An Aggregate U.S. Feed Grain Model*

By Karl D. Meilke

The demand for U.S. feed grain is estimated using a six-equation simultaneous model. Four different utilizations of feed grain and feed grain price as well as the number of animal units fed are estimated. The reduced form of the model is used to provide forecasts of the endogenous variables for the 1973 and 1974 crop years. The structural equations are estimated using two-stage least squares and annual data from 1945 to 1972.

Keywords: Simultaneous equations, demand, model, feed grains, feed concentrates, two-stage least squares, impact analysis.

The feed grain economy is an important sector in U.S. agriculture. Production of the four feed grains—corn, oats, barley, and grain sorghum—contributed $9.0 billion to the $68.8 billion gross farm income in 1972. Likewise, the feed grains were the major input into a livestock sector that produced $35.6 billion of 1972 farm income. Given the importance of the feed grain sector, relatively little research interest has been shown in the demand for feed grains since the early 1950's when a series of U.S. Department of Agriculture studies dealt with the problem (6, 12, 15). However, events of the 1970's have served to refocus attention on the feed grain sector. First, in 1970, corn blight caused corn production to fall to its lowest level since 1964. Second, record high prices for all grains in 1973 led to the depletion of Government feed grain stocks. Third, the drought-reduced crop of 1974 leaves the United States with its lowest total supply of feed grain since the 1957 crop year. Each of the preceding events has led to concern about the future level of feed grain price and utilization. To provide insight into the feed grain market and to help answer some of the questions being asked, a six-equation simultaneous model of feed grain demand is developed in this paper.

The primary objective of the study is to formulate and estimate a model of feed grain demand that can be used to provide short-run forecasts of the four major utilizations of feed grain, the price of feed grain, and the number of grain-consuming animal units on feed. Improved knowledge of the structure of the feed grain market will enable researchers to study the effects of changes in market structure on the entire system. Examples of the specific type of questions the model can provide answers for are:

(1) What is the effect of a change in the Government loan rate for feed grains?
(2) What is the effect of a bumper feed grain crop with different price support levels?
(3) What would be the effect of eliminating concessional exports?
(4) What is the relationship between changes in personal consumption expenditures, animal units, and feed prices?
(5) What effect would a 10 percent decline in the production of feed grains in major importing countries have on the U.S. feed grain economy?
(6) What is the effect on feed grains of a change in the price of high protein feed?
(7) How are the secondary demands for feed grains affected by price changes?
(8) What is the relationship between feed grain prices and livestock prices?

The Statistical Model: Structural Equations

The feed grain market is represented by a six-equation simultaneous model. The six endogenous variables explained by the model are (1) quantity of feed grain fed, (2) quantity of feed grain used in food and industrial products, (3) quantity of feed grain stored, (4) quantity of feed grain commercially exported, (5) price of feed grain, and (6) number of grain-consuming animal units fed. The structural equations in the model and the variable definitions are given in table 1.

In the econometric model the demand for seed and the import levels are not considered. To preserve the equilibrium conditions, production is adjusted by adding imports and subtracting the quantity used for seed. Likewise, the supply of feed grain is not considered in this study. The production of feed grain is taken as predetermined, which implicitly assumes that current feed grain price has no effect on current production.

The structural equations in the model are estimated using two-stage least squares and crop year data from 1945 through 1972. All of the behavioral equations in the system are linear in actual values and overidentified.

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¹For examples of work on the supply side see (10, 11, 19, 20, 21).
Behavioral relationships

1. Feed: \( QF_t, PF_t, AU_t; PHPF_t, PL_t, e_1 \)
2. Food and industry: \( F_t, PF_t, Y_t, DY_t, WPI_t, DC_t, e_2 \)
3. Export: \( QE_t, PF_t, PHPF_t, CE_t, AUF_t, R_t, SF_t, e_3 \)
4. Inventory: \( S_t, PF_t, PS_t, S-1, QG_t, e_4 \)
5. Animal units fed: \( AU_t, PF_t, PF-1, PL-1, Y_t, AU-1, e_5 \)

Identity

6. \( QF_t + QE_t + F_t + S_t + CE_t = S-1 + QG_t \)

Variable identification

\( QF_t \) = quantity of feed grains fed during crop year \( t \). Million tons. (endogenous) (29, 30)

\( PF_t \) = an index of prices received by farmers for feed grain during crop year \( t \). (1945 = 100) (endogenous) (29, 30)

\( AU_t \) = number of grain-consuming animal units produced during crop year \( t \). Million units. (endogenous) (29, 30)

\( PHPF_t \) = an index of wholesale prices of 11 high-protein feeds during crop year \( t \). (1967 = 100) (exogenous) (29)

\( PL_t \) = an index of prices received by farmers for livestock and livestock products during crop year \( t \). (1910-14 = 100) (exogenous) (27)

\( F_t \) = quantity of grain used in producing food and industrial products during crop year \( t \). Million tons. (endogenous) (28, 29, 30)

\( Y_t \) = real personal consumption expenditures during crop year \( t \). Billion dollars. (exogenous) (26)

\( DY_t \) = a zero-one variable equal to zero for 1945 to 1952 and one for 1953 to 1972 multiplied by real personal consumption expenditures during period \( t \). (exogenous)

\( QE_t \) = quantity of feed grains commercially exported during crop year \( t \). Million tons. (endogenous) (29, 30, 31)

\( CE_t \) = quantity of feed grain exported under PL-480 during crop year \( t \). Million tons. (exogenous) (31)

\( R_t \) = an index of production of feed grain per animal unit in Japan and seven European countries. (1960 = 100) (exogenous) (25)

\( PS_t \) = an index of the Government loan rate during crop year \( t \). (1945 = 100) (exogenous) (29, 30)

\( PL-1 \) = livestock and livestock product prices lagged one year. (1910-14 = 100) (exogenous) (27)

\( PF-1 \) = feed grain prices lagged one year. (1945 = 100) (exogenous) (29, 30)

\( DC_t \) = a zero-one variable equal to zero for 1945 to 1952 and one for 1953 to 1972. (exogenous)

\( QG_t \) = quantity of feed grains produced, adjusted for imports and seed use, during crop year \( t \). Million tons. (exogenous) (29, 30)

\( AU-1 \) = number of grain-consuming animal units lagged 1 year. Million units. (exogenous) (29, 30)

\( WPI_t \) = wholesale price index for all commodities during crop year \( t \). (1957-59 = 100) (exogenous) (26)

\( AUF_t \) = an index of number of animal units fed in Japan and seven European countries. (1960 = 100) (exogenous) (25)

\( SF_t \) = July 1 stocks of feed grain in three major exporting countries, Million metric tons. (exogenous) (28)

\( S_t \) = end of crop year carryover stocks of feed grain, Million tons. (endogenous) (29, 30)

\( S-1 \) = end of crop year carryover stocks lagged 1 year. Million tons. (exogenous) (29, 30)

Livestock Feed

In the equation used to estimate the demand for feed grains to be fed, the price of feed, the quantity of feed fed, and the number of animal units are endogenous variables. The prices of high-protein feed and livestock are considered exogenous.

Livestock prices are almost always treated as exogenous in feed grain models, for two primary reasons. First, as Fox has argued, the supply and consumption of livestock products are nearly predetermined in the short run, and for this reason livestock prices can be treated as exogenous in feed grain models (8, p. 101). Second, not treating livestock prices as exogenous results in considerable expansion of the econometric model. Since the prices of different livestock and livestock products are affected by different demand forces, the model would most likely have to be expanded to take explicit account of several different livestock products. Therefore, in this model livestock prices are treated exogenously in order to limit the scope of the study and because it appears the error involved is quite small. In general we would expect the price of livestock to affect feeding rates while the number of animal units fed accounts for the direct demand shifting influence of changes in the number of animals fed.

The treatment of high-protein feed prices as exogenous is a much more important and crucial decision. In King’s study of the byproduct feed market, he considered the price of feed grains and the price of protein feed to have been determined simultaneously (12, pp. 79-88). King found a strong substitution effect between the feed grains and the protein feeds. Conditions have changed substantially since King’s study, which was based on 1921-41 data. In 1941 less than 10 percent of the protein feed fed was soybean meal. By 1970 almost half of the protein feed fed was soybean meal. This has created a completely different market from the one King was analyzing. No longer can we safely assume that the production of protein feed equals the consumption of protein, since exports are extremely important for soybean meal (9). King recognized this fact when he stated that to the extent that crushing of soybeans depends on the current price of soybean meal, it is not correct to assume that production of meal is predetermined (12, p. 82).
If we are to treat high-protein feed prices as exogenous, the relevant criterion is whether feed grain prices or feed grain consumption affects high-protein feed prices. Fox argues that changes in the price of high-protein feeds are relatively independent of changes in the price of feed grains. He argues that much of the correlation between feed grain and high-protein prices is due to the common effect of changes in livestock prices (7, p. 69). J. P. Houck in his model of the soybean market does not include feed grain prices in his equation relating to the domestic demand for soybean meal (9, p. 17).

Considering the end uses of high-protein feed, it also seems logical to treat the price of high-protein feed as exogenous. When the price of high-protein feed increases, farmers are likely to substitute home-grown feed grain for high-protein feed. On the other hand, the major user of protein feed is the formula feed industry (14, pp. 75-76). Therefore a change in the price of feed grain will have little effect on protein feed use because of feed manufacturers’ inability to substitute grain feed for protein feed and still maintain a balanced ration.

The statistical estimate of the quantity of feed grains fed is presented below:

\[
Q_{F_t} = -90.41 - .439 PF_t + 2.01 AU_t \\
(39.34) (.097) (0.37) \\
[-.411] [1.92] \\
+.157 PHPF_t + .066 PL_t \\
(.061) (.053) \\
[.14] [1.7] \\
\]

**Standard error of estimate = 5.92**

**D.W. = 1.58**

**R^2 = .93**

The standard error of each variable’s coefficient is presented in parentheses and the elasticity at mean values in brackets, below its estimated coefficient.

The signs on all of the variables in equation (1) are correct and the coefficients are large in relation to their standard errors, except for the price of livestock. The independent variables in equation (1) explain 93 percent of the variation in the dependent variable.3

The direct price elasticity of -.41 is similar to estimates calculated by Foote and Meinken (6, 15). The elasticity with respect to animal units, 1.92, is also similar to that estimated by Foote when he used quantity of feed grains fed as a dependent variable.4 The elasticity of feed grains fed with respect to high-protein feed is .14. This is a much lower estimate than the elasticity King found, .63 or .47, depending on the form of the equation, using 1921-41 plus 1946-54 data (12, pp. 86-87). It is also lower than the estimate of .41 by Chuang and Judge using 1927-59 data, with 1942-46 omitted (3, pp. 15-18). The results of these studies would lead to the conclusion that the cross elasticity between the quantity of feed grains fed and the price of high-protein feeds has been declining since the 1920’s. This conclusion seems reasonable given the increased awareness of the nutritional value of protein in animal rations. Farmers are less likely to reduce the quantity of protein in their rations than they were in earlier years.

The elasticity of feed grains fed with respect to current livestock price is estimated to be approximately .17. This is somewhat lower than the elasticities of .39 calculated by King and of .37 by Chuang and Judge (3, 12). The lower livestock price elasticity indicates that farmers have changed their feeding practices in the postwar period. This has implications in regard to the amount of the time that will be required to produce additional livestock products generated by an increase in livestock prices. In particular, high livestock prices are likely to last longer as farmers wait to increase their livestock herd rather than increasing production per animal.

In summary, it appears that the elasticity of feed grains fed with respect to feed grain prices and changes in number of animal units has been quite constant for the past 50 years. The elasticity with respect to high-protein feed prices and livestock prices has fallen since the prewar period.

**Food and Industry**

Each year approximately 10 percent of the feed grain crop is used to produce products that are consumed either by consumers or in industry. Barley is employed primarily by the alcoholic beverage industry and oats are made into breakfast foods.

Although comprising only about 10 percent of the corn crop, corn used for food accounts for about 80 percent of the feed grains used in food and industry. Corn products are classified as either wet process or dry process products. Dry processed corn products include dry breakfast food, cornmeal, and corn flour. Wet processed products include cornstarch, corn sugar, corn syrup, and corn oil. The production of dry processed corn products has decreased in importance since 1945 when they comprised 50 percent of the total corn products to 30 percent in 1970. The total production of cornmeal has increased slightly since 1945, but has declined on a per capita basis from 17.6 pounds per person to 7.4 pounds per person in 1970. Cornmeal is reasonable, an elasticity close to 2 seems quite high. Most of the past studies of feed grain demand also found a high elasticity for this variable.
generally assumed to have negative income elasticity. The use of corn in wet processed products has increased substantially since 1945. The production of cornstarch has nearly doubled and the production of corn oil has more than doubled in the past 26 years.

The total consumption of feed grains in food and industrial products declined from 1945 until 1953. This decline was due primarily to the reduction in the consumption of corn meal and corn sirup. Since 1954 the increase in the consumption of corn oil, corn hominy, and a shift in the consumption of corn sirup have caused total consumption of all grain products to increase. Due to the change in direction in the total consumption of feed grain products, slope and intercept dummy variables are included in the feed grain consumption function. We would expect the income elasticity in the early years to be negative when it is dominated by the decline in corn meal consumption and to be positive in the later years. The income variable used in the statistical analysis is real personal consumption expenditure, during the crop year. The income variable includes the effect of increases in individual income and population.

We would expect the use of feed grains in food and industrial products to be responsive to changes in business conditions. The wholesale price index of all commodities is used as a proxy for the level of business conditions. If the proxy is fairly accurate, we would expect the use of feed grains for processed products to increase when the wholesale price index rises and to decrease when it declines. In general, we would expect the price elasticity of demand for feed grains used in food and industrial products to be close to zero since grain comprises only a small proportion of the total product costs. The estimated function is

\[
F_t = 12.41 - .014 PF_t - .008 Y_t + .027 DY_t \\
(3.05) (.009) (.018) (.016) \\
[-.13] [-.18] [
\ \ \ \ \ \ \ \ \ \]^{**} \\
+ .012 WPI_t - 7.44 DC_t \\
(0.028) (3.78) \\
[.10]
\]

Standard error of estimate = .55  
D.W. = 1.62  
\[R^2 = .93\]

*mean of 1945-52  
** = mean of 1953-72

All of the variables in equation (2) have the expected signs, but the standard errors for the variables relating to the wholesale price index and real personal consumption expenditures are quite large. This may be due in part to high multicollinearity between these two variables.

The direct price elasticity is very inelastic as was to be expected, although it is somewhat higher than that of —.03 calculated by Brandow for corn (1, p. 65). Part of the divergence can be explained by the fact that Brandow did not estimate industrial demand and assumed it to be -.10. The coefficients on the personal consumption variable and the slope dummy variable include the direct demand shifting influences of increasing income and population growth. In 1945-52, the effect of increasing personal consumption expenditures had a negative but statistically insignificant effect on the amount of feed grains utilized in food and industrial products. In the later period the income elasticity is positive, reflecting the increased importance of wet process corn products. The elasticity of feed grain consumption with respect to the wholesale price index of .10 indicates that the use of feed grain is affected by business conditions.

**Exports**

Since this study is concerned primarily with U.S. agriculture, the influence of foreign developments is introduced into the model with a simple, single equation, export function. If the primary purpose had been to study the feed grain export market, a more detailed model would have had to be constructed. To be realistic, a model concentrating on exports would probably have to include an equation for each major importing and exporting country (17, 18).

The quantity of feed grains exported from the United States is subject to a great number of influences. In particular, it is subject to changes in import duties and quotas in foreign nations. U.S. exports are also affected by development decisions made in foreign countries on whether to be self-sufficient in the production of feed grains, livestock products, or both. Historically, Europe and Japan have been the major importers of U.S. feed grains. To represent conditions in importing countries in the statistical model, an index of feed grain production (expressed in terms of total digestible nutrients) divided by grain-consuming animal units was constructed for Japan and seven European countries. We would expect U.S. exports to increase when the ratio of total digestible nutrients to animal units falls in the major importing countries. An index of the number of animal units in Japan and the seven European countries is also included in the analysis. For countries not self-sufficient in the production of feed grains, we would expect increases in the number of animal units to increase U.S. exports. The July 1 stock of feed grain in three principal

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4 The prices for the various feed grain products are not collected at either the wholesale or retail level. Therefore, it is not possible to include an index of product prices in the consumption function.

6 This statement is true only under certain conditions which appear to hold in this case. For a discussion of the problem see Bronfenbrenner (2).

7 The seven European countries are Spain, Netherlands, Italy, West Germany, Belgium, Luxembourg, and the United Kingdom.
exporting countries is contained in the export equation. U.S. exports should decline when stocks in the other major exporting countries are high.

With the establishment of the Food for Peace program in 1954, some feed grains are exported under concessional sales programs. The goal of the program is to provide low-cost feed grains to the least developed countries. The concessional export program was planned so as not to displace commercial sales, but it was felt that there may be a spillover effect. If there is, we would anticipate concessional exports of feed grains to reduce commercial exports.

The price of high-protein feed is incorporated in the statistical model to allow for the possibility of importing countries substituting high-protein feeds for feed grains under favorable price conditions. We would expect the direct price elasticity of demand for feed grain exports to be fairly high since importing countries can purchase their grain requirements from other sources. Equation (3) presents the results of estimating the export demand function.

\[
(3) \quad QE_t = 22.90 - 0.137 PF_t + 0.172 \frac{PH_t}{PF_t} - 0.260 CE_t
\]

\[
\begin{align*}
(11.87) & \quad (0.044) \quad (0.029) \quad (0.471) \\
[-1.20] & \quad [1.39] \quad [-0.43]
\end{align*}
\]

\[
- 0.245 R_t + 0.143 AUF_t - 0.200 SF_t
\]

\[
(0.077) \quad (0.054) \quad (0.199)
\]

\[
[-2.04] \quad [1.16] \quad [-0.19]
\]

Standard error of estimate = 2.67

\[
D.W. = 1.95
\]

\[
R^2 = 0.93
\]

The direct price elasticity is fairly high, as expected. Brandow, using different techniques, estimated the price elasticity of feed grains for export to be -1.3 in the late 1950's (J, p. 55). The model indicates a substitution effect between the feed grains and high-protein feeds. Importers are apparently willing to substitute high-protein feeds for feed grains when feed grain prices are high. The sign on the concessional export variable is negative, indicating that concessional exports may have replaced some commercial sales.

Conditions in Japan and the major importing countries of Europe are important in determining the quantity of U.S. feed grain exported. The elasticity of U.S. exports with respect to a change in the ratio of TDN/animal units in Europe and Japan is -2.04. An increase in the number of animal units fed in the importing countries will also increase U.S. exports approximately 1.16 percent for every 1 percent change in the animal units index, holding \( R_t \) constant.

Inventory

The end-of-year carryover of feed grain supplies is held by farmers, commercial grain traders, and the Government. Since 1945 the role of the Government in storing feed grain has varied greatly. Under the provisions of the farm price support program a participating farmer may obtain a loan from the Commodity Credit Corporation (CCC) using his grain as collateral. The farmer may repay the loan by delivery of his feed grain to the CCC or by repaying the loan plus carrying charges.

The stock of Government-stored feed grain increased rapidly from zero in 1947 to a peak of 74.7 million tons in 1960. After 1960, tightened production controls and lower loan rates resulted in smaller Government carryovers. By the end of the 1974 crop year stocks held by the Government will be negligible.

The quantity of feed grain accumulated in Government stocks is largely determined by the relation between market prices and loan rates. When market prices are high relative to loan rates, the Government inventory declines; conversely, Government inventories increase during periods of high loan rates and low market prices.

The commercial inventory of feed grain is held in farm storage facilities, country elevators, terminal markets, and warehouses to provide working stocks for the milling industry, formula feed industry, and exports. Over the study period the share of total carryover held by commercial interests varied from 12 to 100 percent.

Although it would seem preferable to estimate separate demand functions for commercial and Government stocks, only one combined inventory function is used in the model. This approach is taken because of the tendency for a linear Government inventory function to predict negative inventory levels during the recent period of very high prices. In general little predictive ability is lost by using the single stock function.

In the statistical model the end-of-year carryover is assumed to be a function of feed grain price, Government loan rate, total production, and inventory lagged 1 year. High feed grain prices indicate limited supplies and a high cost of holding inventories; therefore, we would expect year-end holdings to decline when prices have been high. The Government loan rate represents the Government’s willingness to purchase grain. In years of high loan rates, farmers will tend to sell their grain to the Government rather than feeding it or selling it in the open market. Generally some fraction of total production is necessary to provide pipeline or working stocks. This influence is taken into account by including total production in the inventory equation.

\[\text{For an example of separate Government and commercial inventory equations see (14).}\]

\[\text{The theoretical arguments for this type of specification are presented in (13, 11, pp. 80-81).}\]
Since feed grain can be stored over long periods of time and the CCC is unable to sell grain at will, a lagged stock variable is also included in the model.

Equation (4) explains the demand for carryover stocks.

\[
S_t = 20.48 - .436 PF_t + .250 PS_t + .652 S_{-1} \\
(24.73) \quad (.108) \\
[-1.04] \\
+ .069 QG_t \\
(.070) \\
[.22] \\
\]

Standard error of estimate = 7.47

\[D.W. = 1.51\]

\[R^2 = .88\]

The estimated inventory demand equation indicates that a feed grain price increase of 1 percent results in a 1.04 percent decrease in the end-of-year carryover. Similarly, a 1 percent increase in the Government loan rate results in a .71 percent increase in the level of carryout.\(^\text{11}\)

**General Comments**

The structural equations estimated appear reasonable. All of the variables have the expected signs and most of the variable coefficients are statistically significant at a 5 percent level of significance. Serial correlation does not appear to be a problem in any of the structural equations (4). Likewise, multicollinearity among independent variables included in any single equation is minimal with the exception of the wholesale price index and real personal consumption expenditures in equation (2). Consequently, the structural equations are used in the next section to provide forecasts of the endogenous variables.

**Reduced Form Equations**

To use the feed grain model for policy analysis, the reduced form of the simultaneous model is derived. The reduced form of the model expresses each of the endogenous variables as a linear function of all the predetermined variables. The coefficients of the solved reduced form equations preserve all of the behavioral relationships and identities that are built into the structure. For example, any change in the supply of feed grain will be apportioned completely among the four market outlets. In general, the reduced form equations show the equilibrium impact of a change in any exogenous variables on each endogenous variable, while maintaining the relationships among variables implicit in the structure (32, p. 161). For this reason the reduced form coefficients are often referred to as impact or short-run multipliers.

\[AU_t = 49.11 - .066 PF_t - .031 PF_{-1} + .041 PL_{-1} \\
(12.91) \quad (.026) \quad (.029) \quad (.016) \\
[-.065] \quad [-.030] \quad [.106] \\
+ .014 Y_t + .484 AU_{-1} \\
(.007) \quad (.134) \\
[.044] \\
\]

Standard error of the estimate = 1.83

\[D.W. = 1.90\]

\[R^2 = .91\]

All of the variables in the equation have the correct signs and 91 percent of the variation in the dependent variable is explained. Farmers appear to respond more quickly to changes in feed price than to livestock price as indicated by the statistically significant coefficient on the current price of feed grain. The current price of livestock was also included as a variable in equation (5) but its estimated coefficient was very small and statistically insignificant, so it was dropped from the final equation. The elasticities calculated by Foote for lagged livestock and feed grain prices for the prewar period are somewhat larger than the ones obtained here (6).

**Animal Units Fed**

Animal units enter a derived demand function for feed in much the same was as population enters a consumption function. The basic difference is that the number of animal units a farmer produces is a decision variable under his control. Therefore, in the statistical model the number of animal units fed during a year is treated as an endogenous variable.

In general, the same factors that affect the number of animal units fed also affect the quantities of feed fed per animal unit, although with different time lags. The decision as to how many animal units a farmer will produce is assumed to be made on the basis of current and 1-year-lagged feed grain and livestock prices. Real personal consumption expenditure is included in the animal units equation to reflect the general demand conditions in the economy. The number of animal units produced lagged 1 year is also included in equation (5) since the increase or decrease in animal units produced from one year to the next will depend to a certain extent on the amount produced in the last period.

We would expect positive coefficients on the personal consumption expenditure, livestock price, and lagged animal units variables, and negative coefficients for the feed price variables.

\(^\text{11}\) Note that the Durbin-Watson test for serial correlation is biased when a lagged dependent variable is included in the equation (16). For an alternative test see (5).
Table 2 contains the reduced form coefficients for each of the six equations in the simultaneous model. Each endogenous variable is expressed as a function of the 17 exogenous variables in the system. To check the predictive ability of the reduced form equations, Theil's inequality coefficients are calculated for each equation (table 3)\(^{12}\) (23, pp. 15-36). If the inequality coefficient equals zero then the forecast values are equal to the actual values, while an inequality coefficient equal to one implies that the prediction procedure leads to the same error as no-change extrapolation. Table 3 indicates that the inequality coefficients for all of the equations are quite small. Furthermore, Theil has shown that the inequality coefficient is made up of three components: that due to bias, that due to the difference of the regression coefficient from unity, and that due to residual variation.\(^{13}\) Table 3 reveals that most of the error made in the reduced form predictions of the endogenous variables is due to residual variance. Each of the reduced form equations performs well during the time period used to estimate the structural equations and will hopefully provide good forecasts in the future.

\(^{12}\)The actual values of the lagged endogenous variables are used in calculating the predicted values. The formula for the inequality coefficient is

\[
U = \sqrt{\frac{\sum (P_i - A_i)^2}{\sum A_i^2}}
\]

where \(P_i\) = predicted values and \(A_i\) = actual values.

Table 2. Reduced form of the simultaneous system

<table>
<thead>
<tr>
<th>Endogenous variables</th>
<th>Predetermined variables</th>
<th>Constant</th>
<th>AU-1</th>
<th>S-1</th>
<th>PF-1</th>
<th>PL-1</th>
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<th>DYt</th>
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<td>.0716</td>
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<td>.0236</td>
<td>-6.4235</td>
<td>.0105</td>
</tr>
<tr>
<td></td>
<td>(PL_t)</td>
<td>-.0038</td>
<td>-.0187</td>
<td>-.0142</td>
<td>.0139</td>
<td>-.0081</td>
<td>.0114</td>
<td>-.0421</td>
<td>.0530</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(PHPF_t)</td>
<td>-.0038</td>
<td>-.0051</td>
<td>-.1233</td>
<td>.1212</td>
<td>-.0705</td>
<td>.0898</td>
<td>-.3648</td>
<td>.4592</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(PS_t)</td>
<td>-.0008</td>
<td>-.0039</td>
<td>-.0030</td>
<td>.0029</td>
<td>-.0017</td>
<td>.0024</td>
<td>-.0089</td>
<td>.0112</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(R_t)</td>
<td>-.0249</td>
<td>-.1233</td>
<td>.1559</td>
<td>.0924</td>
<td>-.0538</td>
<td>.0754</td>
<td>-.2782</td>
<td>.4196</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(AUGt)</td>
<td>-.0078</td>
<td>.1329</td>
<td>-.0296</td>
<td>-.2165</td>
<td>.1261</td>
<td>-.1767</td>
<td>-.3480</td>
<td>.1100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(SF_t)</td>
<td>.0671</td>
<td>.2840</td>
<td>.2158</td>
<td>-.2119</td>
<td>.1234</td>
<td>-.1730</td>
<td>.6382</td>
<td>-.8032</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Theil inequality coefficients for the reduced form equations, 1945-72

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inequality coefficient</th>
<th>RMS</th>
<th>Percent of error due to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bias</td>
</tr>
<tr>
<td>AU(_t)</td>
<td>.0084</td>
<td>1.78</td>
<td>Neg.</td>
</tr>
<tr>
<td>QF(_t)</td>
<td>.0176</td>
<td>3.99</td>
<td>Neg.</td>
</tr>
<tr>
<td>Ft</td>
<td>.0200</td>
<td>.47</td>
<td>Neg.</td>
</tr>
<tr>
<td>St</td>
<td>.0471</td>
<td>4.48</td>
<td>Neg.</td>
</tr>
<tr>
<td>QE(_t)</td>
<td>.0736</td>
<td>2.18</td>
<td>Neg.</td>
</tr>
<tr>
<td>PF(_t)</td>
<td>.0392</td>
<td>8.32</td>
<td>Neg.</td>
</tr>
</tbody>
</table>

Neg. - less than .001.

RMS - root-mean-square prediction error.

*Regression coefficient obtained by regressing the actual values on the predicted values.
The solved reduced form equations of the model can be used to examine the impact of selected changes in the predetermined variables on the jointly determined variables. For example, the effect of an increase of 50 million tons in the supply of feed grains can be found by multiplying the figures in table 2 under $QG_t$ by 50. The effect of a 50-million-ton increase in supply is to increase the amount fed to livestock by 23.0 million tons, exports by 5.5 million tons, inventory by 21.0 million tons, and use in food and industry by .5 million tons. The model predicts a 40.2 percentage point decline in the feed grain price index. Additional examples of the impact on jointly determined variables of changing predetermined variables can be obtained by using the reduced form coefficients contained table 2.

**Projections**

In this section the reduced form equations are used to project the value of the dependent variables for the 1973 and 1974 crop years. By inserting known values for the predetermined variables into the simultaneous model, the system of equations can be solved to forecast the values of the unknowns. Although the exogenous variables in the model are considered to be "knowns," not all of their values are known with certainty at this time. Some economic variables move so slowly along secular trends that their future values can be projected with considerable accuracy; others, such as livestock prices, are subject to more fluctuation (22). The values of the predetermined variables used in the feed grain model to project price and utilization rates for 1973 and 1974 are shown in table 4.

Four variables—real personal consumption expenditure, the wholesale price index, foreign feed grain production divided by livestock production, and the index of foreign animal units—are estimated by projecting trends and using outlook information where it is available. The projected production of feed grain is based on farmers’ planting intentions. The index of price support loan rates is calculated from announced support levels. Concessional exports and the stock of feed grain in major exporting countries are assumed to remain at their recent average of about 1.5 and 15.0 million tons per year. The high-protein feed price index and the prices of livestock and livestock products are assumed to fall slightly from their 1973 levels in 1974. The values of the lagged endogenous variables are taken from the May 1974 issue of *Feed Situation*.

Table 5 presents the results of using the model to predict feed grain utilization and price in 1973 and 1974. The projected utilizations in 1973 are all quite close to their actual values. The largest error is for commercial exports where the prediction error is 5.7 percent.

The model underestimated the feed grain price level in 1973. This is not surprising, but the size of the error—almost 30 percent—is disturbing. One of the reasons for the model’s underestimation of price is the decline of 52 points in the price of high-protein feed between 1972 and 1973. Table 2 indicates that a decline of this magnitude in the price of protein feed should have caused the feed grain price index to decrease 15 points, which it obviously didn’t do. The model’s projections for 1974 are interesting nevertheless.

14 Values of the predetermined variables for 1973 are based on data available in July 1974. Consequently the actual values of the predetermined variables for 1973 should be quite close to those in table 4.

15 If this model is to be used to forecast more than 1 year in advance, production estimates will have to come from other sources. Work is currently under way in the Department of Agricultural Economics, University of Minnesota, to develop supply models that could be used in conjunction with this model for longer range forecasts (10, 11, 19, 20, 21).

### Table 4. Projected values of predetermined variables used in projection exercises, 1973 and 1974

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Projected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>Billion 1958 dollars</td>
<td>55.6, 560.6</td>
</tr>
<tr>
<td>$PHPF_t$</td>
<td>Percent</td>
<td>220.0, 216.0</td>
</tr>
<tr>
<td>$PL_t$</td>
<td>Percent</td>
<td>490.0, 475.0</td>
</tr>
<tr>
<td>$PS_t$</td>
<td>Percent</td>
<td>105.7, 110.0</td>
</tr>
<tr>
<td>$CE_t$</td>
<td>Million tons</td>
<td>1.5, 1.5</td>
</tr>
<tr>
<td>$WPI_t$</td>
<td>Percent</td>
<td>155.3, 170.8</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Percent</td>
<td>83.0, 95.0</td>
</tr>
<tr>
<td>$OG_t$</td>
<td>Million tons</td>
<td>202.0, 172.0</td>
</tr>
<tr>
<td>$AUF_t$</td>
<td>Percent</td>
<td>138.0, 139.0</td>
</tr>
<tr>
<td>$FS_t$</td>
<td>Million metric tons</td>
<td>15.0, 15.0</td>
</tr>
<tr>
<td>$S-1$</td>
<td>Million tons</td>
<td>32.4, 22.6</td>
</tr>
<tr>
<td>$AU-1$</td>
<td>Million units</td>
<td>115.4, 114.2</td>
</tr>
<tr>
<td>$PF-1$</td>
<td>Percent</td>
<td>129.6, 191.0</td>
</tr>
<tr>
<td>$PL-1$</td>
<td>Percent</td>
<td>465.2, 490.0</td>
</tr>
</tbody>
</table>
Table 5. Projected values of the jointly determined variables for 1973 and 1974

<table>
<thead>
<tr>
<th>Variables</th>
<th>Units</th>
<th>1973 Actual*</th>
<th>1973 Predicted</th>
<th>Percentage error</th>
<th>1974 Unconstrained</th>
<th>1974 Constrained¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFₜ</td>
<td>Mil. tons</td>
<td>155.9</td>
<td>156.3</td>
<td>0.3</td>
<td>139.6</td>
<td>134.2</td>
</tr>
<tr>
<td>PFₜ</td>
<td>Percent</td>
<td>191.0</td>
<td>135.6</td>
<td>28.6</td>
<td>157.0</td>
<td>164.9</td>
</tr>
<tr>
<td>AUₜ</td>
<td>Mil. units</td>
<td>114.2</td>
<td>119.0</td>
<td>4.2</td>
<td>116.1</td>
<td>115.5</td>
</tr>
<tr>
<td>Sₜ</td>
<td>Mil. tons</td>
<td>22.6</td>
<td>22.3</td>
<td>1.3</td>
<td>6.2</td>
<td>12.0</td>
</tr>
<tr>
<td>Fₜ</td>
<td>Mil. tons</td>
<td>15.1</td>
<td>15.6</td>
<td>3.3</td>
<td>15.5</td>
<td>15.5</td>
</tr>
<tr>
<td>QEₜ</td>
<td>Mil. tons</td>
<td>419</td>
<td>39.5</td>
<td>5.7</td>
<td>33.1</td>
<td>32.0</td>
</tr>
</tbody>
</table>

*Preliminary, based on data in *Feed Situation*, May 1974.

¹These values are obtained by restricting the 1974 carryout (inventory) to equal 12.0 million tons.

especially when compared with the model’s 1973 forecast.

The model’s unconstrained forecasts for 1974 indicate an end-of-year carryover of only 6.2 million tons. Under this condition grain prices are predicted to increase about 16 percent over the model’s predicted price in 1973. A grain carryover of 6.2 million tons seems rather unrealistic, since more grain than this is probably needed simply to provide pipeline stocks. Consequently the model was rerun, using 12.0 million tons as an estimate of the minimum carryout stocks needed in 1974. These results are given in the last column of table 5. With the 1974 carryover constrained to equal 12.0 million tons, the model predicts prices 21.6 percent higher than in 1973. Even at these higher prices, 32.0 million tons of feed grain are exported. Feed grains fed are predicted by the model to fall to 134.2 million tons which would be the lowest level of feeding since 1968. (USDA projections in November 1974 indicated less than 127 million tons might be fed.) The 1974 feed grain price projections by the model probably represent a lower bound for actual price changes because of the assumption that high-protein and livestock prices will decline slightly in 1974. Any substantial increase in either of these variables would lead to higher feed grain prices than predicted above.

Summary

The six-equation simultaneous feed grain model appears to be a useful tool for predicting future levels of feed grain utilization and price. The model also provides a framework within which the effects of changes in specific variables on the entire system can be evaluated.

Future work should be concentrated in two areas: (1) some of the more volatile exogenous variables, primarily high-protein feed prices and livestock prices, should be made endogenous to the system to make the model less dependent on outside information for short-run forecasts; and (2) a supply model should be incorporated with the demand model to make long-run projections.

References