Measuring Dynamic Efficiency under Uncertainty: An Application to German Dairy Farms

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Abstract

The existing literature on dynamic efficiency is deterministic and ignores uncertainty when deriving dynamic efficiency measures, even though it is known that uncertainty affects the optimal adjustment path and the optimal use of quasi-fixed factors. Here, we contribute to closing this gap by developing a model that takes the dynamic efficiency measurement and the optimal investment under uncertainty jointly into consideration. We apply this model to German farm-level panel data to investigate whether West German dairy farms use their variable and quasi-fixed factors in a technically and allocative efficient way in the long run, and to explore the role of uncertainty within the optimal factor allocation process. We find empirical evidence for the importance of considering uncertainty when deriving (dynamic) efficiency measures: neglecting uncertainty within the estimation procedure will overestimate the average inefficiency score. This is not only interesting from an academic point of view; it has further implications for the analysis of the relative performance of specific farm types like cash crop or other livestock farms.

Keywords: Efficiency, shadow cost approach, uncertainty, dairy sector

JEL codes: D61, D81, Q12
1 Introduction

The dairy sector is the most important farming sector in the European Union (EU). Within the 2003 Common Agricultural Policy (CAP) reform and the ensuing 2008 health check, the decoupling of direct payments from production levels, the further reduction of intervention prices and the stepwise increases of the milk quotas induced considerable adjustment processes at the farm level. Thereupon, milk and other commodity prices have become more volatile, which in turn has introduced further pressure on dairy farms. Particularly for dairy farms, price uncertainty is rather new compared to other sectors such as hog fattening (Keane and O’Connor 2009). In the long-term perspective, the respective EU dairy sectors are characterized by considerable increases in farm-level productivity, and only small increases in the demand for dairy. As a result of these issues, dynamic adjustments of dairy farms have taken place, including specialization, farm growth and farm shrinkage or even closure. For example, the number of dairy farms in Germany declined from 1,216,700 in 1960 to 84,268 in 2011, while the average farm size increased from an average of 5 cows per farm in 1960, to 41 cows per farm in 2011. This begs the question of how adjustment pressure, for instance induced by increases in price volatility, is related to farm-level decision-making with regard to the optimal factor allocation in the long run. It can be conjectured that an increase of risk may reduce the propensity to invest in production capacity. Moreover, the role of the milk quotas during the adjustment process is unclear. Certainly the production limitation introduced additional costs of adjustments for the quasi-fixed capital stock of growing farms, and the devaluation of the milk quotas through the stepwise increase in quantities will likely have two effects. First, it may lead to a reduction of investment costs and second, it may reverse the trend of capital overuse caused by the introduction of the milk quotas. Jointly with the increase in price volatility for milk, as well as for inputs such as concentrates, this can be judged as a fundamental change of production decisions.

It is widely acknowledged that structural change and farms’ adjustment processes are closely related to the efficiency of firms within a sector (e.g., Goddard et al. 1993). According to the efficient structure hypothesis, firms with superior performance and higher efficiency increase their market share at the expense of less efficient firms, thereby increasing concentration. At the same time, however, it can be observed that inefficient firms persist in the market, at least in the short run (Emvalomatis et al. 2011). A vast body of literature relates technical and economic efficiency to structural characteristics of dairy farms such as size, specialization, organization or financial structure (e.g., Curtiss 2002; Mosheim and Lovell 2009; Lambert and Bayda 2005). The existing literature on the relation between farm size and efficiency offers mixed results. Hadley (2006) reports that farm or herd size has a significantly positive effect on the technical efficiency of UK farms. Alvarez and Arias (2004) reach a similar conclusion for Spanish dairy farms. In contrast, Latruffe et al. (2005) find a U-shaped relationship between technical efficiency and the size of Polish farms.

The aforementioned studies apply a static view of efficiency that does not take into account intertemporal linkages of production and (dis)investment decisions. This view is emphasized by the concept of dynamic efficiency (e.g., Silva and Stefanou 2007). Measuring dynamic
efficiency acknowledges that changes in the level of quasi-fixed production factors entail adjustment costs that may prevent firms from realizing otherwise optimal investments or disinvestments. As a result, firms may appear overcapitalized or undercapitalized. This finding suggests the importance of distinguishing between short run and long run efficiency. Rungsuriyawiboon and Stefanou (2007; 2008) establish a dynamic efficiency model by integrating the static shadow cost approach and the dynamic dual model of intertemporal decision making; their model accounts for technical and allocative inefficiencies of variable inputs and net investments. Recently, Rungsuriyawiboon and Hockmann (2012) use this modeling approach to investigate structural change and technical change in Polish agriculture.

The existing contributions to dynamic efficiency measurements, however, are built on the assumption of static expectations of prices and returns, that is, current prices and outputs are assumed to persist. Decision makers are not allowed to revise their expectations and uncertainty of prices and yields does not play a role. However, this assumption is clearly unrealistic. Farmers rather make their production and investment decisions in an uncertain economic environment. This is particularly true for the dairy sector in the EU, where reduced price support and increasing quota levels have led to increasing milk and factor price volatility in the last decade (e.g., Keane and O’Connor 2009; Jongeneel et al. 2010). Why is uncertainty an issue when analyzing dynamic efficiency, farm adjustments and the interplay between them? It is well-known that risk has an impact on the optimal demand of variable inputs, and even more on the demand of quasi-fixed production factors (e.g., Pindyck 1991; Serra et al. 2010). Real options theory asserts that the effect of adjustment costs on the expansion path of capital is amplified if investment decisions have to be made under uncertainty (Abel et al. 1996). Empirical evidence for this theoretical finding is provided, for example, by Pietola and Myers (2000), who analyze dynamic adjustment in the Finnish pork industry. The effect of uncertainty on optimal factor demand should, in turn, have an influence on the measurement of dynamic efficiency, since the latter is based on optimal factor demand functions. We conjecture that if deterministic factor demand equations are used as a benchmark, farms’ adjustments may appear seemingly inefficient. The purpose of this paper is to explore this conjecture in greater detail.

Our objectives are twofold: First, we want to empirically analyze the dynamic efficiency of German dairy farms, since it is an important driver of structural change. We thus explore the role of uncertainty for optimal factor allocation and investigate whether West German dairy farms use their variable and quasi-fixed factors in a technically and allocatively efficient way. Moreover, we relate observed differences in inefficiency levels to structural parameters. The second contribution of the paper is to provide an estimation procedure that allows us to incorporate uncertainty in the measurement of dynamic efficiency, which has thus far not been done. The basic idea is to merge models of investment under uncertainty and (deterministic) dynamic efficiency analysis. Based on a dynamic programming equation for a cost-minimizing firm, optimal dynamic demand functions for the variable inputs and net investments are derived. Decomposing economic efficiency is achieved by a shadow cost approach, which means distinguishing between a firm’s actual costs and behavioral (or shadow) costs. The actual cost function refers to the perfect minimization of cost, whereas behavioral costs are associated with the observed input levels of the firm. In the presence of
inefficiencies, shadow costs for production factors will deviate from actual (market) prices. In contrast to existing models we allow for factor price risk, and thus factor price volatility emerges as a variable in the theoretical factor demand equations, as well as in their empirical counterparts. This enables us to test whether disregarding price uncertainty will cause an omitted variable bias in the measurement of dynamic efficiency.

The paper is organised as follows. The theoretical model of dynamic efficiency under uncertainty is derived in Section 2. In Section 3, we present the empirical application, focusing on West German dairy farms. The empirical model is described in Section 4. In Section 5, we present the results for West German dairy farms with respect to efficiency and uncertainty. Section 6 summarizes and provides some concluding remarks.

2 Theoretical approach: dynamic efficiency under uncertainty

Our model derivation follows Rungsuriyawiboon and Stefanou (2007). The dynamic efficiency model links the static shadow cost approach with a dynamic dual model of inter-temporal decision making. A representative firm is assumed to minimize its expected discounted sum of all future costs over an infinite planning horizon, subject to production sequence and capital accumulation. Expanding upon the approach of Rungsuriyawiboon and Stefanou (2007), we allow for non-static expectations of input prices and the output level. The value function \( J(\cdot) \) of this optimization problem is given by:

\[
J(w(0), c(0), y(0), K(0)) = \min_{t} E_0 \int_{0}^{\infty} e^{-r t} \left[ \sum_{n} (w_n(t) \cdot x_n(t)) + \sum_{m} (c_m(t) \cdot K_m(t)) \right] \, dt
\]  

(1)

where \( t \) denotes time, \( E_0 \) is the conditional expectation operator, and \( x_n(t) \) represents the \( n \)th variable factor, with \( [x_1(t), \ldots, x_n(t)] \in \mathbb{R}_+^n \) and \( [w_1(t), \ldots, w_n(t)] \in \mathbb{R}_+^n \) denoting the respective factor prices with \( n = 1, \ldots, \bar{m} \). Further, \( K_m(t) \) represents the \( m \)th quasi-fixed input level with \( [K_1(t), \ldots, K_m(t)] \in \mathbb{R}_+^m \) and the respective factor prices denoted by \( [c_1(t), \ldots, c_m(t)] \in \mathbb{R}_+^m \). Moreover, \( I_m(t) \) is gross investment in the \( m \)th quasi-fixed factor with \( m = 1, \ldots, \bar{m} \), and \( r \) is a constant discount rate.

The cost minimization is subject to the production sequence:

\[
y(t) \leq F\left(x(t), K(t), \dot{K}(t)\right)
\]  

(2)

where we assume a single output \( y \) and the firm’s technology is represented by a production function \( F\left(x(t), K(t), \dot{K}(t)\right) \). Including the net investment \( \dot{K}(t) \) reflects the presence of

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1 Time dependency is suppressed wherever possible.
internal adjustment cost in terms of foregone output (Stefanou 1989). The optimization is further subject to the equation of motion of the stock of quasi-fixed inputs, with constant depreciation rate being \( \delta \).

\[
\dot{K}_m(t) = (I_m(t) - \delta \cdot K_m(t))
\]

(3)

Uncertainty is introduced by a vector, \( z(t) \), containing the state variables \( \ln y(t) \), \( \ln w_n(t) \), and \( \ln c_m(t) \), which are assumed to follow an arithmetic Brownian motion:

\[
dz = \alpha \cdot dt + \nu \cdot dv
\]

(4)

wherein \( \alpha \) denotes the drift parameter, \( \nu \) is the respective variance parameter, and \( dv \) is a standard Wiener increment with \( E\{dv\} = 0 \), \( E\{(dv)^2\} = dt \), and \( E\{dv_i, dv_j\} = 0 \) for all \( i \neq j \).

Solving the optimization problem as given in (1)-(4) by stochastic dynamic programming yields the optimal factor allocation under perfect efficiency (cf. Pietola and Myers 2000). To measure technical and allocative inefficiency, we employ a dynamic shadow cost approach (cf. Rungsuriyawiboon and Stefanou 2007). This procedure involves three major steps. First, the behavioral or shadow value function \( J^b \) is defined using the shadow prices defined as follows: \( w_n^b = \lambda_n w_n \). The shadow prices deviate from actual variable input prices by \( \lambda_n \), which measures relative allocative inefficiency such that values \( \lambda_n > 1( < 1) \) indicate that less (more) of the \( n \)th input is used compared to the cost-minimizing (efficient) allocation. The optimal behavioral factor demands, \( x_n^b \) and \( \dot{x}_m^b \) imply cost-minimizing factor usages under shadow prices; however, the behavioral demand may differ from the actual demand by technical inefficiency, denoted by \( \tau_{K_m} \geq 1 \) and \( \tau_{x_n} \geq 1 \) (input-oriented measures) such that

\[
\dot{K}_m^b = \left( \frac{1}{\tau_{K_m}} \right) \cdot \dot{K}_m \quad \text{and} \quad x_n^b = \left( \frac{1}{\tau_{x_n}} \right) \cdot x_n.
\]

Hence, under possible inefficiency, the optimized actual factor demand functions are expressed by \( x_n^o = \tau_{x_n} \cdot x_n^b \) and \( \dot{K}_m^o = \tau_{K_m} \cdot \dot{K}_m^b \), given by:

\[
\dot{K}_m^o = \tau_{K_m} \cdot \dot{K}_m^b = \tau_{K_m} \cdot \left( J^b_{K_m, ln c_m} \right)^{-1} \left( r J^b_{ln c_m} - c_m \cdot K_m - \sum_{m \neq m} \left( K_m^b \cdot J^b_{K_m, ln c_m} \right) - \frac{1}{2} \cdot \Omega^b_{ln c_m} \right).
\]

(5)

Therein, \( \Omega^b_{ln c_m} \) accounts for uncertainty of the output level and the input prices. This term is the derivative of the \((1 + n + m) \times (1 + n + m)\)-matrix \( \Omega^b \) with respect to \( \ln c_m \), where \( \Omega^b = \sum_{j=1}^{l} \sum_{j'=1}^{l} J^b_{ij, j'j''} \cdot \sigma_{jj''} \cdot J^b_{ij, j'j''} \) denotes the respective second partial derivatives of \( J^b \) with respect to vector \( z \), \( j \) and \( j' \) index the respective state variables, and \( \sigma_{jj'} \) represents the respective variance and co-variance parameters. To simplify the model derivation, we assume that the drift rate is zero.

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2 The production function is assumed to be concave in \( \dot{K}_m(t) \), which implies increasing marginal adjustment costs: the loss in production is assumed to be larger for faster adjustments in the capital stock, and as a result, the firm will tend to adjust more slowly such that \( \dot{K}_m F_{k^*} < 0 \) and \( F_{k^*} < 0 \) holds (Stefanou 1989).
Next, the actual value function is defined. Herein, the actual prices and quantities are considered such that the actual input levels ($x_{o}^{m}$ and $K_{m}^{o}$) are the optimal ones. The details of the derivation are presented in Appendix A (cf. equations (A.8)-(A.10)). The solution represents a fully efficient input use. Finally, the optimized actual value function is expressed in terms of the behavioral value function ($J^{b}$). This is done to define inefficiency as the deviation between the actual and behavioral value functions. Accordingly, the actual terms are substituted by their behavioral counterparts. For instance, $K_{m}^{o}$ is substituted by $\tau_{K_{m}} \cdot \hat{K}_{m}^{b}$ to introduce the technical inefficiency of capital use. In this step, the allocative inefficiency parameter of net investments $\mu_{m}$ is introduced by multiplying it with the behavioral marginal value of the capital stock $J^{b}_{K_{m}}$.

Inserting the derivatives into the factor demand equations from the second step, we obtain optimized actual factor demand functions in terms of the behavioral value function under uncertainty. The $n^{th}$ optimized actual variable factor demand in terms of the behavioral value function is given by:

\[ x_{o}^{m} = \frac{1}{w_{n}} \left\{ r \sum_{m} \tau_{\beta_{m}} \cdot J^{b}_{m, \ln w_{n}, \ln w_{a}} - r \sum_{m} \left[ \tau_{\beta_{m}} \sum_{m} J^{b}_{K_{m}, \ln w_{n}} \cdot J^{b}_{m, \ln c_{m}, \ln w_{n}} \right] 
\right. 
\]

\[ + \sum_{m} \left[ \frac{\tau_{\beta_{m}}}{\mu_{m}} J^{b}_{m, \ln c_{m}, \ln w_{n}} \cdot (\hat{K}^{b}_{m, \ln c_{m}, \ln w_{n}}) \right] - \sum_{m} \frac{1}{2} \sum_{m} \left[ \tau_{\beta_{m}} \cdot J^{b}_{m, \ln c_{m}, \ln w_{n}} \cdot J^{b}_{m, \ln c_{m}, \ln w_{n}} \right] \]

\[ + \frac{1}{2} \sum_{m} \left[ \frac{\tau_{\beta_{m}}}{\mu_{m}} J^{b}_{m, \ln c_{m}, \ln w_{n}} \cdot (\hat{K}^{b}_{m, \ln c_{m}, \ln w_{n}}) \right] - \sum_{m} \frac{1}{2} \sum_{m} \left[ \tau_{\beta_{m}} \cdot J^{b}_{m, \ln c_{m}, \ln w_{n}} \cdot J^{b}_{m, \ln c_{m}, \ln w_{n}} \right] \]

\[ + \frac{1}{2} \sum_{m} \left[ \frac{K_{m}^{o}}{\mu_{m}} \cdot \Omega^{b}_{m, \ln c_{m}, \ln w_{n}} \cdot J^{b}_{m, \ln c_{m}, \ln w_{n}} \right] \}

(6)

We normalize the factor prices by using the first variable factor prices as a numeraire to satisfy the property of linear homogeneity in prices of the cost function (cf. Maietta 2000), and the shadow prices are redefined accordingly $w_{n}^{b} = \left( \lambda_{n} w_{n} / \lambda_{w_{1}} \right) = \lambda_{n} w_{n}^{1}$ such that $\lambda_{n}$ accounts for price distortions of the $n^{th}$ variable factor relative to the numeraire factor. Values of $\lambda_{n} > 1$ ($< 1$) imply that the ratio of the shadow price of the $n^{th}$ variable factor relative to

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3 Note that any derivatives higher than the second order of $J^{b}$ are ignored.
the numeraire variable factor is higher (lower) than the respective actual prices ratio, i.e., underuse (overuse) of the \( n^{th} \) variable factor in relation to the numeraire. The optimized actual demand for the numeraire is given by re-arranging the optimized version of the behavioral HJB equation:

\[
\begin{align*}
\tau_h \cdot \left( x_h^b \right) &= \tau_h \cdot \left( rJ^b - \sum_{n=2}^{N} (w_n^b \cdot x_n^b) - \sum_m (c_m \cdot K_m) - \sum_m (J_k^b \cdot K_m^b) - \frac{1}{2} \cdot \Omega^b \right)
\end{align*}
\]

(7)

where \( x_h^b \) denotes the behavioral demand for the numeraire variable factor. The details can be found in Appendix A. In line with Rungsiyawiboon and Stefanou (2007), and due to expected singularity problems, only the estimation of the variable factor demand is based on the optimized actual in terms of the behavioral value function as given in (6) and (7). Thus, we do not present the optimized actual investment demand in terms of the behavioral, and (5) is the base for the estimation.

3 Measuring dynamic efficiency for West German dairy farms

We use farm-level data of West German\(^4\) dairy farms in the national farm accountancy data network (FADN) from 1996 to 2010 (henceforth: BMELV Testbetriebsnetz). We select specialized dairy farms where more than 75% of the total revenues are realized from dairy production to avoid distortions in the relation of the expenditures to dairy production. Farms must remain at least 5 years in the panel, and outliers are removed by applying common rules: we remove observations below the 1% percentile and above the 99% percentile using the main variables that are explored below. In a further selection step, we consider only farms with at least one observation with positive investment rates; that is, observations with zero or negative investments were excluded. Thus, the employed data set is unbalanced, with 4,213 observations (1,269 farms that are 3.3 years in the panel on average).

The model is specified for one output (\( y \)): milk production per farm (in tons); one quasi-fixed input (\( K \)): the number of dairy cows per farm; and two variable inputs–feed and other inputs. Accordingly, one quasi-fixed input price (\( c \)), and two variable input prices (\( w_2, w_1 \)) are considered. Feed (\( x_2 \)) consists of purchased concentrates and roughage. The implicit quantity of feed is obtained as the ratio of feed expenditures to the yearly feed price index (\( w_2 \)) provided by the Federal Statistical Office. Other inputs (\( x_1 \)) include insemination, veterinary service, energy, seeds, fertilizer, pesticides and hired labor; they are used as the numeraire. The quantity of other inputs is the ratio of the aggregated expenditures for other inputs to the price index for other inputs (\( w_1 \)). The latter is calculated using the Törnqvist price index. Here, we either use prices at the farm level or price indices from official statistics depending on availability at the farm level. The capital price (\( c \)) is approximated by the farms’ imputed replacement costs and the costs for purchased cows divided by the number of cows. The imputed costs are calculated as replacement rate times herd size to obtain the average number

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\(^4\) Our analysis focuses on farms in West Germany since specialized dairy farms are concentrated in West Germany; in East Germany the majority of farms are mixed farms.
of cows being replaced by year; the term is then multiplied with the price that is approximated by the average book values per cow. We opted for this approach since the replacement rate is directly related to the farms’ herd management ability, which in turn directly influences the optimal allocation of all factors. The variables are summarized in Table 1.

Table 1 Summary statistics of the main variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>Milk production per farm [metric tons]</td>
<td>283.14</td>
<td>129.55</td>
</tr>
<tr>
<td>x_2</td>
<td>Purchased feed [€]</td>
<td>128.86</td>
<td>61.28</td>
</tr>
<tr>
<td>x_1</td>
<td>Other inputs [€]</td>
<td>696.98</td>
<td>1,147.73</td>
</tr>
<tr>
<td>K</td>
<td>Livestock capital [# of cows]</td>
<td>44</td>
<td>18</td>
</tr>
<tr>
<td>c</td>
<td>Capital price [€ per cow]</td>
<td>241</td>
<td>146</td>
</tr>
</tbody>
</table>

Note: The database is the BMELV-Testbetriebsnetz, 1996-2010.

Uncertainty enters the empirical model through three variables: the price variance of purchased feed, livestock input price, and milk production. Purchased feed is represented by feed concentrates, and its price volatility $\sigma_{\ln w}^2$ is measured by a General Autoregressive Conditional Heteroscedasticity (GARCH) model using a monthly price series for concentrates. The resulting measure varies over time, not by farm. Milk production volatility $\sigma_{\ln y}^2$ is calculated by the variance of the farms’ milk production. Note that milk output is only available at the farm level with a short time series dimension, and thus a GARCH specification is not appropriate and the measure is time-invariant. The third uncertainty parameter is the volatility of livestock capital price $\sigma_{\ln c}^2$. This parameter is difficult to measure since only bookkeeping values of livestock are available; market prices for livestock are not available over the entire period. Since beef, milk and livestock prices are highly correlated, we proxy $\sigma_{\ln c}^2$ by the variance of the farms’ milk price such that the variance differs among farms, but we allow for two volatility regimes to capture changes of price volatility due to changes in the CAP. Beginning in 2005, the reduction of market intervention and further decoupling of direct payments, accompanied with an increase of the total milk quota quantity until 2014/15 all lead to a significant increase in the volatility of commodity prices (Keane and O’Connor 2009; Jongeneel et al. 2010). The development of the milk price in Germany as shown in Figure 1 confirms this, and accordingly we define the low-uncertainty period from 1997-2004 and the high-volatility period from 2005-2010.\(^5\) Furthermore, the theoretical idea of co-variances, for instance between the livestock capital and the feed price, would enter the model; however, they are found to be low and are thus neglected in the empirical model.

\(^5\) We index the volatility measure by $i$ and $t$; the respective period of the volatility regimes (1997-2004 and 2005-2010) is not indexed separately.
4 Empirical model

The value function as given in (1) must be specified so that fulfilling validity-properties ensures an optimal solution such that the output and price uncertainty enter the factor demand function. Thus, $J$ is assumed to be concave and $J_k$ assumed to be linear in $w$ and $c$, respectively. Since our model is stochastic, $J_z$ is additionally assumed to be quadratic and $J_{zz}$ is assumed to be linear in $w$ and $c$, respectively. In addition, $\alpha$ should be non-increasing and convex in $w$ and $c$. Furthermore, following the standard procedure would involve the specification of up to 4th order properties of the value function; however, Pietola and Myers (2000) have proven that if the properties hold, uncertainty enters the optimal decision rules. Thus, following Epstein (1981) and Pietola and Myers (2000), the behavioral value function is given by:

$$J^b(z, K) = a_0 + b_k \cdot K + b_y \cdot \ln y + b_{w_z} \cdot \ln w^b_z + b_c \cdot \ln c + A_{kK} \cdot \frac{1}{2} K^2 + A_{yk} \cdot K \cdot \ln y$$

$$+ A_{wy} \cdot \frac{1}{2} (\ln y)^2 + A_{wy} \cdot \ln y \cdot \ln w^b_z + A_{wy} \cdot \frac{1}{2} (\ln w^b_z)^2 + A_{y} \cdot \ln y \cdot \ln c$$

$$+ A_{wz} \cdot \ln w^b_z \cdot \ln c + A_{\alpha} \cdot \frac{1}{2} (\ln c)^2 + M_{cK}^{-1} \cdot c \cdot K + A_{wzK} \cdot w^b_z \cdot K$$

where $a_0$ is an unknown constant term, the $b$-parameters indicate the respective first-order parameters and the respective $A$ - and $M$ -parameters are second-order parameters of the value function. In contrast to non-stochastic models, the last term $\left(M_{cK}^{-1} \cdot c \cdot K + A_{wzK} \cdot w^b_z \cdot K \right)$ enters the behavioral value function to ensure $J^b_z$ is quadratic and $J^b_{zz}$ is linear in $w$ and $c$. Note that $w^b_z = \left(\lambda_2 w_2 / \lambda_1 w_1 \right) = \lambda_2 w_2$ holds.
Inserting the respective derivatives of $J^b$ as given in (8) into the optimized actual factor demand functions (5), (6) and (7) yields stochastic factor demand equations in terms of the value function parameters. Due to space limitations, these equations can be found in Appendix B as equations (B.1), (B.2) and (B.3). Based on these equations, the empirical system of equations is derived; it is recursive in net investment demand, which serves as an explanatory variable in the variable input demand equations. Furthermore, the system is recursive in variable input demand since the demand for purchased feed $x_2$ is an explanatory variable for the demand of other inputs, $x_i$.

The empirical net investment demand equation is given by:

$$I = \tau K \left[ M_{cK} r \left( \alpha \cdot \frac{1}{c_{it}} + A_{cy} \cdot \ln y_{it} + A_{cw_2} \cdot \frac{1}{c_{it}} \ln w_{21,t} + A_{cx} \cdot \frac{1}{c_{it}} \ln c_{it} \right) 
+ (r - M_{cK}) \cdot K_{it} \right] + \sigma_{ln,ct}^2$$

(9)

wherein $I$ represents investment in the quasi-fixed factor (livestock capital), and is the empirical equivalent to $\dot{K}$. Then, the parameter $\alpha = \left( h_{c} + A_{cw_2} \ln \lambda_{21} \right)$. As shown in (9), uncertainty enters the model through an interaction term: uncertainty of the quasi-fixed factor price $\sigma_{ln,ct}^2$ times the quasi-fixed factor level $K_{it}$. Referring to Aiken and West (1990), we use the so-called simple slopes to interpret the interaction terms. This evaluation, however, requires $\sigma_{ln,ct}^2$ and $K_{it}$ to enter the model in addition to their interaction $\left( \sigma_{ln,ct}^2 \cdot K_{it} \right)$; thus, the uncertainty term is further considered as an explanatory variable (though not directly coming from the theoretical modeling approach). Besides interpretation, it is notable that we are – in contrast to static dynamic efficiency models – able to identify technical inefficient levels of livestock capital in use $\tau K$.

For the variable factor demand equations we proceed analogously. The demand for feed is given by:

---

6 We use variables that are related to an interaction term in mean centered form (cf. Jaccard and Turrisi 2003) to ensure a mean of zero that further eases the interpretation of the interaction terms.
\[ x_{2,it} = \frac{1}{\mu} \left[ \tau_K \left( A_{wz} \cdot r \cdot \frac{1}{w_{21,it}} + \alpha_2 \left( (r - M_{cK}) \cdot K_{it} + (\alpha_1 + A_{cw2}) \cdot M_{cK} \cdot r \cdot \frac{1}{c_{it}} \right) + r M_{cK} \cdot A_{cy} \cdot \ln y_{it} \cdot \frac{1}{c_{it}} + M_{cK} \cdot A_{cy} \cdot r \cdot \ln c_{it} \cdot \frac{1}{c_{it}} + M_{cK} \cdot A_{cw2} \cdot r \cdot \ln w_{21,it} \cdot \frac{1}{c_{it}} \right) + \left( M_{cK} \cdot \frac{r - 1}{r} \right) \cdot I_{it} - \frac{1}{2} \cdot K_{it} \cdot \sigma_2^2 \ln c_{it} + \frac{1}{2r} \cdot I_{it} \cdot \sigma_2^2 \ln c_{it} \right) + b_K \cdot M_{cK} \cdot A_{cw2} \cdot r \cdot \frac{1}{w_{21,it} \cdot c_{it}} + A_{kk} \cdot M_{cK} \cdot A_{cw2} \cdot r \cdot \frac{1}{w_{21,it} \cdot c_{it}} - A_{kk} \cdot M_{cK} \cdot A_{cw2} \cdot I_{it} \cdot \frac{1}{w_{21,it} \cdot c_{it}} - \alpha_3 \cdot \frac{1}{2} \cdot K_{it} \cdot \sigma_2^2 \ln w_{21,it} - \alpha_2 \cdot \frac{1}{2r} \cdot I_{it} \cdot \sigma_2^2 \ln w_{21,it} \right]^{10} 
\]

where \( \alpha_2 = (A_{wz2}\lambda_{21}) \), \( \alpha_3 = (\tau s A_{wz2}) \) and \( \alpha_4 = (A_{wz2} \tau s A_{wz2}) \). The demand for other inputs is given by:

\[ x_{1,it} = \tau_s \left[ r \cdot \left( \alpha_5 + b_K \cdot K_{it} + A_{kk} \cdot \frac{1}{2} \cdot K_{it}^2 + A_{cy} \cdot \ln y_{it} \cdot K_{it} + \alpha_2 \cdot w_{21,it} \cdot K_{it} + \alpha_1 \cdot \ln c_{it} \right) + b_y + A_{wz2} \cdot \ln \lambda_{21} \cdot \ln y_{it} + A_{cy} \cdot \frac{1}{2} \cdot (\ln y_{it})^2 + A_{wz2} \cdot \ln y_{it} \cdot \ln w_{21,it} \right] + A_x \cdot \frac{1}{2} \cdot (\ln c_{it})^2 + A_y \cdot \ln y_{it} \cdot \ln c_{it} + \left( b_y + A_{wz2} \ln \lambda_{21} \right) \cdot \ln w_{21,it} + A_{wz2} \cdot \frac{1}{2} \cdot (\ln w_{21,it})^2 + A_{wz2} \cdot \ln w_{21,it} \cdot \ln c_{it} + (M_{cK} \cdot r - 1) \cdot c_{it} K_{it} \right]^{11}
\]

where \( \alpha_5 = (a_0 \cdot r + b_{wz} \cdot \ln \lambda_{21} \cdot r + A_{wz2} \cdot r \cdot \frac{1}{2} \cdot (\ln \lambda_{21})^2) \). The structural parameters are retrieved from the estimated coefficients. To measure time- and individual-specific efficiency scores, we specify \( \tau_k, \tau_s \) and \( \mu \) as functions of variables hypothesized to influence efficiency levels. Technical efficiency of the quasi-fixed factor is defined as:

\[ \tau_k = f \left( z_1, z_2, z_3, D_1, D_2 \right) \]

wherein \( z_1, z_2 \) and \( z_3 \) refer to grassland share, dairy cow per hectare and age of the farm manager, respectively. The variables \( D_1 \) and \( D_2 \) indicate whether the farm is located in
Southern Germany, and whether the farm manager has obtained higher agricultural education, respectively. The technical efficiency of variable factors is given by the following definition:

\[ \tau_x = f(z_2, z_4, z_5, \text{trend}) \]  

(13)

wherein \( z_4 \) refers to the average milk yield per cow, and \( z_5 \) represents the quantity of purchased feed per cow. Furthermore, we define \( \mu \) as:

\[ \mu = f(z_1, z_2, z_6, D, \text{trend}) \]  

(14)

wherein \( z_6 \) refers to expenditures for feed per cow. Age and education represent managerial ability (Kumbhakar et al. 1991; Hallam and Machado 1996), and the number of cows per hectare and grassland share both capture production intensity levels. The regional dummy accounts for local factor availability (like land or quota). The descriptive statistics of all variables can be found in Appendix C, Table C1.

The equations as given in (12), (13) and (14) are inserted into (9), (10) and (11), respectively. To guarantee that the efficiencies are non-negative, we use the exponential transformation. We assume that the discount rate \( r \) is 0.05. The resulting empirical model is a non-linear recursive system of equations. We considered firm-specific means of all explanatory variables according to the Chamberlain approach (Wooldridge 2010) to reduce the potential influence of unobserved heterogeneity effects since panel-specific estimators could not be used. For convergence reasons, allocative efficiency of variable factors \( \lambda_{21} \) is a scalar, and the value function parameter \( b_y \) is set to 1.

5 Results: efficiency of West German dairy farms under uncertainty

Starting with the parameter estimates of the value function, we further analyze technical and allocative efficiency over time and by farm size to examine possible differences between them. With regard to the question of how adjustment pressure, for example as induced by increases in input and milk price volatility in the long run is related to the optimal factor allocation, we first interpret the impact of uncertainty on factor allocation. Second, we elaborate on the effect of uncertainty on the efficiency measurement itself.

Value function parameter  The demand equations are estimated by non-linear least squares using Huber-White standard errors, and the results can be found in Table 2. The R-squared values for net investment demand, feed demand and other input demand are 0.502, 0.904 and 0.827, respectively. The majority of the first- and second-order parameters of the value function as given in (8) is significant at the 1% level. The signs of the value function parameters reveal that nearly all properties of the value function are met. Concavity of the value function in factor prices is satisfied since \( A_{w_2n_2} \) and \( A_{w_1t} \) is negative (-3.246). The first-order parameters with respect to input prices \( (b_{w_2} \) and \( b_{t} \) ) should be positive, thus indicating that the behavioral value function is non-decreasing in input prices.
This is only fulfilled for \( b_{w_2} = 0.00003 \). Convexity in the use of the quasi-fixed input is satisfied since \( A_{KK} \) is positive.

\[
\text{Table 2 Results: first- and second-order value function parameters}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expected sign (theory)</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>--</td>
<td>4.55E+04 ***</td>
<td>1.17E+04</td>
</tr>
<tr>
<td>( b_K )</td>
<td>( \leq 0 )</td>
<td>-3.25E+00</td>
<td>7.10E+00</td>
</tr>
<tr>
<td>( b_{w_2} )</td>
<td>( \geq 0 )</td>
<td>3.02E+05 ***</td>
<td>5.98E+04</td>
</tr>
<tr>
<td>( b_x )</td>
<td>( \geq 0 )</td>
<td>-5.36E+01 ***</td>
<td>2.51E+01</td>
</tr>
<tr>
<td>( A_{KK} )</td>
<td>( &gt; 0 )</td>
<td>3.96E+01</td>
<td>9.48E+01</td>
</tr>
<tr>
<td>( A_{yK} )</td>
<td>--</td>
<td>-4.93E+01</td>
<td>7.29E+01</td>
</tr>
<tr>
<td>( A_{yy} )</td>
<td>--</td>
<td>3.05E+04 ***</td>
<td>9.98E+03</td>
</tr>
<tr>
<td>( A_{w_2,y} )</td>
<td>--</td>
<td>5.10E+05 ***</td>
<td>9.97E+04</td>
</tr>
<tr>
<td>( A_{w_2,w_2} )</td>
<td>( &lt; 0 )</td>
<td>-5.55E+02</td>
<td>7.21E+02</td>
</tr>
<tr>
<td>( A_{xy} )</td>
<td>--</td>
<td>-1.37E+01</td>
<td>3.05E+01</td>
</tr>
<tr>
<td>( A_{w_2,w_2} )</td>
<td>--</td>
<td>4.26E+00</td>
<td>9.14E+00</td>
</tr>
<tr>
<td>( A_{xc} )</td>
<td>( &lt; 0 )</td>
<td>-2.51E+01</td>
<td>3.71E+01</td>
</tr>
<tr>
<td>( M_{cK} )</td>
<td>--</td>
<td>2.90E+02</td>
<td>2.90E+02</td>
</tr>
<tr>
<td>( A_{w_2,K} )</td>
<td>--</td>
<td>-7.10E+00 ***</td>
<td>4.19E-01</td>
</tr>
</tbody>
</table>

Note: *** denotes statistical significance at the 1% level.

From the structural parameters we can retrieve the adjustment rate of the quasi-fixed factor, 
\[
\left[ r - M_{cK} - 1/2 \sigma_{w_c}^2 \right] \] (Epstein and Denny 1983), which amounts to 0.012 per year. This low rate suggests the sample dairy farms adjust sluggishly to their long-run equilibrium level with respect to the quasi-fixed factor (livestock capital). A low adjustment rate for cows has also been found by Howard and Shumway (1988) for the U.S. dairy industry; Stefanou and Chang (1988) found that the adjustment of dairy cows is more sluggish than the adjustment of durable equipment for Pennsylvania dairy farms.

**Perfect efficiency** First we test for the perfect efficiency hypothesis using a Likelihood Ratio test. The restricted model is estimated by setting all efficiency parameters in (9), (10) and (11) equal to one. The likelihood values are compared with the full model such that the null is rejected in favor of the model specification we used. In other words, the sample farms operate inefficiently.

Technical efficiency of the quasi-fixed factor \( 1/\tau_K \) varies between 0.189 and 1, with an average of 0.892. Technical efficiency of variable inputs \( 1/\tau_x \) amounts to 0.782, with a minimum score of 0.10 and a maximum of 1. These findings are in line with previous studies.
in the static and dynamic context. For instance, Kumbhakar and Heshmati (1995) report an average technical efficiency of about 0.847. Cuesta (2000) reports a value of 0.827, and Reinhard and Thijssen (2000) find an average of 0.84. Based on dynamic approaches, Serra et al. (2011) and Emvalomatis et al. (2011) report values of 0.896 and 0.782, respectively.

The allocative efficiency of net investment in the quasi-fixed factor $\mu$ amounts to 0.432 on average, ranging from 0.004 to 5.082. This implies an overuse of net investment in livestock capital, and indicates that the behavioral marginal value of the capital stock $J^b_k$ is less than its actual one, on average. The respective allocative term for the variable inputs is $\lambda_{21}$, which represents the price distortion of feed relative to other inputs, and its estimated average value of about 1.41 implies that the ratio of the shadow prices of feed relative to other inputs is higher than the ratio of actual prices. It may be conjectured that purchased feed compared to other inputs is underused. Note that $\lambda_{21}$ (also $\mu$) is not restricted to the zero-one interval, and is to be interpreted in terms of an overuse or underuse of the resources, respectively. The literature offers mixed results, and the majority do not even report allocative measures for quasi-fixed inputs. Reinhard and Thijssen (2000) found that feed and nitrogen fertilizer is overused compared with intermediate inputs for Dutch dairy farms. For Italian dairy farms, Maietta (2000) found an underuse of forage crops and purchased feed compared to hired labor. A direct comparison of the absolute values of the allocative efficiency scores is only possible with relatively measured terms.

According to our specification of the efficiency terms, they vary over time. In Figure 2, the annual average efficiency scores evaluated at the mean of their explanatory variables other than time are shown (cf. equations (12), (13) and (14)). The technical efficiency terms $1/\tau_k$ and $1/\tau_s$ increase between 1997 and 2010 from 0.833 to 0.963 and from 0.653 to 0.930, respectively. Among other factors, technical progress, improved organization, and herd management likely impact the efficiency scores. For instance, one could think of employed breeding potentials over time. Also, the allocative efficiency of net investment $\mu$ has increased over time; that is, there has been a decrease in the overuse of net investment in the quasi-fixed factor (livestock capital). One possible reason behind this could be the increase in the volatility of input and output prices in recent years (cf., among others, Jongeneel et al. 2010), which in turn is negatively related to investment. In addition, until 2005 strong increases in milk quota prices could be observed, but after 2006, along with the increase in the total quota amount, the prices declined considerably in West Germany. This might also contribute to the reduction in capital overuse. Furthermore, the quota policy may have introduced policy risk to farmers, which in turn may also lead to more reluctant investment behavior.

---

7 In 2012 an average price of about 8 €-cent per kg was reported for West Germany, while in 2006 the price was, on average, 51 €-cent per kg (Deutscher Bauernverband 2013).
Figure 2 Average annual efficiency scores

![Figure 2](image)

**Efficiency, herd size and average milk yield**

Herd size is measured using the number of dairy cows per farm. We distinguish between 3 categories using the respective tertiles: small (<35 cows), medium (35-50 cows) and large farms (>50 cows). Milk yield measured in tons per cow and year is analogously categorized as low (< 5.9 tons per cow and year), medium (5.9-6.8 tons per cow and year) and high (> 6.8 tons per cow and year). The average efficiency scores per category and for the whole sample are shown in Table 3.

**Table 3 Average efficiency scores by category**

<table>
<thead>
<tr>
<th>Category</th>
<th>Level</th>
<th>Technical efficiency</th>
<th>Allocative efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>quasi-fixed factor</td>
<td>variable factors</td>
</tr>
<tr>
<td>Herd size</td>
<td>&lt; 35</td>
<td>0.885</td>
<td>0.782</td>
</tr>
<tr>
<td>[# of cows]</td>
<td>35 - 50</td>
<td>0.891</td>
<td>0.768</td>
</tr>
<tr>
<td></td>
<td>&gt; 50</td>
<td>0.899</td>
<td>0.794</td>
</tr>
<tr>
<td>Milk yield</td>
<td>&lt; 5.9</td>
<td>0.891</td>
<td>0.798</td>
</tr>
<tr>
<td>[tons per cow, year]</td>
<td>5.9 - 6.8</td>
<td>0.885</td>
<td>0.767</td>
</tr>
<tr>
<td></td>
<td>&gt; 6.8</td>
<td>0.899</td>
<td>0.779</td>
</tr>
<tr>
<td>Mean</td>
<td>--</td>
<td>0.892</td>
<td>0.782</td>
</tr>
</tbody>
</table>

The results indicate that the differences in the category “herd size” with respect to average technical efficiency are rather small, but larger farms show a slightly higher score. Note the differences in technical efficiency of the quasi-fixed factor are only significant between small and large farms, whereas the differences in technical efficiency of variable factors are only significant between medium and large farms. In addition, large farms show a higher minimum technical efficiency score of the quasi-fixed factor (0.223 versus 0.189).
Further, small and medium-sized farms exhibit a higher level of allocative efficiency compared to large farms, where the difference is significant except between medium-sized and large farms. The results with respect to the average milk yield state the following: farms with a high milk yield per cow have a slightly higher mean technical efficiency of the quasi-fixed factor, whereas farms with a low milk yield have a higher technical efficiency of variable factors. The differences in technical efficiency of the quasi-fixed factors are only significant between medium and high average milk yields, whereas the differences in technical efficiency of variable factors are all significant except between medium and high average milk yields. In addition, the allocative efficiency of net investment increases with decreasing average milk yields (significant differences). The results of these two categories reveal a weak correlation between technical efficiency and herd size, as well as milk yield for our sample. The direct comparison to other studies is difficult since the size categories differ by region. Mosheim and Lovell (2009) found increasing technical and allocative efficiency scores for U.S. dairy farms with herd sizes of <30 cows up to >2,000 cows. A positive relationship for technical efficiency is further found by Hadley (2006) for U.K. dairy farms, by Alvarez and Arias (2004) for Spanish dairy farms, and by Kumbhakar et al. (1991) for U.S. dairy farms.

Impact of uncertainty The results reveal a significant impact of uncertainty on optimal factor allocation. While the demand for feed is negatively related to the variance of the concentrate price although not significantly, investment is negatively related to the variance of the milk price, which was used as a proxy for factor price variance. This finding is in line with the theoretical idea of Dixit and Pindyck (1994), and has been empirically confirmed, for instance by Pietola and Myers (2000) and Hinrichs et al. (2008).

Since the variance variable $\sigma^2_{w.c,it}$ enters the net investment demand equation within the interaction term $\left(\sigma^2_{w.c,it} \cdot K_{it}\right)$, one must refer to the concept of simple slopes according to Aiken and West (1990) to examine the uncertainty effect on investment at different levels of livestock capital. Here we distinguish according to the mean herd size using the mean and the standard deviation of the herd-size: first, the mean herd size with one standard deviation measure below the mean (26 cows); second, the mean herd size (44 cows); and third, the mean plus one standard deviation (62 cows). The simple slopes are shown in Table 4. The negative effect of uncertainty on the net investment demand is more pronounced for larger farms, while it is a positive effect for smaller farms, though the effect is only significant for mean-sized farms.

<table>
<thead>
<tr>
<th>Mean herd size</th>
<th>Simple slope</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean – standard deviation: 26 cows</td>
<td>0.118</td>
<td>0.617</td>
</tr>
<tr>
<td>Mean: 44 cows</td>
<td>-0.454***</td>
<td>0.128</td>
</tr>
<tr>
<td>Mean + standard deviation: 62 cows</td>
<td>-1.027</td>
<td>0.734</td>
</tr>
</tbody>
</table>

Note: *** denotes statistical significance at the 1% level.
The core question remains how large the omitted variable bias is on the estimates of the coefficients and the predicted efficiency scores if the impact of uncertainty on the optimal factor allocation is ignored. Hence, we estimate two types of the investment equations: one with uncertainty variables (a), and one without uncertainty variables (b). The results of both models are shown in Table 5. The estimated coefficients differ in their absolute values, though not in their significance levels; however, the uncertainty model (a) shows comparatively lower standard errors. These findings indicate a considerable omitted variables bias through ignoring the uncertainty variable. Moreover, we conduct a Likelihood Ratio test, which reveals a statistically significant better performance of the model with uncertainty (a) at the 1% level. In addition to the better model performance from an econometric viewpoint, the implications of the omitted variables bias are more severe in this context. Comparing the mean technical efficiency of the quasi-fixed factor use shows a significantly higher value if uncertainty is considered (0.892 versus 0.845). That is, ignoring the impact of uncertainty will lead to the conclusion that firms are less efficient even though it is not the case. Thus, farms appear seemingly technically inefficient.

| Table 5 Investment demand and technical efficiency: results with and without uncertainty |
|-------------------------------------------------|------------------|------------------|
| Explanatory variables                        | (a) with uncertainty | (b) without uncertainty |
|                                                | Estimate  | Standard error | Estimate  | Standard error |
| Inverse capital price                         | -0.076 ** | 0.041           | -0.136 ** | 0.077           |
| Inverse capital price × Output                | -0.020    | 0.020           | -0.004    | 0.034           |
| Inverse capital price × input price relation  | 0.006     | 0.008           | 0.015     | 0.016           |
| Inverse capital price × capital price         | -0.037 *  | 0.022           | -0.085 *  | 0.052           |
| Livestock capital                             | 0.021     | 0.025           | -0.014    | 0.040           |
| Capital price uncertainty × livestock capital | -1.448    | 0.949           | --        | --              |
| Capital price uncertainty                     | -0.454 ***| 0.113           | --        | --              |
| Technical efficiency                          | 0.892     | --              | 0.845     | --              |

Note: *, ** and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Applying model versions (a) and (b) to the feed and other input demand equation reveals a similar effect. Comparing the allocative efficiency scores for the quasi-fixed input $\mu$ and for the variable inputs $\lambda_{21}$ reveals that if uncertainty is disregarded, the farms are also seemingly inefficient, that is, their inefficiency scores are higher in the model without uncertainty variables.
6 Concluding remarks

This paper examines the dynamic efficiency of West German dairy farms and the effect of uncertainty in the optimal factor allocation process. Thus far, uncertainty has been ignored when deriving dynamic efficiency measures, which also prevented the identification of technical efficiency measures for quasi-fixed inputs. Our model closes these gaps, and we combine investment under uncertainty with (deterministic) dynamic efficiency analysis using a static shadow cost approach and a stochastic dynamic dual model of investment. To operationalize the complex theoretical model structure, we use one quasi-fixed input (livestock capital) and two variable inputs (purchased feed and other inputs). The results may vary if a different number of quasi-fixed inputs such as land and milk quota is considered. Recently, Rungsuriyawiboon and Hockmann (2012) extend the model of Rungsuriyawiboon and Stefanou (2007) to capture multiple quasi-fixed inputs. Further disaggregated variable inputs could also provide further insights. Another simplification the paper introduces is the treatment of the costs attached to adjusting the quasi-fixed capital stock. Here, they are expressed by a simple adjustment rate to be used in the linear accelerator model rather than more sophisticated adjustment cost functions, which have been suggested by e.g., Hamermesh and Pfann (1996). Our results show that West German dairy farms are technically efficient with regard to the quasi-fixed factor at 0.892. The technical efficiency of variable inputs amounts to 0.782, on average. Both measures show a considerable increase over time. Further, the allocative efficiency of net investment is 0.432 on average, and the allocative efficiency of the variable factors (“feed”) in relation to the numeraire factors (“other”) amounts to 1.41.

With regard to the initial question of how changes in the policy environment impact the adjustment process in the dairy sector and the optimal factor allocation of farms, we state the following. The consideration of uncertainty is crucial for deriving (dynamic) efficiency measures: neglecting uncertainty within the estimation procedure will overestimate the average inefficiency score. Thus, farms appear seemingly inefficient. Further, the results reveal a significant interaction between price uncertainty and livestock capital. We show that uncertainty has a negative impact on farm-level investments in herd size; this effect increases with the number of cows per farm. The demand for feed is negatively related to the variance of the concentrate price, though not significantly. The allocative efficiency of net investment has increased over time; that is, there is a decrease in the overuse of net investment in the quasi-fixed factor (livestock capital). This process goes along with the changes in the policy environment, particularly the increase in the volatility of input and output prices. In addition, after 2006 milk quota prices declined considerably in West Germany. Thus, this provides indirect evidence on overinvestment in dairy capital—mainly introduced by investing in milk production rights.

This is not only interesting from an academic point of view; it has further implications for the analysis of the relative performance of specific farm types like cash crop or other livestock farms. Due to external and internal conditions such as the socio-economic environment or farmer characteristics, the level of uncertainty may vary among farm types. As a consequence, the impact of uncertainty on efficiency level estimates may differ as well. Hence, an adequate
measurement of dynamic efficiency is important for obtaining meaningful results with regards to the evaluation of the relative farm-type performance.

Acknowledgments

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References
ZMP (various volumes) Marktbilanz Milch. Bonn
Appendix A: Derivation of the dynamic efficiency model under uncertainty

The optimization problem for a representative firm is expressed in terms of the value function $J(\cdot)$ as follows:

$$J(w(0), c(0), y(0), K(0)) = \min_{I_n(t)} \int_0^\infty e^{-\alpha t} \left[ \sum_n (w_n(t) \cdot x_n(t)) + \sum_m (c_m(t) \cdot K_m(t)) \right] \cdot dt$$  \hspace{1cm} (A.1)

subject to

$$y(t) \leq F(x(t), K(t), \dot{K}(t))$$  \hspace{1cm} (A.2)

$$\dot{K}_m(t) = (I_m(t) - \delta \cdot K_m(t)) \quad K_m(t) > 0$$  \hspace{1cm} (A.3)

$$dz = \alpha \cdot dt + \gamma(t) \cdot dv$$  \hspace{1cm} (A.4)

To measure inefficiency, the actual and behavioral value functions are distinguished. The behavioral value function guarantees the cost-minimizing relation under shadow prices. In contrast, the actual value function represents a fully efficient input use. The behavioral is equivalent to the actual value function in the presence of perfect efficiency; they differ in the presence of inefficiency. The Hamilton-Jacobi-Bellman (HJB) equation corresponding to the behavioral value function is:

$$r J^b(w_n^b(t), c_m(t), K_m(t), y(t)) = \sum_n (w_n^b(t) \cdot x_n^b(t)) + \sum_m (c_m(t) \cdot K_m(t))$$

$$+ \sum_m (J^b_m \cdot (I_m(t) - \delta \cdot K_m(t))) + \sum_j J^b_j \cdot \alpha + \frac{1}{2} \cdot \Omega^b$$  \hspace{1cm} (A.5)

wherein $J^b_m = \mu^b_m \cdot J^a_{K_n}$. Therein, $J^a_{K_n}$ denotes the marginal value of actual (superscript a) capital, $J^b_{z_j}$ denotes derivatives of $J^b$ with respect to vector $z$ and $\gamma^b(t) \geq 0$ is the behavioral short run marginal cost of production. Differentiating the behavioral HJB equation in (A.5) with respect to $\ln c_m$ and $\ln w_n$ yields the optimal behavioral factor demand equations. Using $\dot{K}_m^b = \left(1/\tau_{K_n}\right) \cdot \dot{K}_m$ and $\dot{x}_n^b = \left(1/\tau_{x_n}\right) \cdot x_n$ in the optimal behavioral factor demand equations yields the optimized actual net investment and variable factor demand equations:

$$\dot{K}_m^a = \tau_{K_n} \cdot \dot{K}_m^b = \tau_{K_n} \cdot \left(J^b_{K_n, \ln c_m}\right)^{-1} \left( r J^b_{\ln c_m} - c_m \cdot K_m - \sum_{m \neq m} (\dot{K}_m^b \cdot J^b_{m, \ln c_m}) - \frac{1}{2} \cdot \Omega^b_{\ln c_m} \right)$$  \hspace{1cm} (A.6)

$$\dot{x}_n^a = \tau_{x_n} \cdot \dot{x}_n^b = \tau_{x_n} \cdot \left( r J^b_{\ln w_n} - \sum_{m} (J^b_{K_m, \ln w_n} \cdot \dot{K}_m^b) - \frac{1}{2} \cdot \Omega^b_{\ln w_n} \right)$$  \hspace{1cm} (A.7)
The optimized actual HJB corresponding to the optimized actual value function is given by:

\[
rf^a = \sum_{n} \left( w_n \cdot x_n^a \right) + \sum_{m} \left( c_m \cdot K_m \right) + \sum_{m} \left( J_{K_n}^a \cdot \dot{K}_m^a \right) + \frac{1}{2} \cdot \Omega^a
\]  

(A.8)

where \( \Omega^a = \sum_{j=1}^{1+\frac{\alpha}{2}} \sum_{j=1}^{1+\frac{\alpha}{2}} J_{i,j}^a \sigma_{i,j} \cdot \). Therein \( J_{i,j}^a \) denotes the respective second partial derivatives of \( J^a \) with respect to vector \( z \). Differentiating (A.8) with respect to \( \ln c_m \) and \( \ln w_n \), yields the optimized actual net investment and variable factor demand equations under perfect efficiency:

\[
\dot{K}_m^a = \left( J_{K_n, \ln c_m}^a \right)^{-1} \left( rJ_{\ln c_m}^a - c_m \cdot K_m - \sum_{m' \neq m} \left( \dot{K}_{m'}^a \cdot J_{K_n, \ln c_m}^a \right) - \frac{1}{2} \cdot \Omega^a_{\ln c_m} \right)
\]  

(A.9)

\[
x_n^a = \frac{1}{w_n} \left( r \cdot J_{\ln w_n}^a - \sum_{m} \left( J_{K_n, \ln w_n}^a \cdot \dot{K}_m^a \right) - \frac{1}{2} \cdot \Omega^a_{\ln w_n} \right)
\]  

(A.10)

The optimized actual value function is expressed in terms of the behavioral value function. To achieve this, in the optimized actual HJB equation (A.8), \( J_{K_n}^a \) is substituted by \((1/\mu_m) \cdot J_{K_n}^b\), \( x_n^a \) by (A.7), \( \dot{K}_m^a \) by (A.6) and \( \Omega^a \) by \( \Omega^b \). The optimized actual HJB equation is expressed in terms of behavioral value function:

\[
rf^a = \sum_{n} \left[ \frac{\tau_n}{\beta_n} \left( rJ_{\ln w_n}^b - \sum_{m} \left( J_{K_n, \ln w_n}^b \cdot \dot{K}_m^b \right) - \sum_{m' \neq m} \left( \dot{K}_{m'}^b \cdot J_{K_n, \ln c_m}^b \right) - \frac{\Omega_{\ln c_m}^b}{2} \right) \right] + \frac{\Omega^b_{\ln w_n}}{2} + \sum_{m} \left( c_m \cdot K_m \right)
\]  

(A.11)

Differentiating (A.11) and substituting the derivatives into equation (A.9) yields the \( m^{th} \) optimized actual net investment demand in terms of the behavioral value function.\(^8\)

---

\(^8\) Higher than second order derivatives of \( J^a \) are ignored.
\[
\left\{ \frac{1}{r} \sum_{m} \frac{\tau_{Kn}}{\lambda_{n}} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \right) + \frac{1}{r} \left( 1 - \frac{\tau_{Kn}}{\mu_{m}} \right) c_{m} - \frac{\tau_{Kn}}{r \mu_{m}} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \right) \right\} + \sum_{m} \frac{\tau_{Kn}}{\mu_{m}} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \cdot K_{m} \right) - \frac{\tau_{Kn}}{r \mu_{m}} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \cdot K_{m} \right) - \frac{\Omega_{K_{m, \ln \epsilon_{m}}}^{b}}{2r} \right\} K_{m}^{\circ} = \\
\sum_{n} \frac{\tau_{Kn}}{\lambda_{n}} \cdot rJ_{ln \epsilon_{m}, \ln \epsilon_{m}}^{b} - \sum_{n} \frac{r \tau_{Kn}}{\lambda_{n}} \sum_{m} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \cdot K_{m} \right) - \sum_{n} \frac{\tau_{Kn}}{2 \lambda_{n}} \cdot \Omega_{ln \epsilon_{m}, \ln \epsilon_{m}}^{b} \\
+ \sum_{n} \frac{\tau_{Kn}}{\lambda_{n}} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \cdot K_{m} \right) + \sum_{n} \frac{\tau_{Kn}}{2 \lambda_{n}} \sum_{m} \left( J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot J_{K_{m, \ln \epsilon_{m}}}^{b} \cdot c_{m} \cdot K_{m} \right)
\] 

(A.12)

Similarly, differentiating (A.11) and inserting the derivatives into equation (A.10) yields the \( n^{th} \) optimized actual variable factor demand in terms of the behavioral value function.\(^9\)

\[^9\] Higher than second order derivatives of \( J^{s} \) are ignored.
\[ x''_n = \frac{1}{w_n} \left[ r \sum_n^{\tau_{x_n}} \cdot J^b_{\ln w_n, \ln w_n} - r \sum_n^{\tau_{x_n}} \sum_m^{J^b_{K_n, \ln w_n, \ln w_n}} - \frac{1}{2} \sum_n \left( \frac{J^b_{K_n, \ln w_n, \ln w_n}}{\mu_m} \cdot \Omega^b_{\ln w_n, \ln w_n} + J^b_{K_n, \ln w_n, \ln w_n} \right) \right] \]

\[ + \frac{1}{2} \sum_n \left( \frac{\tau_{x_n}}{\mu_m} \cdot J^b_{K_n, \ln w_n, \ln w_n} \right) + \frac{1}{2} \sum_m \left( \frac{\tau_{K_m}}{\mu_m} \cdot c_m \cdot K_m \cdot J^b_{K_n, \ln w_n} \right) \]

\[ + \frac{1}{2} \sum_m \left( \frac{\tau_{K_m}}{\mu_m} \cdot J^b_{K_n, \ln w_n, \ln w_n} \right) - \frac{1}{2} \sum_n \left( \frac{\tau_{x_n}}{\mu_m} \cdot \Omega^b_{\ln w_n, \ln w_n} \right) - \frac{1}{2} \sum_m \left( \frac{\tau_{K_m}}{\mu_m} \cdot J^b_{K_n, \ln w_n, \ln w_n} \right) \]

\[ + \frac{1}{2} \sum_m \left( \frac{\tau_{K_m}}{\mu_m} \cdot J^b_{K_n, \ln w_n, \ln w_n} \right) + \frac{1}{2} \sum_m \left( \frac{\tau_{K_m}}{\mu_m} \cdot c_m \cdot K_m \cdot J^b_{K_n, \ln w_n} \right) \]

Rearranging the optimized version of the behavioral HJB equation in (A.5) yields the behavioral demand for the numeraire. The optimized actual demand for the numeraire variable factor in the presence of uncertainty is given as:

\[ x''_b = \tau_{x_n} \cdot J^b_{\ln w_n, \ln w_n} - \sum_n (W_n^b \cdot \dot{x}_n) - \sum_m (c_m \cdot K_m) - \sum_m (J^b_{K_n, \ln w_n, \ln w_n}) \]

**Appendix B: Factor demand equations in terms of the value function parameters**

Using the value function specification as presented in (8) considering one quasi-fixed input and two variable inputs, the stochastic factor demand equations (A.6), (A.13) and (A.14) are given as follows.

The net investment demand (A.6) in terms of the value function parameters is given by:

\[ \dot{K} = \tau_K \left[ \left( M_{\lambda K} \cdot \ln w_{21} + \frac{1}{c} \cdot \ln y + A_{w_2} \cdot \frac{1}{c} \cdot \ln w_{21} + A_{c} \cdot \frac{1}{c} \cdot \ln c \right) + \left( r - M_{\lambda K} \right) \cdot K \right] \]

The optimized actual demand for the variable inputs (A.13) and (A.14) in terms of the value function (8) are given by:
\[ x_2 = \frac{1}{\mu} \left[ \tau_{K} \left( A_{w_2} r \cdot \frac{1}{w_{21}} + (A_{w_2,K} \lambda_{21} r - M_{c,k} A_{w_2,K} \lambda_{21}) \cdot K + A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2} r \cdot \ln y \cdot \frac{1}{c} \right) 
+ \left( b, A_{w_2,K} + A_{w_2} \ln \lambda_{21} A_{w_2} + A_{w_2} A_{w_2,K} \lambda_{21} M_{c,k} r \cdot \frac{1}{c} + A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2} r \cdot K \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} \right) 
+ A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2} r \cdot \ln c \cdot \frac{1}{c} + b, M_{c,k} A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2} r \cdot \ln w_{21} \cdot \frac{1}{c} \right) 
+ A_{w_2,K} \lambda_{21} M_{c,k} A_{w_2} r \cdot \ln y \cdot \frac{1}{w_{21}} \cdot c + \left( A_{w_2,K} \lambda_{21} \cdot M_{c,k} \frac{1}{r} - A_{w_2,K} \lambda_{21} \right) \cdot \dot{K} - A_{w_2,K} \lambda_{21} \frac{1}{2} \cdot K \sigma_{\ln c}^2 
- A_{w_2,K} \lambda_{21} \cdot \dot{K} \sigma_{\ln c}^2 
+ r \left( A_{w_2,K} \tau_{\ln c} \cdot \frac{1}{w_{21}} + \tau_{\ln c} A_{w_2,K} r \cdot K - \tau_{\ln c} A_{w_2,K} M_{c,k} A_{w_2} \cdot \frac{1}{c} \right) \right) - A_{w_2,K} \lambda_{21} \frac{1}{2} \cdot \dot{K} \sigma_{\ln w_2}^2 \right] \] (B.2)

and

\[ x_1 = \tau_{\ln c} \left[ r \left( a_y + b, y \ln \lambda_{21} A_{w_2} \left( \frac{1}{2} \left( \ln \lambda_{21} \right)^2 \right) + b, K \cdot K + A_{w_2} \frac{1}{2} \cdot K^2 + A_{w_2} \cdot \ln y \right) 
+ A_{w_2,K} \lambda_{21} \cdot w_{21} K + \left( b, y + A_{w_2} \ln \lambda_{21} \right) \cdot \ln y + A_{w_2} \frac{1}{2} \cdot \left( \ln y \right)^2 \right) 
+ \left( A_{w_2} \ln \lambda_{21} + b, y \right) \cdot \ln c + A_{w_2} \frac{1}{2} \cdot \left( \ln c \right)^2 \right) \right) 
+ \left( A_{w_2} \ln \lambda_{21} \right) \cdot \ln w_{21} + A_{w_2} \frac{1}{2} \cdot \left( \ln w_{21} \right)^2 \right) \right) \left( M_{c,k} r - 1 \right) \cdot c K - b, \frac{1}{\tau_{K}} \cdot \dot{K} 
- \frac{1}{\tau_{K}} A_{w_2,K} \cdot \dot{K} - A_{w_2} \frac{1}{\tau_{K}} \cdot \ln y \dot{K} - A_{w_2,K} \lambda_{21} \frac{1}{\tau_{K}} \cdot w_{21} \dot{K} - M_{c,k} \frac{1}{\tau_{K}} \cdot c \dot{K} 
- M_{c,k} \frac{1}{2} \cdot K \sigma_{\ln c}^2 \right) 
- A_{w_2,K} \lambda_{21} \frac{1}{2} \cdot \dot{K} w_{21} \sigma_{\ln w_2}^2 - A_{w_2,y} \cdot \sigma_{\ln w_2, \ln y} - A_{w_2,y} \cdot \sigma_{\ln w_2, \ln y} 
- A_{w_2} \cdot \sigma_{\ln c, \ln w_2} - A_{w_2} \lambda_{21, w_2} \frac{1}{2} \cdot \sigma_{\ln c, \ln y} - A_{w_2} \lambda_{21, w_2} \frac{1}{2} \cdot \sigma_{\ln y}^2 - A_{c,c} \frac{1}{2} \cdot \sigma_{\ln c}^2 \right] \right) \left( \lambda_{21} \cdot w_{21} x_2 \right) \] (B.3)
## Appendix C: Additional data information

### Table C1 Summary statistics of main variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk production per farm [metric tons]</td>
<td>283.14</td>
<td>129.55</td>
<td>50.64</td>
<td>1,613.02</td>
</tr>
<tr>
<td>Purchased feed [€]</td>
<td>128.86</td>
<td>61.28</td>
<td>18.14</td>
<td>248.33</td>
</tr>
<tr>
<td>Other inputs [€]</td>
<td>696.98</td>
<td>1,147.73</td>
<td>58.39</td>
<td>12,065.22</td>
</tr>
<tr>
<td>Livestock capital [# of cows]</td>
<td>44</td>
<td>18</td>
<td>10</td>
<td>201</td>
</tr>
<tr>
<td>Capital price [€ per cow]</td>
<td>241</td>
<td>146</td>
<td>0.22</td>
<td>1,898</td>
</tr>
<tr>
<td>Grassland share [%]</td>
<td>0.729</td>
<td>0.248</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Dairy cows per hectare [# of cow per hectare]</td>
<td>0.99</td>
<td>0.32</td>
<td>0.24</td>
<td>2.69</td>
</tr>
<tr>
<td>Age [years]</td>
<td>46</td>
<td>10</td>
<td>19</td>
<td>101</td>
</tr>
<tr>
<td>Average milk yield [tons per cow, year]</td>
<td>6.39</td>
<td>1.10</td>
<td>3.04</td>
<td>10.97</td>
</tr>
</tbody>
</table>

*Note:* The database is the BMELV-Testbetriebsnetz, 1996-2010.