Journal of Agribusiness 20,2 (Fall 2002):155–161

A General Framework for Grain Blending and Segregation

Eswar Sivaraman, Conrad P. Lyford, and B. Wade Brorsen

The Hennessy and Wahl model of optimal grain blending and segregation (GBS) is extended to the case where it is not possible to separate components within a load. Analytical solutions are not available when segregation is the optimal strategy, and so solutions are obtained with nonlinear optimization. The model is then used to determine the optimal sorting of hard red winter wheat by protein content. Most of the benefits from sorting can be obtained with only two bins.

Key Words: blending, grain, segregation

Various grain blending and segregation (GBS) strategies are used by the grain industry to sort or blend wheat of variable quality. The goal of these GBS efforts is to achieve the grade or quality levels that will result in the highest returns when the wheat is marketed. Efforts to improve U.S. wheat quality have been a topic of federal legislation in both 1986 (the Grain Quality and Improvement Act) and 1990 (Title XX of the Food, Agriculture, Conservation, and Trade Act). A number of studies have focused on methods to improve U.S. grain quality (Hill, 1990; Mercier, 1993; U.S. Office of Technology Assessment, 1989; Wilson and Dahl, 1999). GBS strategies constitute an important means to meet end-user quality needs, and a better understanding of these strategies is needed to improve grain quality.

Recently, Hennessy and Wahl (1997) developed a GBS model with specific reference to linearly separable grain attributes like dockage. Dockage is said to be linearly separable because 100 bushels of grain with 1% dockage, for example, can be completely separated into 99 bushels of zero dockage grain and 1 bushel of cleanings. On the other hand, pure protein is not achievable, so protein is not a linearly separable attribute. A key contribution of Hennessy and Wahl's effort was a better understanding of the influence of the premium/discount schedule on GBS decisions for dockage. They showed the convexity or concavity of the objective or price function determines whether it is optimal to clean or commingle wheat.

Eswar Sivaraman is a Ph.D. candidate in the Department of Industrial Engineering and Management, Oklahoma State University; Conrad P. Lyford is an assistant professor in the Department of Agricultural Economics, Texas Tech University; and B. Wade Brorsen is a regents professor and Jean and Patsy Neustadt Chair in the Department of Agricultural Economics, Oklahoma State University.

An earlier version of this paper was presented at the Southern Agricultural Economics Association meeting in Fort Worth, Texas, January 29–31, 2001. Assistance and advice from Francisca G.-C. Richter, Phil Kenkel, and Kim Anderson are gratefully acknowledged.

In this study, we extend the work of Hennessy and Wahl to model decisions based on grain properties like protein content, test weight, etc., which, unlike dockage, cannot be linearly separated. We present an optimization model that is applicable to all price function schedules. In the key case of a convex objective function, the analytical solution of Hennessy and Wahl no longer holds and the solution must be found by solving a nonlinear optimization problem. Although premiums are really linear-by-steps functions of quality, as steps become small, these functions can be approximated by continuous nonlinear functions. Thus, although we consider stepped as well as concave and convex functions in the theory section, the empirical portion analyzes a continuous nonlinear function only. The general framework is used to determine optimal GBS strategies for wheat based on protein content.

A Grain Blending and Segregation Framework

Let $\Pi(p)$ denote the price of wheat defined as a function of the percent protein, p. Let $D = [p_1, p_3]$ represent the range of protein values considered. Assume the range D contains only one inflection point, p_2. In deriving the optimal solution, a key consideration is whether the price schedule, $\Pi(p)$, is convex or concave over the range of protein values in D. Note that if a section of the price schedule is linear, then that section is considered concave.[1]

The average quality content across a bin of wheat is denoted by P_{avg}. For any range of protein values D_i representing a subset of D, the average quality is computed as:

$$(1) \qquad P_{avg}(i) = \frac{\int_{D_i} p f_w(p)\, dp}{F_i}, \qquad F_i = \int_{D_i} f_w(p)\, dp,$$

where $f_w(p)$ is the probability density function of protein level, and so $P_{avg}(i)$ is the average protein content evaluated over the range specified by D_i. The optimal GBS problem is how to segregate wheat into N different storage categories such that the premiums received for the average protein levels in the various categories, namely, $P_{avg}(i)$, maximize profit. This is consistent with the overall situation faced by grain elevators which are constrained by the total number (N) of receiving points or "dump pits" at their elevator. However, it is unlikely elevators will know the probability distribution of protein level with certainty. More realistically, as the crop progresses through time and elevators receive wheat, they come up with estimates for this distribution after receiving several loads of wheat.

For the general model discussed above, $D = [p_1, p_3]$. Let $d_0 = p_1$, and $d_N = p_3$. We introduce this notation so that $D_i = [d_{i-1}, d_i]$, $D_N = [d_{N-1}, p_3]$, and $\bigcup_{i=1}^{N} D_i = D$. Thus, the problem now reduces to finding optimal values of N, d_1, d_2, ..., d_{N-1}. In this discussion it is assumed the grain in each bin is "well-blended," such that a random sample drawn from storage category i will have a protein level of $P_{avg}(i)$.

[1] The case of discontinuous/step models for the premium function will be discussed later.

The optimal GBS strategy is the solution to:

$$\max_{d_0, d_1, \ldots, d_N, N} \sum_{i=1}^{N} \Pi\left(P_{avg}(i)\right) F_i - C(N), \tag{2}$$

where $C(N)$ is the cost associated with operating the N storage categories. The optimization model represented by equation (2) can be solved incrementally by specifying the value of N, and then solving for the storage category boundaries. The iterative process stops when the marginal revenue from one extra storage category is not worth the cost of maintaining an extra storage category or bin. The process employed next is to find the revenue-maximizing strategy for each number of bins. The gains in revenue are then compared to the costs of segregating grain, as estimated by Vandeburg, Fulton, and Dooley (2000).

The shape of the objective function determines the optimal GBS decision, as mentioned earlier. Following Hennessy and Wahl (1997), these solutions can be categorized by their overall forms as follows:

- **CASE 1:** $\Pi(p)$ is *strictly* convex over D. The GBS problem can be readily formulated and solved as specified by equation (2).[2] When the number of segregations or bins is greater than the number of loads, the revenue-maximizing strategy is to put each load in a separate bin. This result is in contrast to the findings of Hennessy and Wahl, who conclude that when cleaning is costless and the profit function is convex, it is optimal to completely clean into pure cleanings and top-quality grain. Further, with the linearly separable case considered by Hennessy and Wahl, the revenue-maximizing strategy never uses more than two bins.

- **CASE 2:** $\Pi(p)$ is concave over D.[3] Under concavity, the optimal GBS policy is to store all the grain in one bin, which is the same solution as found by Hennessy and Wahl. The proof follows readily from Jensen's inequality (Varian, 1992, p. 182), which states that for a (strictly) concave function, $f(\mathrm{E}[x]) \geq \mathrm{E}[f(x)]$, where E is the expectation operator and f is the concave function of interest. Thus, in the context of our discussion, the best sale premium we can achieve is for the protein content averaged over the entire load, i.e., P_{avg}.

- **CASE 3:** $\Pi(p)$ is part concave and part convex. For the concave region $[p_1, p_2]$, apply results from Case 2 above. For the convex region $[p_2, p_3]$, the problem is solved via the optimization approach as outlined for Case 1 above.

- **CASE 4:** $\Pi(p)$ is a discontinuous/stepped premium schedule. In their handling of stepped price function models, Hennessy and Wahl provide a very simple and elegant approach whereby the effective premium (discount) schedule is derived directly from a plot of the price function. However, their technique is applicable

---

[2] The implicit constraints that follow from the definition of a probability measure are: $\sum_i F_i = 1$, $\sum_i P_{avg}(i) F_i = P_{avg}$, $d_0 = p_2$, and $d_N = p_3$.

[3] Note that a linear function is concave.

only for linearly separable attributes (e.g., dockage), with linear separability (LS) being a key component of their proof. For GBS decisions based on attributes like protein content, LS does not hold, and the solution is found using the formulation in equation (2). With a stepped premium schedule, the objective function is discontinuous.

Data and Empirical Analysis

The optimization model in (1) is used with parameters estimated from two data sources. First, data on Kansas City spot hard wheat basis (cents per bushel) over 1993–1999 were obtained from data published by the Kansas City Board of Trade (KCBT). The basis typically increases as protein increases, and thus reflects protein premiums. The second data set includes over 1,200 values of actual truckload-level grain quality samples measured at wheat elevators in Oklahoma over 1990–1999 (Kenkel).

These data sets were used to estimate the distribution function of protein. With the KCBT data, all wheat with less than 11% protein receives the same price. The basis for wheat with less than 11% protein was obtained by taking the mean of all observations with less than 11% protein. The observations with greater than or equal to 11% protein were used to estimate a quadratic function of protein. The null hypothesis of a linear equation was tested and rejected. The estimated price schedule based on the KCBT data is:

$$p < 11\%: \Pi(p) = 22.57 \text{ (cents), and}$$

$$p \$ 11\%: \Pi(p) = 206.846 - 41.174p + 2.236p^2 \text{ (cents).}$$

As the price distribution indicates, the price function is concave in (0, 11), and strictly convex in [11, 4). Based on the Oklahoma truckload data, protein is distributed normally with mean 11.26 and standard deviation 1.42. The null hypothesis that protein percentages were normally distributed was tested and not rejected using a Kolmogorov-Smirnov test. Since, in practice, protein only varies to a limited extent, we use $D = [5, 17]$, within which virtually all of the distribution is captured. Yoon, Brorsen, and Lyford (2002) develop a nonparametric algorithm for a similar problem, but their model has many local optimums and is slow to solve, so it is much less practical to use than employing a normal distribution for the characteristics as is done here.

Protein is important because of the various end-uses of wheat. For example, the major end-use of wheat in Asian markets (Ahmadi-Esfahani and Stanmore, 1994) is based primarily on protein content, e.g., white salted noodles (9%–10% protein), instant noodles (10.5%–11%), yellow egg noodles (12%), leavened bread (11.5%–14.5%), etc. As with any hedonic price, a protein premium is determined by supply and demand for protein. Supply in particular can vary by year, and consequently protein premiums vary. Indeed, in 1996, Attaway (1998) reports protein in Oklahoma wheat was so high that no protein premiums were offered. Both the price schedule

and the distribution of protein used here represent averages across years. Therefore, the actual benefits of sorting by protein content would vary across years.

Following the earlier discussion in Case 2, the optimal policy for the concave section of the price schedule is to completely blend, i.e., [5%, 11%) is one storage category. For the convex section of the price schedule, the optimal GBS problem with a fixed value of N is:

$$\text{(3)} \qquad \max_{d_0, d_1, \ldots, d_N} \sum_{i=1}^{N} \Pi\left(P_{avg}(i)\right) F_i$$

$$\text{s.t.:} \quad d_0 = 11\%; \quad d_N = 17\%; \quad d_i \neq d_j \ \forall\, i < j$$

and the constraints that define $\Pi(\cdot)$, $P_{avg}(i)$, and F_i. In (2), costs were assumed to depend only on the number of bins. In (3), the number of bins is fixed and so all costs are fixed; thus the optimal solution is the revenue-maximizing solution.[4] The approach could easily handle additional complexities such as a nonlinear cost function and constraints on the size of bins/segregations if needed. The problem as stated in equation (3) was modeled in GAMS, and the solutions are shown in table 1.

Optimal GBS Solution

Table 1 reveals several points of interest. The expected sale premium increases with the number of storage categories, and mirrors our finding that the revenue-maximizing policy is to store each wheat load in a separate bin—a solution which obviously is not feasible. To this end, the model specified in equation (2) can be modified to include storage/maintenance costs. For most elevators, the number of bins is fixed and the optimal solution can be taken directly from table 1.

As theory implies, the small gain in going beyond two segregations is a direct result of the shape of the price schedule. While the portion of the schedule above 11% protein is convex due to the positive coefficient on the quadratic term, the degree of convexity is small, and so gains from more than two bins are small. This result is consistent with practice at most elevators where two bins are used to sort wheat by protein content. All wheat earning a premium is placed in one bin, while the remaining wheat is placed in the other bin.

Vandeburg, Fulton, and Dooley (2000) estimate the costs of handling value-added grains, and thus include estimates for the cost of segregation. Their estimate of the cost of segregation for value-added grain is at least 4¢/bushel (the analysis is done under different scenarios). The revenue from one segregation as calculated from table 1 is only 2.7¢/bushel. Attaway (1998), using crude approximations, estimated similar gains of 3¢/bushel from sorting wheat by protein content. Most local elevators do not pay protein premiums and do not segregate by protein, which is consistent

[4] Because we do not have information on the cost of building and maintaining separate storage categories within an elevator, costs are zero in the analysis.

Table 1. Expected Sale Premiums Achieved from Optimal Segregation with One to Six Bins

N [a]	Storage Categories	Expected Sale Premium per Unit Load of Wheat (¢/bushel)
0	[5, 17]	30.288
1	[5, 11), [11, 17]	32.947
2	[5, 11), [11, 12.392), [12,392, 17]	33.617
3	[5, 11), [11, 12.26), [12.26, 13.52), [13.52, 17]	33.750
4	[5, 11), [11, 12.26), [12.26, 13.52), [13.52, 14.78), [14.78, 17]	33.843
5	[5, 11), [11, 11.587), [11.587, 12.187), [12.187, 12.856), [12.856, 13.716), [13.716, 17]	33.874

[a] N / the number of categories for the convex region. Including the concave section, there are $N + 1$ categories.

with the costs estimated by Vandeburg, Fulton, and Dooley. Larger elevators that do segregate must either have lower costs due to economies of size or higher premiums due to meeting the needs of a specialty market.

Conclusion

A general mathematical programming model is developed to determine optimal grain blending and segregation strategies. The model maximizes expected sale premiums based on grain attributes like protein. Analytical results show the effect of different price schedules: (*a*) concave price schedules encourage complete commingling, and (*b*) the optimal segregation with a convex/stepped price schedule can be obtained by solving a mathematical programming problem.

The model is then used to determine optimal segregation by protein for hard red winter wheat. Results show most of the benefits of segregation may be achieved with just two bins. This result is consistent with most elevator operations where only two segregations are used for protein. Many small elevators do not segregate at all, and given available estimates of the cost of segregation, their choice seems rational.

References

Ahmadi-Esfahani, F. Z., and R. G. Stanmore. (1994, December). "Quality premiums for Australian wheat in the growing Asian markets." *Australian Journal of Agricultural Economics* 38, 237–250.

Attaway, R. D. (1998, May). "Wheat grading accuracy and potential profit for segregation and cleaning." Unpublished M.S. thesis, Department of Agricultural Economics, Oklahoma State University, Stillwater.

Hennessy, D. A., and T. Wahl. (1997, August). "Discount schedules and grower incentives in grain marketing." *American Journal of Agricultural Economics* 79, 888–901.

Hill, L. (1990). *Grain Grades and Standards: Historical Issues Shaping the Future.* Urbana, IL: University of Illinois Press.

Kansas City Board of Trade. (1993–1999). *Kansas City Board of Trade Review.* Various issues.

Kenkel, P. (1990–1999). Truck tailgate grain quality samples, unpublished data. Department of Agricultural Economics, Oklahoma State University, Stillwater.

Mercier, S. (1993). "The role of quality in wheat import decisionmaking." Publication No. AER-670, U.S. Department of Agriculture/Economic Research Service, Washington, DC.

U.S. Office of Technology Assessment. (1989, February). "Enhancing the quality of U.S. grain for international trade." United States Congress, Washington, DC.

Vandeburg, J., J. Fulton, and F. Dooley. (2000). "The cost effects of value-added grains on the grain marketing system: Implications for local cooperatives." Paper presented at the 2000 annual meetings of the American Agricultural Economics Association, Tampa, FL.

Varian, H. (1992). *Microeconomics*, 3rd edition. New York: W. W. Norton and Co., Inc.

Wilson, W., and B. Dahl. (1999, Spring/Summer). "Quality uncertainty in international grain markets: Analytical and competitive issues." *Review of Agricultural Economics* 21(1), 209–224.

Yoon, B. S., B. W. Brorsen, and C. P. Lyford. (2002, December). "Value of increasing kernel uniformity." *Journal of Agricultural and Resource Economics* 27(2), forthcoming.