Unbiased and Consistent Estimation of Risk Preferences: A Monte Carlo Simulation

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Introduction

Agents’ risk attitudes directly impact their decision making. A significant amount of effort in the literature has been devoted to estimating risk preferences from agents’ production decisions. However, whether risk preferences can be indeed recovered is being debated in the literature. We conduct a Monte Carlo experiment to investigate this issue and discuss potential factors that might affect estimation performance.

The Experiment Design

The experiment design in this study largely follows Lence’s (2009) setup with some modifications. Producers are assumed to maximize their expected utility (EU) conditional on random, end-of-period wealth:

\[
\ln(W_t) = \ln(p_t) - r_t s + W_{t-1}
\]

where $p_t$ denotes the end-of-period output price and $W_t$ output, both of which are stochastic; $r_t$ is input price vector; $W_{t-1}$, the initial wealth, is generated from $W_0 = 18.9 + 69.2z$, where the random variable $z$ falls in the interval $[0, 1]$ and follows the standard Beta distribution. The production function is:

\[
\ln(S_t) = \alpha_t a_t \ln(X_t) + \epsilon_t
\]

Parameters $\alpha_t, a_t$ and $a_t$ are set as 3, 0.2, and 0.6, respectively.

<table>
<thead>
<tr>
<th>Risk Preferences</th>
<th>Sample Size</th>
<th>Utility</th>
<th>Technology</th>
<th>with known $\gamma_1$</th>
<th>with known $\gamma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HARA</strong></td>
<td></td>
<td>$\hat{\gamma}_0$</td>
<td>$\hat{\gamma}_1$</td>
<td>$\hat{\alpha}_0$</td>
<td>$\hat{\alpha}_1$</td>
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<tr>
<td><strong>DRRA</strong></td>
<td>100</td>
<td>3.292</td>
<td>5.236</td>
<td>2.833</td>
<td>0.020</td>
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<tr>
<td></td>
<td></td>
<td>(-18.06,309.70)</td>
<td>(0.51,48.54)</td>
<td>(25.5,13.13)</td>
<td>(0.19,02.22)</td>
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<tr>
<td></td>
<td>500</td>
<td>0.011</td>
<td>2.085</td>
<td>0.201</td>
<td>0.604</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.029</td>
<td>2.661</td>
<td>0.601</td>
<td>6.021</td>
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<td></td>
<td>10,000</td>
<td>5.015</td>
<td>2.036</td>
<td>0.220</td>
<td>0.600</td>
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<td><strong>CRRA</strong></td>
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<td>6.836</td>
<td>2.835</td>
<td>0.205</td>
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<td></td>
<td>(-18.86,375.71)</td>
<td>(0.79,67.67)</td>
<td>(25.7,13.13)</td>
<td>(0.19,02.22)</td>
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<td>4.267</td>
<td>4.001</td>
<td>2.855</td>
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<td>2.901</td>
<td>3.510</td>
<td>2.861</td>
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<td>0.026</td>
<td>3.018</td>
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<tr>
<td><strong>IRRA</strong></td>
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<td>8.556</td>
<td>2.839</td>
<td>0.201</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-17.25,437.12)</td>
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<td>(25.6,13.15)</td>
<td>(0.19,02.22)</td>
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<tr>
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<td>42.870</td>
<td>5.984</td>
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</tr>
</tbody>
</table>

Estimation

Recovery of the utility function parameters is based on the following first order conditions (FOC) of the EU maximization problem:

\[
\epsilon_{y,a} = \ln(x_{y,a}) - \ln(x_{a}) - a_t \ln(x_{a}) - \alpha_t \ln(x_{y,a})
\]

(6) $\epsilon_{y,a} = \ln(Y_0 + W_{0,a}) - \ln(Y_0 + W_{a}) - \beta_t \ln(Y_0 + W_{0,a}) - \gamma_t (Y_0 + W_{0,a})^\gamma_t - \gamma_t Y_0 - \gamma_t W_{0,a}$

where $W_{0,a} = W_{0,a} + p_t x_{a} - p_t x_{a} - p_t x_{a}$, the multiplicative term $(Y_0 + W_{0,a})^\gamma_t$ in (7) is a scaling factor used to avoid the solution of $\gamma_t$ for $Y_0$ of the original FOCs.

The GMM is used to estimate parameters $[a_0, a_t, \gamma_t, \gamma_t, \gamma_t]$. Instruments used are $[1, W_{0,a}, p_t x_{a}, Y_0, Y_1]^T$.

Results and Conclusions

Two million observations were generated in each scenario (DRRA, CRRA, and IRRRA) and used in the estimation at different sample sizes. The table on the left reports the median and the 2.5% and 97.5% quantiles (in parentheses) of the results obtained from valid estimations (without singularity issues).

- All risk preference parameters in the flexible HARA utility function can be consistently estimated, though at a slower convergence rate for $\gamma_0$.
- Technology parameters can be estimated with high precision across all sample sizes (bias in DRRA is due to log transformation of distribution and can be corrected accordingly).
- The right panel of the figure below shows that the GMM objective function for $Y_1$ (for a sample of 1,000 obs.) has a steep curve, which means the algorithm will easily converge and produce estimates in a relatively small range. However, the left panel gives a fairly flat surface for a large set of $Y_0$, suggesting relatively large shifts in solutions may be produced (see wider ranges in the table). But the curve becomes much steeper at the sample size of 10,000 (not shown).
- Estimates converge faster if one parameter is set at true value (the last two columns). Parameters of the widely used power utility function (i.e., when $Y_0$ is known) can be estimated with good precision.