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Can Income Equality Increase Competitiveness?

by

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Can Income Equality Increase Competitiveness?

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Abstract

This paper explores the relationship between income distribution, prices, production efficiency and aggregate output in a decentralized search economy. We show that income distribution determines how competitive the market is, and thereby affects production efficiency and aggregate output. It is shown that it is generally possible to engineer a judicious transfer of income from high to low income individuals which simultaneously increases income equality, competitiveness, and aggregate output.

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1 Introduction

According to conventional wisdom, while income redistribution may be desirable on grounds of fairness and social equity, the downside is that it reduces efficiency by distorting incentives. Thus income equality and efficiency are often viewed as incompatible goals. In this paper we argue that, although this view may give the whole picture in the idealized setting of perfectly competitive markets, it is at least incomplete in the more realistic world of imperfect information. In particular, we argue that in the presence of informational imperfections, a transfer of income from the rich to the poor may actually boost efficiency by making markets more competitive.

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Our approach is informed by the theoretically and empirically compelling recognition, pioneered by Stigler (1961), that the need of imperfectly informed consumers to invest in search and other costly information gathering activities plays an important role in determining market structure, price formation and firm profitability.\(^1\) In particular, theoretical considerations suggest that informational frictions endow firms with market power, the extent of which depends on consumers' incentive to search for lower prices; The more motivated consumers are to search, the more pressure they exert on firms to lower prices, hence the more competitive the market is. Conversely, the lower the consumers’ propensity to search, the less competitive the market is.

The main insight of this paper is that changes in income distribution affect market structure by affecting the incentives of market participants to acquire information. Specifically, a transfer of income from rich to poor consumers affects the overall level of consumer search activity in diverse ways. On the one hand, by increasing the income of poorer individuals and enabling them to afford higher prices, the transfer reduces their incentive to search. This effect reduces competitiveness in the market. On the other hand, the transfer increases the incentive to search of wealthier individuals, whose income is reduced by the transfer. This has the opposite effect of increasing competitiveness. Hence, the effects of income distribution on competitiveness may generally be quite complex.

Nevertheless, we find that there generally exist income transfer schemes which simultaneously increase both income equality and competitiveness. Moreover, the transfer schemes which can achieve the twin goals of equality and competitiveness have natural and desirable properties and we derive a very simple rule which the policy maker can consult to determine whether a specific transfer scheme is pro-competitive.

We show that pro-competitive income transfers increases productivity and aggregate output by motivating firms to invest more efficiently. This is because the market power enjoyed by firms distorts their investment incentives and motivates them to overinvest in production capacity. And the less competitive the market is, the more severe the consequences of this distortion are. Therefore in our model, equality-increasing, pro-competitive transfers simultaneously increase income equality and output. The market-structure, information theoretic orientation of this approach to understanding the relationship between income equality and efficiency distinguishes it from more conventional analyses. While existing theories generally appeal to a variety of macro and political economic factors to link equality and productivity (see e.g. the survey of Benabou (1996)), here greater income equality, by nurturing a more competitive market environment, can increase productivity by motivating more efficient investment.

2 The Model

Consider an economy with a continuum of households and firms. The measure of households is normalized to 1. Each household is endowed with \( L \) units of labor. \( L \) is distributed heterogeneously between households according to the exogenous distribution function \( \Lambda(L) \) which is assumed to be continuously differentiable over its entire support.\(^2\) Each household is both a worker and a consumer. It inelastically supplies its labor endowment to a perfectly competitive labor market and purchases consumption goods with the wages it earns. The total number of units of labor in the economy is normalized to be 1.

There are two consumption goods. One good, termed the numeraire good, is consumed in infinitely divisible units and is produced and sold in a Walrasian (perfectly competitive) market. Consumers are perfectly informed about the price of this good. The other good, termed the search good, is produced and sold by price setting firms.

\(^2\)The interpretation is that each household is endowed with one unit of time, but the labor productivity of this unit differs across individuals.
in a decentralized search market. In this market, consumers are imperfectly informed about prices; They know the price \textit{distribution}, but do not know which firm charges what price. To learn the price of any individual search good firm, a consumer must engage in costly search, as described in detail below. The price of the search good, denoted \( p \), is denominated in units of the numeraire good. We denote by \( F(p) \) the cumulative distribution of the price of the search good. That is, \( F(p) \) is the proportion of firms whose price is less than or equal to \( p \). The search good is produced in indivisible units and a household demands at most one unit of it.

2.1 Consumers

To buy the search good, a consumer must engage in costly search. Each search reveals the price of one randomly selected firm and costs the consumer \( k > 0 \) units of utility.\(^3\) Consumers may sequentially visit as many firms as they wish.

All consumers have identical preferences. A consumer derives constant marginal utility from the numeraire good. Its utility from the search good is \( s > 0 \) for the first unit and zero from any additional unit. More specifically, a consumer’s utility function is given as:

\[
U(c_1, c_2, n) = c_1 + sc_2 - hk
\]  

where \( c_1 \) is the (continuous) quantity consumed of the numeraire good, \( c_2 \in \{0, 1\} \) is the number of units, zero or one, consumed of the search good, and \( h \) is the number of search good firms which the consumer visits before she buys. Thus \( hk \) is the loss of utility from searching \( h \) times.

Each consumer faces the budget constraint \( c_1 + pc_2 \leq m^i \), where \( m^i \) is his income, denominated in units of the numeraire good, and \( p \) is the price paid for the search good.

Consumers make the following interrelated decisions. First, given the distribution

\(^3\)Formulating the cost of search in terms of utility ensures that the optimal search strategy is stationary. If the cost of search were formulated in terms of goods, the consumers’ reservation price would not be stationary, since its wealth would decrease after each search. This would greatly complicate the analysis.
of prices for the search good, \( F(p) \), and its income, \( m \), a consumer decides whether to search (which involves a utility loss of \( k \) per search) or to forego consumption of the search good and spend all its income on the numeraire good. If it decides to search at least once, the consumer decides on a stopping rule that specifies which prices to accept and which prices to reject. We refer to this two tiered decision process as a consumer’s decision rule.

It is well known (e.g., Weitzman, 1979) that under our assumptions the optimal stopping rule - given that the consumer decides to search - is characterized by a reservation price, with the property that the consumer keeps searching until it finds a price which is less than or equal to the reservation price. Upon finding such a price, it buys a unit and spends any remaining income on the numeraire good.

### 2.2 Production

Labor is the sole input used to produce each good. A firm producing the numeraire good can produce any quantity, using one unit of labor per unit of output. To produce, a search good firm must invest a fixed cost of \( v < 1 \) units of labor. This investment endows it with the capacity to produce any quantity at constant marginal cost, which we normalize to zero. There is free entry into the search good industry. The equilibrium measure of search good firms is determined such that the profit of each firm, net of the investment \( v \), is zero.

### 2.3 Income Distribution, Inequality and Transfers

Let \( \Delta(m) \) be the distribution of income. Since the labor market is perfectly competitive and the marginal product of labor in production of the numeraire good is 1, the equilibrium wage (in terms of the numeraire good) per unit of labor is 1.\(^6\) Hence

\(^4\)The assumption that \( v < 1 \) ensures that a positive quantity of the numeraire good is produced in equilibrium.

\(^5\)Note that the number of firms is infinite and yet each firm employs a measure \( v \) of workers. To see how this can be modeled explicitly see Burdett and Judd (1983).

\(^6\)As noted in footnote 4, a positive quantity of the numeraire good is always produced in equilibrium.
\( \Delta(m) = \Lambda(L) \). We denote by \( \underline{m} \) and \( \overline{m} \) the lower and upper bound respectively of the support of \( \Delta(m) \) (i.e. the lowest and highest incomes in the economy) and by \( \delta(m) \) the density of \( \Delta(m) \).

The goal of this paper is to establish the effects of income distribution on market competitiveness, efficiency and aggregate output. To focus on realistic redistribution programs, we restrict attention to income transfers with the following desirable properties. The first of these is that the transfer be self-financing. By this we mean that the total income transferred, summed over all households, is zero (a zero sum transfer). Second, policy considerations would typically require that poorer households receive proportionally more (or contribute proportionally less) than richer households. Another realistic requirement is that the transfer be rank preserving. That is, if household \( i \)'s pre-transfer income was higher than that of household \( j \), then household \( i \) continues to be wealthier after the transfer. We shall refer to an income transfer with these properties as a “progressive transfer.” Formally:

**Definition 1** Let the function \( t(m) \) be the post-transfer income of a household with pre-transfer income \( m \). Then, \( t(m) \) is a “progressive transfer” if and only if

(i) \( \int [t(m) - m] d\Delta(m) = 0 \) (self-financing).

(ii) \( \partial \left( \frac{t(m) - m}{m} \right) / \partial m < 0 \) (poorer households receive more or contribute less than richer ones).

(iii) \( t'(m) > 0 \) (rank preservation).

Let \( \Delta(m) \) and \( \Phi(m) \) denote the pre and post-transfer income distributions, respectively. Since by property (iii) \( t(m) \) is strictly increasing, \( \Phi[t(m)] = \Delta(m) \).

An important property of progressive transfers, proved in the proof of the following lemma, is that \( \Phi \) and \( \Delta \) cross exactly once. That is, there exists \( z, \underline{m} < z < \overline{m} \) such that for \( m < z \), \( \Phi(m) < \Delta(m) \) and for \( m > z \), \( \Phi(m) > \Delta(m) \); see figure 1. The single crossing property implies the following:

**Lemma 1** A progressive transfer increases equality.
Proof: We begin by proving the single crossing property. Properties (i) and (ii) imply that $t(m) > m$. Suppose to the contrary that $t(m) - m \leq 0$. Then by property (ii) $t(m) - m < 0$ for all $m > m$, implying that $\int [t(m) - m]d\Delta(m) < 0$, contradicting (i). Thus $t(m) > m$.

Since $\int [t(m) - m]d\Delta(m) = 0$ and $t(m) > m$, there must be $\tilde{m}$ satisfying $t(\tilde{m}) < \tilde{m}$. Since $t(m) > m$, $t(\tilde{m}) < \tilde{m}$ and $t$ is continuous, it follows from the Mean Value Theorem that there is an interior income level, $z$, at which $z = t(z)$. Then by property (ii), $t(m) > m$ for all $m < z$ and $t(m) < m$ for all $m > z$. That is, $\Delta$ and $\Phi$ cross exactly once, at $z$. Hence, and since $\Phi$ is increasing then for all $m < z$, $\Phi(m) < \Phi[t(m)] = \Delta(m)$ and for $m > z$, $\Phi(m) > \Delta(m)$. This proves that $\Phi$ and $\Delta$ cross exactly once.

The single crossing property implies that $\Phi$ second order stochastically dominates $\Delta$. As shown by Atkinson (1970), this implies that $\Phi$ is “more equal” than $\Delta$. Thus a progressive transfer unambiguously increases income equality.

\[\square\]

3 Equilibrium

Denote the distribution of consumer reservation prices as $G(x)$ and its density as $g(x)$ and let $n$ denote the measure of search good firms.

In equilibrium, given their income and the price distribution $F(p)$, consumers choose optimal decision rules. And given $G(x)$, and $F(p)$ each firm chooses its price to maximize its profit. There are three markets: the market for each consumption good and the labor market. Since the marginal cost of the search good is zero, the total quantity of labor devoted to its production is the aggregate investment, $n\nu$. All other labor is employed in the production of the numeraire good. Thus the equilibrium quantity of the numeraire good, $y$, is $y = 1 - n\nu$. In equilibrium, all markets clear. More formally,
**Definition 2** An *equilibrium* consists of a price distribution for the search good, $F(p)$, a distribution of consumer reservation prices, $G(x)$, the quantity of the numeraire good, $y$ and the measure of operative search good firms, $n$, such that:

(a) Given $F$ and her income, $m$, a consumer’s decision rule maximizes her expected utility.

(b) utility maximization by consumers reproduces $G$.

(c) Given consumers’ decision rules, $G$ and $F$, no firm can increase its profits by altering its price.

(d) All markets clear.

(e) Each search good firm earns zero profits.

### 4 Analysis

The most a consumer is willing to pay for the search good is its marginal utility from a unit, $s$. Also, no consumer can pay more than its income for a unit. Thus the most a consumer with an income of $m$ will pay for the search good is $\min\{m, s\}$. Suppose for a moment that the search good were sold by a monopoly. Then, since $1 - \Delta(m)$ is the measure of consumers with an income of at least $m$, the monopoly’s revenue from a price $p \leq s$ is $p[1 - \Delta(p)]$ and zero for a price $p > s$. Let $p^m$ denote the monopoly price. We make the standard assumption (Bulow and Roberts, 1989) that the monopoly’s marginal revenue is strictly decreasing and equals the marginal cost at an interior point, $\bar{p}$. That is:

**Assumption 1:**

$$\frac{\partial^2 \{p[1 - \Delta(p)]\}}{\partial p^2} < 0$$

and $m < \bar{p} < m$ where $\bar{p} \equiv \arg\max\{p[1 - \Delta(p)]\}$.

Thus, $p^m = \min\{\bar{p}, s\}$.

Observe that a consumer who searches will optimally continue to search until it finds a price not exceeding its reservation price (otherwise it would not be optimal
to search even once). Thus, in any equilibrium, all consumers who search consume a unit of the search good (and of course consumers who do not search consume only the numeraire good).

As is typically the case in equilibrium search models, there exists a degenerate “Diamond” (Diamond, 1971) equilibrium for our model in which all firms charge the monopoly price $p^m$ and only consumers whose income is greater or equal to $p^m$ consume the search good.\(^7\) However, there also exists a more plausible dispersed price equilibrium in which different firms charge different prices for the search good and the average price is less than the monopoly price. We shall restrict our attention to this equilibrium\(^8\). Accordingly, the term “equilibrium” will henceforth refer to the dispersed price equilibrium. We proceed to derive and characterize this equilibrium.

Let $A$ be the lowest price in the market and $B > A$ the highest price. Let $\bar{p}$ satisfy: $\int_A^B F(x)dx = k$. The following lemma states that $\bar{p}$ is the reservation price of all searching consumers whose income is above $\bar{p}$ and that the reservation prices of searching consumers whose income is below $\bar{p}$ is just their income.

**Lemma 2** In a dispersed price equilibrium, the reservation price of a searching consumer with income $m^i$ is $\min\{m^i, \bar{p}\}$.

**Proof:** By the reservation price property (e.g., Weitzman, 1979), $\bar{p}$ is the reservation price of a consumer whose income is at least $\bar{p}$. Since a consumer cannot pay more than its income, its reservation price is $\min\{m^i, \bar{p}\}$. ■

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\(^7\)There are a few exceptions in which price dispersion is the unique equilibrium. Benabou (1992) and Fishman (1992) show that price dispersion is the only equilibrium if prices are eroded by inflation and it is costly for firms to change prices. Albrect and Vroman (1998) show that the Diamond equilibrium is eliminated in models with asymmetric information.

\(^8\)The dispersed price equilibrium seems to be more plausible than the Diamond equilibrium because it has the intuitive feature that the average price of the search good is lower, the lower the cost of search and approaches the competitive price as the cost of search goes to zero. By contrast, the Diamond equilibrium has the unintuitive feature that the price of the search good is independent of the cost of search as long as the latter is positive.

\(^9\)This property is due to the fact that the marginal utility from the numeraire good is constant and therefore independent of income. If the marginal utility from the numeraire good were decreasing, all consumers with different incomes would have different reservation prices. As we discuss in the concluding section, this complication would not significantly change our main results.
Models of equilibrium search typically assume that in equilibrium all consumers buy the search good. However, in our general equilibrium framework this assumption must be justified since consumers have the option of spending their entire income on the numeraire good. We shall proceed assuming that in equilibrium all consumers do indeed search and provide the parameter values which justify this assumption below. Our working paper (Fishman and Simhon, 2003) analyzes the more complex case in which not all consumers search and shows that our main findings continue to apply in that case as well.

An immediate implication of Lemma 2, above, is that when all consumers search, the reservation price distribution, $G(x)$, is simply the income distribution function, truncated at $\bar{p}$.

**Corollary 1:** If all consumers search, then:

$$G(x) = \begin{cases} 
\Delta(x) & \text{if } x < \min\{s, \bar{p}\} \\
1 & \text{if } x \geq \min\{s, \bar{p}\}
\end{cases}.$$

The preceding result is used to construct the equilibrium price distribution in the following lemma, whose proof is in the Appendix.

**Lemma 3** Let $[A, B]$ denote the support of $F(p)$. If all consumers search, then

1. $A = m$, where $m$ is the lowest income (the lower bound of the support of $\Delta$).

2. $B = \min \{\bar{p}, p^m\}$.

3. $F(p) = \frac{p^2\delta(p)}{B^2\delta(B)}$.

In equilibrium, a firm whose price is $p$ sells only to consumers whose reservation price is greater or equal to $p$. Hence, a firm’s market share is greater, the lower its price. In equilibrium all firms earn equal profit. The equilibrium price distribution, $F(p)$, has the property that the trade-off between a firm’s market share (which decreases with its price) and its price per unit are so balanced that all prices generate identical profits.
The following assumption proves useful in economizing on tedious and repetitious calculations.

**Assumption 2:**

\[
\begin{align*}
(i) \; \bar{p} & \leq \min\{s, \bar{p}\} \\
(ii) \; s & \geq m + \frac{\bar{p}^2 \delta(\bar{p})}{m^2 \delta(m)}k.
\end{align*}
\]

The assumption that \( \bar{p} \leq s \) means that the monopoly price is not constrained by \( s \). The assumption that \( \bar{p} \leq \bar{p} \) means that reservation price of the richest consumers is greater than the monopoly price.

The second part of Assumption 2 is shown in the proof of Lemma 4 to ensure that in equilibrium all consumers search. Our working paper (Fishman and Simhon, 2003) provides a complete analysis of the cases when Assumption 2 does not hold and shows that our main results apply qualitatively to those cases as well.

**Lemma 4** There exists a unique dispersed price equilibrium in which all consumers search.

**Proof:** The proof is in the Appendix

**5 Inequality and output**

We now have in place the infrastructure required to address the primary concern of this paper: How do progressive transfers affect competitiveness and aggregate output?

As is evident from Lemma 3, the equilibrium price distribution, \( F(p) \), is determined by the income distribution, \( \Delta(m) \). Thus a change of the income distribution leads to changes in the equilibrium price distribution and the revenues of the search good industry. This is because in our search market setting, how aggressively firms compete and hence their profits depends on consumers’ willingness to invest in costly
search. Consumers’ willingness to search in turn depends on the distribution of income. In particular, a transfer of income from rich to poor individuals has two opposing effects on consumers’ overall search activity. Low income individuals, whose income is increased by the transfer, have to search less to find a price they can afford. This lowers their incentive to search. On the other hand the transfer increases the motivation to search of wealthier individuals, whose income declines.\footnote{Under our assumption of constant marginal utility from the numeraire good, poorer individuals search more only because it is harder for them to find an affordable price. If marginal utility from the numeraire good is decreasing, another reason for poorer individuals to search more is that they obtain a higher marginal utility from the increased consumption of the numeraire good which is enabled by paying a lower price for the search good.} Hence, whether the effect of income redistribution is to increase competition and lower industry revenues or to reduce competition and increase revenues depends on which of those effects is dominant.

Suppose the search good industry’s revenues are decreased. Then, since each search good firm must invest $\nu$ and since the marginal cost is zero, the zero profit condition implies that the equilibrium number of firms, $n$, must also decrease. Since each consumer always consumes one unit of the search good, the equilibrium quantity produced of the search good is independent of $n$. However, since each additional search good firm costs the economy an additional $\nu$ units of labor, the amount of labor which is available to produce the numeraire good decreases with $n$. Thus, reducing industry revenues increases output of the numeraire good without decreasing output of the search good. Hence a progressive transfer which reduces industry revenues simultaneously increases equality and aggregate output.

The following proposition is our main result. It states that there is a very simple rule which determines whether a progressive transfer increases competitiveness (hence increasing aggregate output) or decreases competitiveness (hence reducing aggregate output).

**Proposition**  A progressive transfer decreases the revenues of the search good sector and increases output if $z < B$ and increases revenues and decreases output if $z > B$. 


The immediate implication of the proposition is that the policy maker can always design a progressive transfer to simultaneously increases equality and aggregate output by choosing \( z < B \).

Before proving the proposition, we discuss the intuition for it. Recall that in equilibrium all consumers whose income is less than \( B \) search until they find a price which does not exceed their income while all consumers whose income is greater than \( B \) accept any price. Suppose that \( z > B \) so that the transfer reduces the income of only the latter consumers. Rank preservation (property (iii) of Definition 1) implies that those consumers’ post transfer incomes are still greater than \( B \) and so would continue to accept \( B \) if the price distribution were unchanged. Therefore, those consumers propensity to search is unaffected by the transfer. On the other hand, the transfer reduces the propensity to search of searching consumers, the beneficiaries of the transfer, whose income is less than \( z \). Hence, a transfer with \( z > B \) reduces the overall propensity to search, which reduces competition and increases industry profit. And as we argued above, higher profit leads to lower output.

By contrast, when \( z < B \) households with income between \( z \) and \( B \) reduce their reservation price and are induced to search more intensely than before the transfer. This puts pressure on high priced firms to reduce prices. This in turn exerts pressure for price reductions in the entire industry, increasing competition, reducing industry profit, and increasing output.

We now sketch the proof of Proposition 1 and provide a detailed proof in the Appendix. Consider a firm which charges the highest price in the market, \( B \). The measure of consumers which accept this price is \( [1 - \Delta(B)] \) and that firm’s share of these customers is \( 1/n \). Hence, its revenues is \( B[1-\Delta(B)]/n \). And since in equilibrium all firms earn equal revenues (as marginal cost is zero, profit equals revenues minus entry costs) the revenues of the search good industry is \( B[1-\Delta(B)] \). Similarly, the post - transfer revenue of the search good sector is \( B_{\Phi}[1-\Phi(B_{\Phi})] \), where \( \Phi(m) \) is the post - transfer income distribution and \( B_{\Phi} \) is the value of \( B \) under \( \Phi \). The functions \( m[1 - \Phi(m)] \) and \( m[1 - \Delta(m)] \) are drawn in figure 2. As shown in the proof of
Lemma 1, the former curve lies below the latter for \( m > z \). As indicated by figure 2a, if \( z < B \), the aggregate revenue of the search good sector under \( \Phi \), \( B\Phi[1 - \Phi(B\Phi)] \), is smaller than the aggregate revenue under \( \Delta \), \( B[1 - \Delta(B)] \). Thus designing the transfer with intersection point \( z \) below \( B \) ensures that firms' revenues decline. By contrast, choosing a transfer with intersection point \( z \) above \( B \), as in figure 2b, ensures that firms’ revenues increase.

6 Discussion of the Assumptions

We have developed a simple model of a search economy in which simple and realistic income transfer schemes can simultaneously increase equality, efficiency and output by making markets more competitive. We conclude by commenting on the robustness of the model with respect to our main assumptions.

The first comment concerns our assumptions about consumer preferences. Our analysis was simplified considerably by the assumption of constant marginal utility from the numeraire good, as expressed by the utility function (1). This ensures that all consumers with sufficiently high incomes have the same reservation price (Corollary 1). However, our main results do not depend on this simplifying assumption. We have also solved the model for the more general utility function of the type \( U(c_1, c_2, n) = u(c_1) + sc_2 - hk \), where \( u() \) is a strictly concave function. Then, Corollary 1 no longer obtains because all consumers with different incomes have different reservation prices, which considerably complicates the analysis. Nevertheless, the basic message of Proposition 1 continues to apply: a progressive transfer decreases industry revenue and increases aggregate output if \( z \) is sufficiently low, though not necessarily below \( B \).\(^{11}\) The reasoning is the same as above; the transfer increases competitiveness if some high income consumers which are hurt by the transfer are lead to search more intensively.

Our second comment concerns the relationship between competitiveness and out-

\(^{11}\)The precise technical condition and its proof are available from the authors upon request.
put in our model. In our formulation, increased competitiveness resulting from greater equality increases production efficiency and aggregate output by reducing wasteful investment in production capacity - wasteful because, under a constant returns to scale technology (constant marginal cost), production efficiency requires the number of operative firms to be as small as possible. This is a convenient way to link competitiveness with productivity in our model, but it is certainly not the only one. The usual efficiency distortion associated with monopoly power - underproduction of the search good - does not apply to our model in its present formulation because consumers have unit demand. But it could be introduced by letting consumers have downward sloping demand (as in Reinganum’s (1979) search model). A second alternative, as in Albrecht and Axell (1984) and Fershtman, Fishman and Simhon (2003), would arise if potential producers of the search good are characterized by heterogeneous marginal production costs. In that case only firms whose production cost is below the highest equilibrium price would be operative. Under such a formulation, a decrease in firms’ market power and lower prices would enhance efficiency and increase aggregate output by forcing the exit of the least efficient firms. Again, the result would be that greater equality leads to more efficient investment in search good production. Each of these alternatives, and others, should generate qualitatively similar results to those obtained here, but at considerably greater analytical expense.
References


Proof of Lemma 3

1. We start by claiming that \( A \geq \underline{m} \). Otherwise, a firm charging \( A \) could raise its price to \( A + \min\{\underline{m} - A, k/2\} \) without losing customers, thus increasing its profits. Hence \( A \geq \underline{m} \). If \( A > \underline{m} \), it could not be optimal for consumers whose income is less than \( A \) to search. Thus, if all consumers search, \( A \leq \underline{m} \). Hence \( A = \underline{m} \). This proves part 1 of the Lemma.

2. First of all, since no consumer will pay more than \( s \) for the search good, \( B \leq s \). Consumers with reservation price \( B \) and above buy at the first firm that they encounter and therefore search exactly once. Therefore, the measure of these consumers that arrive at every firm is \( [1 - G(B)]/n \). Hence, the revenue of a firm that charges \( B \) is \( B[1 - G(B)]/n \). By the corollary, \( 1 - G(x) = 1 - \Delta(x) \) for \( x < \min\{s, \bar{p}\} \) and \( 1 - G(x) = 0 \) for \( x > \bar{p} \). Hence, that firm’s revenue is \( B[1 - \Delta(B)]/n \) if \( B \leq \bar{p} \) and zero if \( B > \bar{p} \). This proves that \( B \leq \min\{s, \bar{p}\} \). If \( \bar{p} \equiv \text{ArgMax}_x\{x[1 - \Delta(x)]\} \leq \min\{s, \bar{p}\} \), the profit maximizing price for firms whose price is \( B \) is \( \text{ArgMax}_x\{x[1 - \Delta(x)]\} \). If \( \bar{p} > \min\{s, \bar{p}\} \), then, since by Assumption 1, \( x[1 - \Delta(x)] \) is increasing for \( x < \bar{p} \), the profit maximizing price for that firm is \( \min\{s, \bar{p}\} \). Hence, \( B = \min\{s, \bar{p}, \bar{p} \} = \min\{\bar{p}, p^{\underline{m}}\} \).

3. The demand facing a firm whose price is \( p \) is determined as follows. Consumers reach the firms in consecutive waves that follow each other instantaneously. In the first wave, each firm is visited by \( 1/n \) consumers with reservation prices distributed according to \( G \). In this first wave, a proportion \( F(x) \) of the consumers with a reservation price \( x \) find a price for which they buy, and the rest search again. Since the second wave consists of those consumers who first sampled a firm that charged more than their reservation price, and since the initial density of these consumers is \( g(x) \), it follows that in the second wave the per firm density of consumers with reservation price \( x \) is \( g(x)[1 - F(x)]/n \). In the third wave, this density is \( g(x)[1 - F(x)]^2/n \), and in the \( i + 1 \) wave it is \( g(x)[1 - F(x)]^i/n \).

Thus, given \( G \) and \( F \), the profit of a search firm that sets its price at \( p \), denoted \( \pi(p) \) is:\footnote{The revenues of the firm is comprise of two parts; the first, \( p[1 - G(p)]/n \) is the demand in}
\[
\pi(p) = \frac{p}{n} \left[ 1 - G(p) + \sum_{j=1}^{\infty} \int_{0}^{B} g(x)[1 - F(x)]^j dx \right] - \nu.
\]

Substituting \( \int_{0}^{B} g(x)dx = (1 - G(p)) - (1 - G(B)) \), using the fact that the sum of integrals equals to the integral of the sum and rearranging yields:

\[
\pi(p) = \frac{p}{n} \left[ 1 - G(B) + \int_{0}^{B} \frac{g(x)}{F(x)} dx \right] - \nu.
\]

In equilibrium, all firms earn zero profits. It can be easily verified that \( F(x) = \frac{x^2 g(x)}{B^2 g(B)} \) solves the equation \( \pi(p) = 0 \). Furthermore, by the corollary, \( g(x) = \delta(x) \) for \( x < \bar{p} \) and by part 2 of the lemma \( B \leq \bar{p} \). Hence, \( F(x) = \frac{x^2 \delta(x)}{B^2 \delta(B)} \). \( \blacksquare \)

**Proof of Lemma 4:**

**Proof:** All consumers get the same utility, \( s \), by consuming the search good. On the other hand, the expected search cost, \( hk \), declines with income because the poorer the consumer the more times she has to search, on average, to find a unit at an affordable price. Thus, all consumers search if and only if it is optimal for individuals with the lowest income, \( m \), to search. Those individuals search if and only if their expected utility from search is at least as great as their expected utility from not searching and consuming only the numeraire good. Their expected utility from not searching is just \( m \). By Lemma 3, \( A = m \). Thus if the poorest consumers search, they buy only at the lowest price, \( A \), and consume only the search good. Hence, if \( h \) is the expected number of searches required to find the price \( A \), their expected utility from searching is given by \( s - hk \). By Lemma 3, \( F(m) = \frac{m^2 \delta(m)}{B^2 \delta(B)} \) and therefore \( h = 1/F(m) = \frac{B^2 \delta(B)}{m^2 \delta(m)} \).

Therefore, their expected utility from searching is greater or equal to their expected utility from not searching if \( s - \frac{B^2 \delta(B)}{m^2 \delta(m)} k \geq m \) and since \( B \leq \bar{p} \) the last inequality holds if \( s \geq m + \frac{\bar{p}^2 \delta(\bar{p})}{m^2 \delta(m)} k \). Thus the preceding inequality implies that all consumers search in equilibrium.

---

*Footnote:* a Walrasian setting while \( p \sum_{j=1}^{\infty} \int_{0}^{B} g(x)[1 - F(x)]^j dx/n \) is the sale to customers who previously encountered prices greater than their reservation price which they rejected.
By Assumption 1, \( \tilde{p} \) is unique, and by Assumption 2 and Lemma 3, \( B = \tilde{p} \). Hence \( B \) is unique. By part 3 of Lemma 3 this implies that \( F(p) \) is unique and it follows from Lemma 2 and Corollary 1 that \( G(p) \) is also unique. This proves existence and uniqueness. ■

Proof of the proposition:

We will prove the proposition by constructing such a transfer. Let \( t(m) \) be a progressive transfer. For any variable \( x \) corresponding to \( \Delta \), let \( x_{\Phi} \) denote the value of \( x \) under \( \Phi \). The proposition is proved using the following claims.

Claim 1: \( B \) is a continuous function of \( \Delta \).

Proof: Since \( \Delta \) is continuous, it follows from Assumption 1 that \( \tilde{p} = \arg \max m[1 - \Delta(m)] \) changes continuously with \( \Delta \). Since \( p^{m} = \min \{s, \tilde{p}\} \) and the minimum of continuous functions is continuous, the claim is proved.

Let \( t(m) \) have the property that \( z < B \). By the preceding claim, and for a sufficiently small transfer, i.e. \( t(m) - m \) sufficiently small, \( z < B_{\Phi} \). Then since \( B = \tilde{p} = \arg \max m[1 - \Delta(m)] \),

\[
B \left[ 1 - \Delta(B) \right] \geq B_{\Phi} \left[ 1 - \Delta(B_{\Phi}) \right] \tag{2}
\]

and by Lemma 1, for all \( m > z \), \( \Phi(m) > \Delta(m) \) and the fact that \( z < B_{\Phi} \),

\[
B_{\Phi} \left[ 1 - \Delta(B_{\Phi}) \right] > B_{\Phi} \left[ 1 - \Phi(B_{\Phi}) \right]. \tag{3}
\]

Hence, it follows from (2) and (3) that

\[
B \left[ 1 - \Delta(B) \right] > B_{\Phi} \left[ 1 - \Phi(B_{\Phi}) \right]. \tag{4}
\]

In the proof of Lemma 3 it was shown that

\[
\pi(p) = \frac{p}{n} \left[ 1 - G(B) + \int_{p}^{B} \frac{g(x)}{F(x)} \, dx \right] - v.
\]

Thus, zero profits imply \( \pi(B) = B[1 - \Delta(B)]/n - v = 0 \) and it follows that

\[
v_{n} = B[1 - \Delta(B)] \tag{5}
\]

\[
v_{n,\Phi} = B_{\Phi} \left[ 1 - \Phi(B_{\Phi}) \right].
\]

\(^{13}\)Using, for example, the maximum absolute value distance measure.
It follows from (4) and (5) that \( vn_\Phi < vn_\Delta \). Since \( y = 1 - vn \) and \( y_\Phi = 1 - vn_\Phi \), it follows that \( y_\Phi > y_\Delta \). Since by Lemma 1 \( \Phi \) is more equal than \( \Delta \), this completes the proof of the proposition for \( z < B \). The proof for \( z > B \) is the same, and that completes the proof of the proposition. \( \blacksquare \)
Figure 1
Figure 2a

\[ B[1-\Delta(B)] \]

\[ B_\phi[1-\Phi(B_\phi)] \]

\[ m[1-\Delta(m)] \]

\[ m[1-\Phi(m)] \]
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