_{z_1} - l)] \geq 0$. Thus, the second term in the previous sum is positive. As a result, the
first term in the numerator is positive. It was also shown earlier that $E[U'(\pi)(l - \mu_t)] \geq 0$.

Therefore the numerator is positive.

$$U''''(\pi) < 0$$ implies that the first expectation in the denominator is negative. In addition, $U''''(\pi) > 0$ and $f_{z_1 z_1} < 0$ together imply that the second expectation in the denominator is also negative. As a result, the denominator is negative. Putting this all together implies that

$$\frac{\partial z_1}{\partial \sigma_t} < 0.$$  \hspace{1cm} (A9)

In other words, an “increase in the uncertainty” with respect to the input price has the same effect as the introduction of the “overall uncertainty” with respect to the input price, a reduction in the optimal quantity demanded of the input.
Appendix B

Herfindahl index for cattle finishing industry calculation

A Herfindahl index for the cattle finishing industry was calculated using the number of feedlots and inventories by size as reported annually in NASS’s Cattle on Feed Report for 1997 through 2012. Because only data on the first of January of each year were available, the differences between the number of feedlots in each size category from year to year were divided by 12 and monthly data represent a linear trend of the change in the number of feedlots each year. The number of cattle on feed by size of feedlot is also reported on the first of January each year. The percentage of total cattle on feed by size of feedlot is calculated by dividing the number of cattle on feed by size of feedlot by the total number of cattle on feed each January. Monthly estimates were calculated using the monthly number of total cattle on feed multiplied by the percentage of cattle on feed by size of feedlot each January.

The Herfindahl index was calculated using the following steps

Step 1: For each month, divide the number of cattle on feed by size of feedlot by the total number of cattle on feed for each size category.

Step 2: Divide results from step 1 by the number of feedlots for each size category.

Step 3: Multiply squared results from step 2 by the number of feedlots for each size category.

Step 4: Sum results for each size category from step 3.

Step 5: Multiply the result of step 4 by 10000 to obtain a monthly Herfindahl index for the cattle finishing industry.