Pricing-to-Market: Price Discrimination or Product Differentiation?

Nathalie Lavoie\(^1\) and Qihong Liu\(^2\)

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Keywords: Pricing-to-market; Vertical differentiation; Unit values; Price discrimination; Quality upgrading.

JEL Classification: D42, F10, L12.

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Pricing-to-Market: Price Discrimination or Product Differentiation?*

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Abstract

We employ a vertical differentiation model to examine the potential bias in pricing-to-market (PTM) results when using unit values aggregating differentiated products. Our results show that: i) false evidence of PTM (“pseudo PTM”) is always found when using unit values, whether the law of one price holds or not; and ii) the extent to which results are biased due to pseudo PTM increases with the level of product differentiation. Correspondingly, our simulation results suggest that: i) it is possible to get a statistically significant estimate of the exchange rate coefficient, even when there is no real PTM; ii) the probability of a false PTM finding increases with product differentiation. Pseudo PTM is the result of a change in the mix of qualities imported when the exchange rate changes.

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1 Introduction

Movements in exchange rates can have an important influence on an imperfectly competitive exporter’s pricing behavior. Exchange rates create a wedge between the price set by the exporter and the price paid by the importer, and can be used as an instrument of price discrimination. The idea that an exporter can adjust destination-specific markups to accommodate changes in exchange rates was first documented in Mann (1986) and later was termed “pricing-to-market” (PTM) by Krugman (1987). Knetter (1989) developed an empirical model to analyze the presence of PTM. Knetter’s model has since been used extensively, due to its simplicity and data availability, to determine the presence of price discrimination in international trade. Examples of studies include: Knetter (1989, 1993), Marston (1990), Gagnon and Knetter (1995) in the auto industry; Pick and Carter (1994), Carew and Florkowski (2003), Glauben and Loy (2003) in the food and agriculture industry; and Gil-Pareja (2002) for 26 products covering a wide range of industries.\(^1\)

Most PTM studies, such as those listed above, use export unit values as the price variable.\(^2\) Export unit values are calculated as the ratio of value to volume of exports for a specific product category and destination country. Market- or customer-specific price information is typically confidential, making export unit values the next best alternative. The disadvantage of unit values is that they often aggregate data on products employed for very different uses.\(^3\) Thus, findings of PTM that are attributed to price discrimination might alternatively indicate product differentiation when unit values are used (Sexton and Lavoie, 2001). It is important to understand the effect of unit value data on PTM testing because evidence, or lack of evidence, of PTM can be used for policy purposes (e.g., Carter, 1993; Gil-Pareja, 2003). Moreover, PTM can have important effects on the international transmission of monetary and fiscal policy, and can increase exchange rate volatility, relative to a situation where markets are integrated (Betts and Devereux, 2000). The objective of our study is to examine the impact of the use of unit values aggregating differentiated products on the evaluation of pricing-to-market.

Product differentiation has been explicitly modelled in studies evaluating the extent of exchange rate pass-through (e.g., Dornbusch (1987), Feenstra, Gagnon, and Knetter (1996), Yang

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\(^1\)We count over 100 studies under a search “pricing-to-market” by default fields in EconLit, not counting working paper duplicates.

\(^2\)Some exceptions include Goldberg and Verboven (2001) and Gil-Pareja (2003), who use product level data.

\(^3\)Gehlhar and Pick (2002) found that 40 percent of U.S. food exports are characterized by non-price competition, such as product differentiation. For those products, they argue that unit values are a poor measure of prices in international trade.
In these studies, substitution occurs between a good produced by the home firm and a good produced by the foreign firm. Our analysis of product differentiation differs from the above studies in two respects. First, substitution occurs between a set of vertically differentiated goods produced in the home country and sold to the home and a foreign market. Second, we examine specifically how product differentiation affects the test of PTM.

The disadvantages of unit values are acknowledged in many PTM studies using Knetter’s model. Common criticisms of unit values are that they do not account for different qualities shipped to different markets and for changes in product quality over time (Gil Pareja, 2002). However, authors, like Knetter (1989), typically argue that systematic differences in product quality, such as shipping different qualities to different markets, can be captured by country dummies. Similarly, changes in the quality of the product that is common across countries can be captured by time effects. Thus, the impact of product differentiation on the evaluation of PTM is typically argued to be minimal.

While prior authors acknowledge the problems associated with unit values when they reflect different qualities shipped to different countries or across time, we address an issue that to our knowledge has not been addressed before. Namely, we examine destination-specific changes in the product-quality mix and the use of unit values in PTM studies. More specifically, changes in the product-quality mix can be the result of fluctuations in the exchange rate. This is a problem when, as it is often the case, unit values represent a composite of differentiated products exported to a given market. In this case, false detection of PTM may occur.

In this paper, we explain how unit values aggregating differentiated products can result in false PTM findings and we show that the magnitude of the bias depends on the level of product differentiation. For this purpose, we introduce a conceptual model where a monopolist sells vertically differentiated products to a domestic and a foreign market. Two polar scenarios are analyzed. In the first one, there is perfect and costless consumer arbitrage, and the law of one price (LOP) holds for individual products (i.e., before aggregation). In the second scenario, consumer arbitrage is not feasible and markets are segmented. In both scenarios, we find “pseudo PTM,” i.e., PTM that is purely the result of data aggregation and product differentiation rather than changes in the exchange rate.

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4Exchange rate pass-through refers to the extent to which the price to a given importing country adjusts to changes in the exchange rate.

5See also Alston, Carter, and Whitney (1992), and Goldberg and Knetter (1997) for discussions on the use of unit values in the evaluation of PTM.

6Price stickiness and currency invoicing have also been indicated as potential reasons for bias in PTM findings (e.g., Goldberg and Knetter, 1997; Glauben and Loy, 2003; Gervais and Larue, 2004).
price discrimination across markets. In the first scenario, there is pseudo PTM only. In the second scenario, there is “real PTM” as well because markets are segmented. We show that the extent of pseudo PTM increases with the level of product differentiation.\footnote{This is in the same spirit as in Bodnar, Dumas and Marston (2002), who find that higher product substitutability moderates exchange rate pass-through, using a model where an exporting firm and a foreign import-competing firm produce products of various substitutability.} To evaluate the implications for empirical work, we employ Monte Carlo simulations analyzing the relationship between PTM and the level of product differentiation. The results indicate the presence of pseudo PTM for a sufficiently high level of product differentiation when the LOP holds. In both scenarios, a higher level of product differentiation is more likely to lead to a statistically significant evidence of PTM.

The rest of the paper is organized as follows. The conceptual model is presented in section 2 and the two scenarios are analyzed in section 3. Section 4 provides a simulation study and we conclude in section 5.

2 The model

Consider two countries: country 1 and 2. A monopolist in country 1 produces two vertically differentiated products with exogenous qualities \( q_l \) and \( q_h \) \((0 < q_l < q_h)\).\footnote{See footnote 16 (p.9) for a discussion on endogenous qualities.} The two goods are sold domestically and exported to country 2. The marginal cost is \( \frac{1}{2} q_j^2 \) for the product of quality \( q_j \) \((j = l, h)\).\footnote{When marginal cost is linear in quality, it can be shown that only the high-quality product will be sold.}

We model vertical differentiation à la Mussa and Rosen (1978). Consumers are heterogeneous in their preference for quality. A consumer with preference parameter \( \theta \) will enjoy a utility of \( \theta q - p \) if she buys one unit of the product of quality \( q \) at price \( p \), and zero if she buys nothing. There is a continuum of consumers in each country, i.e., \( \theta \in U[0, \theta_i] \) with density \( 1/\theta_i \) in country \( i \) \((i = 1, 2)\).

Let \( \theta_{il} \) \((i = 1, 2)\) denote the consumer in market \( i \) who is indifferent between buying the low-quality product or buying nothing. That is, \( \theta_{il} \) is the value of \( \theta \) that solves \( \theta q_l - \lambda_1 \cdot p_{il} = 0 \), where \( \lambda_1 = 1 \) and \( \lambda_2 = e \), and \( e \) is the exchange rate expressed in units of country 2’s currency per unit of country 1’s currency.\footnote{Throughout this article, prices are expressed in country 1’s currency.} Similarly, \( \theta_{ih} \) is the consumer in market \( i \) who is indifferent between buying the low- or high-quality product, i.e., \( \theta_{ih} \) is the value of \( \theta \) that solves \( \theta q_h - \lambda_1 \cdot p_{ih} = \theta q_l - \lambda_1 \cdot p_{il} \) with \( \lambda_1 = 1 \) and \( \lambda_2 = e \). Thus, consumers with \( \theta \in [0, \theta_{il}] \) will not buy, those with \( \theta \in [\theta_{il}, \theta_{ih}] \) will buy the low-quality product and the others \((\theta \in (\theta_{ih}, \theta_i])\) will buy the high-quality product.
Accordingly, the demands for the low- and high-quality products in country \( i = 1, 2 \) are:

\[
d_{il}(p_{ih}, p_{il}) = \frac{\theta_{ih} - \theta_{il}}{\theta_i} = \frac{\lambda_i(p_{ih}q_{il} - p_{il}q_{hl})}{\theta_i(q_h - q_l)q_l},
\]

\[
d_{ih}(p_{ih}, p_{il}) = \frac{\theta_i - \theta_{ih}}{\theta_i} = 1 - \frac{\lambda_i(p_{hi} - p_{hl})}{\theta_i(q_h - q_l)}.
\]

When there is pricing-to-market, a firm with market power will set different prices (in the same currency) in different markets based on their respective market conditions. Accordingly, Marston (1990) examines PTM by forming the ratio of the export to the home price set by a domestic monopolist and evaluating how it varies with the exchange rate. Similarly, we use the domestic-export price ratio

\[ X = \frac{P_1}{P_2}, \tag{1} \]

where \( P_i \) is the price in country \( i \), expressed in country 1’s currency.\(^{11}\) The PTM effect can be measured as the effect of a change in the exchange rate on \( X \). When there is PTM, a change in the exchange rate will have a non-zero impact on the ratio \( X \). In other words, there is PTM when the exporter responds to a change in the exchange rate by varying the price to one or both markets not proportionally. Alternatively, in our setting, there is no PTM when \( X \equiv 1 \).

We consider two scenarios. In the first one, the LOP holds for each individual product, and the prices of each product in the two markets are equal in the same currency. In the other scenario, markets are segmented and arbitrage between consumers across countries is not feasible. Consequently, each product is sold by the monopolist at different prices in each country.

### 3 Analysis

In this section, we solve for the equilibrium price and quantities in each scenario. The monopolist’s objective is to maximize profit by choosing prices. Using equilibrium prices and quantities, we calculate the unit values of sales to each country, expressed in country’s 1 currency. Unit values then enter the domestic-export price ratio \( (X) \), which is used to determine the presence of PTM.

We begin with the first scenario where the law of one price (LOP) holds for each product.

**Scenario 1. LOP holds**

The monopolist cannot price discriminate between market 1 and 2 in this scenario. Thus \( p_{il} = p_l \) and \( p_{ih} = p_h \) \((i = 1, 2)\), and profit is maximized according to:

\[
\max_{p_l,p_h} (p_l - \frac{1}{2}q_l^2)(d_{1l} + d_{2l}) + (p_h - \frac{1}{2}q_h^2)(d_{1h} + d_{2h})
\]

\(^{11}\)Other studies using this measure include Bergin and Feenstra (2001) and Gervais and Larue (2004).
where \( d_i(p_l, p_h) \) and \( d_{ih}(p_l, p_h) \) are the demand functions for the low- and high-quality product in country \( i \) \((i = 1, 2)\) as derived in section 2. Note that the prices \( p_l \) and \( p_h \) are set by the monopolist in country 1’s currency, whereas consumers’ demand in market 2 is a function of the price in the local currency, i.e., \( p_l \cdot e \) and \( p_h \cdot e \), where \( e \) is the exchange rate. We assume that in equilibrium the monopolist produces both products and sells to both countries.\(^{12}\)

From the first-order conditions, we obtain the equilibrium prices \( p^* \) and \( p^*_h \) and the equilibrium quantities \( d^*_i \) and \( d^*_{ih} \) for market \( i \) \((i = 1, 2)\).\(^{13}\) The unit value \( P_i \) is computed as the weighted average price to market \( i \), i.e.:

\[
P_i = \frac{p^*_l d^*_i + p^*_h d^*_{ih}}{d^*_i + d^*_{ih}} \tag{2}
\]

The presence of PTM is determined by computing \( X = \frac{P_1}{P_2} \) and evaluating whether it is identically equal to one or varies with the exchange rate. Our results are summarized in the next proposition.

**Proposition 1** When the LOP holds for individual products, there is pseudo PTM when using unit values.

**Proof.** We begin with the premise that there is no PTM when \( X = \frac{P_1}{P_2} \equiv 1 \). Substituting equation (2) into equation (1), the domestic-export price ratio can be expressed as

\[
X = \frac{p^*_l \sigma_1 + p^*_h (1 - \sigma_1)}{p^*_l \sigma_2 + p^*_h (1 - \sigma_2)} \tag{3}
\]

where \( \sigma_i = \frac{d^*_i}{d^*_i + d^*_{ih}} \) \((i = 1, 2)\), is the fraction of low-quality product in country \( i \).

For \( X = 1 \), it must be that \( \sigma_1 = \sigma_2 \). Substituting the equilibrium quantities (see the appendix), we have

\[
\sigma_1 = \frac{q_h(\theta_2 + e\theta_1)}{4e\theta_1^2 - q_l(\theta_2 + e\theta_1)} \quad \text{and} \quad \sigma_2 = \frac{q_h(e(\theta_2 + e\theta_1))}{4\theta_2^2 - eq_l(\theta_2 + e\theta_1)}. 
\]

It follows that \( \theta_2 = e\theta_1 \) is required for \( \sigma_1 = \sigma_2 \). Given that \( \theta_1 \) and \( \theta_2 \) are fixed parameters, \( \sigma_1 = \sigma_2 \) cannot hold when \( e \) varies. Thus, \( X \equiv 1 \) does not hold, indicating PTM. We find false evidence of PTM (pseudo PTM) using unit values. \( \blacksquare \)

Pseudo PTM is found because the exchange rate affects the ratio of unit values through a change in the product-quality mix. An appreciation of the foreign currency (decrease in \( e \)) results in an increase in imports of the high-quality variety in country 2 relative to the total quantity of imports. The reverse is true for a depreciation of the foreign currency.

\(^{12}\)It can be easily shown that the monopolist is better off supplying both products than supplying either product in both scenarios.

\(^{13}\)See the appendix for the derivations of the equilibrium prices and quantities.
The shift in the product-quality mix occurs because fluctuations in the exchange rate affect the relative utility obtained from the high-quality variety in market 2. To illustrate, note that

\[
\frac{U_h}{U_l} = \frac{\theta_2 q_h - \epsilon p_h}{\theta_2 q_l - \epsilon p_l}
\]

and the derivative with respect to the exchange rate is

\[
\frac{\partial(U_h/U_l)}{\partial e} = \frac{\theta_2(p_l q_h - p_h q_l)}{(\theta_2 q_l - \epsilon p_l)^2}.
\]

When \(\frac{p_h}{q_h} > \frac{p_l}{q_l}\) – a necessary condition that is always satisfied in equilibrium for the demand of both goods to be positive – this derivative is negative. In other words, even though both goods face the same change in the exchange rate, a decrease in exchange rate raises the relative utility of the high-quality good, which results in a shift in product-quality mix toward the higher quality product, i.e., quality upgrading.\(^{14}\) Conversely, an increase in the exchange rate results in a shift in the product-quality mix towards the lower quality product.\(^{15}\) There is evidence of quality upgrading or downgrading following movements in the exchange rates or government instruments having a similar effect on prices (e.g., ad valorem tariffs). Conley and Peterson (1995) find evidence for a decrease in quality of U.S. export of beef products to Japan following a depreciation of yen in the 1980s. Hummels and Skiba (2004) show that lower ad valorem tariffs and higher transportation cost result in quality upgrading using export data for more than 5000 product categories.

**Remark** Under perfect competition, where export prices are equal to marginal cost, we also find pseudo PTM using unit values.

The intuition for this remark is the same as presented for proposition 1. This result is particularly important given that PTM results are typically interpreted as evidence of imperfect competition (Goldberg and Knetter, 1997).

\(^{14}\)Feenstra (1995) notes that quality upgrading “can refer to either a shift in demand towards higher priced import varieties (i.e., a change in product mix), or to the addition of improved characteristics on each variety.” (p.1572)

\(^{15}\)This result is akin to the Alchian-Allen theorem – a per unit increase in price lowers the price of the high-quality good relative to the low-quality good thus raising the consumption of the high-quality good (Borcherding and Silberberg, 1978). This result has motivated the vast literature on trade restraints and quality upgrading (e.g., Falvey, 1979; Aw and Robert, 1986; Krishna, 1987; Feenstra, 1988; Boorstein and Feenstra, 1991). Exchange rates have the same effect on prices as ad valorem tariffs and taxes. Results in the literature differ on whether ad valorem tariffs have an effect on quality according to the homotheticity of preferences (Krishna, 1992). With non-homothetic preferences, such as those resulting from vertical differentiation models à la Mussa and Rosen (1978), Das and Donnenfeld (1987), Donnenfeld (1988), and Wall (1992) find that ad valorem tariffs result in quality downgrading and Krishna (1990) finds an ambiguous effect on quality. That tariffs result in quality downgrading mirrors our result that a foreign currency depreciation (appreciation) results in quality downgrading (upgrading).
Scenario 2. Market segmentation

In this scenario, the monopolist price discriminates between markets. Each market can be treated independently because of the assumption of constant marginal cost with respect to quantity. The monopolist maximizes profit by setting the price \( p_{ij} \) to country \( i \) \((i = 1, 2)\) for product of quality \( q_j \) \((j = l, h)\) according to:

\[
\max_{p_{il}, p_{ih}} \left( p_{il} - \frac{1}{2} q_l^2 \right) d_{il} + \left( p_{ih} - \frac{1}{2} q_h^2 \right) d_{ih}
\]

for market \( i \) \((i = 1, 2)\), where \( d_{il}(p_l, p_h) \) and \( d_{ih}(p_l, p_h) \) are the demand functions for the low- and high-quality product in country \( i \) as derived in section 2.

Define \( X_l \) as the domestic-export price ratio for the low-quality product \((i.e., \frac{p_{il}^*}{p_{2l}^*})\) and \( X_h \), as that for the high-quality product \((i.e., \frac{p_{ih}^*}{p_{2h}^*})\). A ratio different from one or varying with exchange rates indicates that the monopolist price discriminates. Thus, there is real PTM when \( X_l \equiv 1 \) or \( X_h \equiv 1 \) does not hold. The next proposition summarizes our findings for this scenario.

**Proposition 2** When markets are segmented,

i) There is real PTM for each individual product.

ii) There is both real and pseudo PTM when using unit values.

**Proof.** i) Substituting the expressions for the equilibrium prices (see scenario 2 in the appendix), the domestic-export price ratios can be expressed as

\[
X_l = \frac{(2\theta_1 + q_l)e}{2\theta_2 + e q_l}, \quad \text{and} \quad X_h = \frac{(2\theta_1 + q_h)e}{2\theta_2 + e q_h}.
\]

For \( X_l = X_h = 1 \), it must be that \( \theta_2 = e \theta_1 \). Given that \( \theta_1 \) and \( \theta_2 \) are fixed parameters, \( \theta_2 = e \theta_1 \) cannot hold when \( e \) varies. Thus, \( X_l \equiv 1 \) and \( X_h \equiv 1 \) do not hold, indicating PTM. We label this result “real PTM” given that the non-aggregated prices used in this calculation are set by a discriminatory monopolist.

ii) Because the two markets are independent, fluctuations in the exchange rates affect only the equilibrium prices and quantities in market 2. Thus, a change in the exchange rate would affect the domestic-export price ratio \((X = \frac{P_l}{P_2})\) only through \( P_2 \), which can be expressed as

\[
P_2 = \frac{p_{il}^* \sigma_2}{X_l} + \frac{p_{ih}^* (1 - \sigma_2)}{X_h}.
\]

It follows that

\[
\frac{\partial P_2}{\partial e} = \frac{\partial \sigma_2}{\partial e} \left( \frac{p_{il}^*}{X_l} - \frac{p_{ih}^*}{X_h} \right) - \frac{\partial X_l}{\partial e} \frac{p_{il}^*}{X_l^2} \sigma_2 - \frac{\partial X_h}{\partial e} \frac{p_{ih}^*}{X_h^2} (1 - \sigma_2) < 0.
\]

A change in the exchange rate affects \( P_2 \) through 1) a change in the composition of imports \((\sigma_2)\) generating the pseudo PTM effect, and 2) a change in \( X_l \) and \( X_h \), which reflect real PTM. The
negative sign of the derivative follows from: \[ \frac{\partial \sigma}{\partial e} = \frac{2q_h q_l (\theta_2 - \theta_1)}{(2\theta_2 - v_l) (2\theta_1 - v_l)} > 0, \] \[ \frac{\partial X_l}{\partial e} = \frac{2\theta_2 (2\theta_1 + q_l)}{(2\theta_2 + v_l)^2} > 0, \] \[ \frac{\partial X_h}{\partial e} = \frac{2\theta_2 (2\theta_1 + q_h)}{(2\theta_2 + v_h)^2} > 0, \] and \( (1 - \sigma_2) > 0. \) Thus, \[ \frac{\partial X}{\partial e} > 0 \) (because \[ \frac{\partial P_1}{\partial e} = 0, \] \[ \frac{\partial P_2}{\partial e} < 0 \) due to both real and pseudo PTM. \[ \blacksquare \]

As with scenario 1, a change in the exchange rate affects the composition of imports. This effect does not matter when examining PTM using individual product prices. However, because unit values constitute a weighted average of the price of high- and low-quality good in each market, a change in the exchange rate not only affects the landed prices in country 2, but also the weights associated to those prices through a change in the product-quality mix imported. Thus, PTM findings are the result of two effects: 1) a true PTM effect, because the monopolist does price discriminate in this scenario, and 2) a pseudo PTM due to the use of unit values, which average the price of good \( h \) and \( l \), and the resulting change in the composition of imports following fluctuations in the exchange rate.

This result indicates that one would conclude correctly that there is PTM using unit values as prices. However, there is also pseudo PTM. The extent to which \( X \) departs from one is affected by the aggregation of differentiated products, i.e., the importance of pseudo PTM. In what follows, we examine the relationship between the level of product differentiation and the extent of pseudo PTM for both scenarios. Increasing levels of product differentiation is modelled by fixing \( q_l \) and increasing \( q_h \). The next two corollaries summarize our results.\[16\]

**Corollary 3** Under the LOP, the extent of pseudo PTM increases with the level of product differentiation.

**Proof.** Under the LOP, PTM findings using unit values represent solely pseudo PTM. Thus, let \(|X - 1| \) measure the extent of pseudo PTM. The extent of pseudo PTM increases with product differentiation if \(|X - 1| \) increases with \( q_h \). To show that, we need to show that if \( X - 1 > 0 \), then

\[16\]Our setting assumes that quality is exogenous. Alternatively, quality can be endogenous and the monopolist chooses qualities followed by prices. The qualitative results do not change, i.e., there is always pseudo PTM. If a change in the exchange rate does not induce a change in the qualities (e.g., the change is perceived to be temporary and quality adjustments are costly, as in the short run), qualities are fixed once chosen. Because our findings of pseudo PTM holds for any \( q_h > q_l > 0 \), endogenous qualities do not improve the model. If qualities adjust automatically with a change in the exchange rate (say in the long run), we also find pseudo PTM. One important disadvantage of the endogenous quality model will become obvious with corollaries 3 and 4 – it does not allow us to determine how product differentiation affects the extent of pseudo PTM. This outcome of our model is important because in the construction of unit values, aggregation is performed over products that are more differentiated in some industries than in others.
\[ \frac{\partial X}{\partial q_h} > 0, \text{ and if } X - 1 < 0, \text{ then } \frac{\partial X}{\partial q_h} < 0. \] In the appendix we show that when \( \theta_2 < e \theta_1, X - 1 > 0, \) and \( \frac{\partial X}{\partial q_h} > 0. \) When \( \theta_2 > e \theta_1, X - 1 < 0, \) and \( \frac{\partial X}{\partial q_h} < 0. \) ■

**Corollary 4** Under market segmentation, the extent of pseudo PTM increases with the level of product differentiation.

**Proof.** Recall that \( X_l = \frac{p_{1l}^*}{p_{2l}^*}, X_h = \frac{p_{1h}^*}{p_{2h}^*}, \) and \( X = \frac{p_{1h}^*(1-\sigma_1)}{p_{2h}^* + (1-\sigma_2)} \). Under market segmentation (scenario 2), findings of PTM using unit values represent both real and pseudo PTM. Let \( |X - X_l| \) and \( |X - X_l| \) together measure the extent of pseudo PTM in this scenario. The extent of pseudo PTM increases with product differentiation if \( |X - X_j| (j = h, l) \) increases with \( q_h \). To show that, we need to show that if \( X - X_j > 0 \) then \( \frac{\partial(X - X_j)}{\partial q_h} > 0, \) and if \( X - X_j < 0 \) then \( \frac{\partial(X - X_j)}{\partial q_h} < 0 \). We divide this proof into two cases.

**Case 1.** \( \theta_2 < e \theta_1 \)

Note that when \( q_h = q_l, X_l = X_h = X > 1. \) Moreover, \( \frac{\partial X_l}{\partial q_h} = 0, \frac{\partial X_h}{\partial q_h} < 0, \lim_{q_h \to \infty} X_h = 1, \) and \( \frac{\partial X}{\partial q_h} > 0. \) This implies that when \( q_h > q_l, X - X_j > 0 \) and \( \frac{\partial(X - X_j)}{\partial q_h} > 0 (j = h, l). \) See the appendix for the derivations.

**Case 2.** \( \theta_2 > e \theta_1 \)

When \( q_h = q_l, X_l = X_h = X < 1. \) Moreover, \( \frac{\partial X_l}{\partial q_h} = 0, \frac{\partial X_h}{\partial q_h} > 0, \lim_{q_h \to \infty} X_h = 1, \) and \( \frac{\partial X}{\partial q_h} < 0. \) This implies that when \( q_h > q_l, X - X_j < 0 \) and \( \frac{\partial(X - X_j)}{\partial q_h} < 0 (j = h, l). \) See the appendix for the derivations. ■

To get a sense of how \( X, X_l, \) and \( X_h \) vary with the level of product differentiation \( (q_h) \), we take scenario 2’s model, assign parameter values, and plot these three measures against \( q_h \). We set \( q_l = 0.3, \theta_1 = 1, \theta_2 = 2, e = 3 (\theta_2 < e \theta_1). \) The results are provided in Figure 1. As indicated in the second case of corollary 4, when \( q_h = q_l, X_l = X_h = X > 1, \) indicating PTM but no pseudo PTM. With differentiated products \( (q_h > q_l) \), there is pseudo PTM because \( X \) is different from \( X_h \) and \( X_l \). Moreover, the graph clearly shows the increasing importance of pseudo PTM with greater levels of product differentiation because \( X \) moves away from both \( X_l \) and \( X_h \) when \( q_h \) increases.19

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17 As product differentiation increases, \( X_h \) decreases but never actually reaches a value of 1 because negative quantities of either variety are not allowed. Thus, \( X_h > 1 \) and \( X > X_l > X_h > 1. \)

18 As product differentiation increases, \( X_h \) increases but never actually reaches a value of 1 because negative quantities of either variety are not allowed. Thus, \( X_h < 1 \) and \( X < X_l < X_h < 1. \)

19 In the process of proving corollary 4, we also showed that when there is both real and pseudo PTM, product
4 Simulations

Our theoretical results indicate that when sales to a given market involve differentiated products and unit values are used as prices to evaluate PTM, there is always pseudo PTM. This result applies with or without price discrimination and even under perfect competition. This implies that in regression analyses following Knetter (1989), the exchange rate coefficient may pick up the effects of pseudo PTM. Next we conduct a Monte Carlo simulation to investigate 1) how prevalent are false statistical findings of PTM, and 2) quantify how the level of product differentiation impacts statistical findings of PTM.

We estimate the following model,

$$\ln X_t = \beta_0 + \beta_1 \ln e_t + U_t, \ t = 1 \ldots T$$

(4)

differentiation exaggerates the real level of price dispersion, i.e., the extent to which the prices to the two markets differ. The more $X$, $X_h$, and $X_l$ diverge from 1 (in a positive or negative fashion), the greater the price dispersion. When $\theta_2 < e\theta_1$ and $q_h > q_l$, $X > X_l > X_h > 1$, and $X$ shows a greater price dispersion than is demonstrated by either $X_h$ or $X_l$. When $\theta_2 > e\theta_1$ and $q_h > q_l$, $X < X_l < X_h < 1$, and $X$ again shows a greater price dispersion than is demonstrated by either $X_h$ or $X_l$. 

Figure 1: Domestic-export price ratios and product differentiation

![Figure 1: Domestic-export price ratios and product differentiation](image)
where \( T \) is the number of draws, \( e_t \sim U[a, b] \) is the exchange rate for draw \( t \). \( X_t \) is the domestic-export price ratio generated as \( \frac{P_i(e_t) + \epsilon_t}{P_i(e_t) + \epsilon_t} \), where \( P_i(e_t) \) is the unit value for market \( i \) \((i = 1, 2)\) computed as described in each scenario of section 3, and \( \epsilon_{it} \sim N(0, \sigma^2) \) and are identically and independently distributed across \( i \) and \( t \).

If there is no PTM, the domestic-export price ratio should be independent of the exchange rate and \( \beta_1 \) should be zero. By analyzing the estimate of \( \beta_1 \) under different levels of product differentiation, we can evaluate the effect of product differentiation on pseudo PTM.

We estimate the above model under the two scenarios examined in section 3. For both scenarios, we set \( a = 1.5 \), \( b = 2.5 \), \( \sigma = 1/15 \), \( T=100 \) (the number of draws), \( \theta_1 = 1 \), \( \theta_2 = 2 \), and \( q_h = 0.3 \)^{20}. We conduct 1000 trials for each level of product differentiation \( (q_h) \) to obtain \( \hat{\beta}_1 \) and its p-value. The means of \( \hat{\beta}_1 \) and the percentages of trials with p-value less than 10% are provided in tables 1 and 2.

Table 1: \( \hat{\beta}_1 \) under the LOP scenario

<table>
<thead>
<tr>
<th>( q_h )</th>
<th>number of trials</th>
<th>mean of ( \hat{\beta}_1 )</th>
<th>percentage of trials with p-value &lt; 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>.4</td>
<td>1000</td>
<td>.1717</td>
<td>12.5%</td>
</tr>
<tr>
<td>.5</td>
<td>1000</td>
<td>.3975</td>
<td>42.8%</td>
</tr>
<tr>
<td>.6</td>
<td>1000</td>
<td>.6218</td>
<td>85.6%</td>
</tr>
<tr>
<td>.7</td>
<td>1000</td>
<td>.8841</td>
<td>99.6%</td>
</tr>
</tbody>
</table>

Table 2: \( \hat{\beta}_1 \) under the market segmentation scenario

<table>
<thead>
<tr>
<th>( q_h )</th>
<th>number of trials</th>
<th>mean of ( \hat{\beta}_1 )</th>
<th>percentage of trials with p-value &lt; 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>.4</td>
<td>1000</td>
<td>1.0328</td>
<td>90.6%</td>
</tr>
<tr>
<td>.5</td>
<td>1000</td>
<td>1.0702</td>
<td>99.3%</td>
</tr>
<tr>
<td>.6</td>
<td>1000</td>
<td>1.1361</td>
<td>100%</td>
</tr>
<tr>
<td>.7</td>
<td>1000</td>
<td>1.2267</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1 is consistent with our theoretical results and indicates that when products are sufficiently differentiated, statistically significant results suggesting PTM may be obtained, although there is no real PTM. We obtain false evidence of PTM with over 42% of our trials when \( q_h = .5 \).

\(^{20}\)The parameters are chosen to ensure that all equilibrium prices and demands are nonnegative. Moreover, \( P_i(e_t) + \epsilon_{it} \) must be positive to calculate \( \ln X_t \), and \( \sigma \) is chosen accordingly.
and almost 100% of the trials when \( q_h = .7 \). Table 2 reflects scenario 2 where there is both real and pseudo PTM. For all levels of product differentiation, the significance level \((1 - p\text{-value})\) is higher than in the first scenario. This is intuitive given that there is pseudo as well as real PTM in this case. Both tables indicate that the PTM elasticity \((\hat{\beta}_1)\) increases with product differentiation, and so does the proportion of statistical PTM findings – a result consistent with corollaries 3 and 4.\(^{21}\)

5 Conclusion

In this study, we examine the extent to which a false detection of pricing-to-market (pseudo PTM) arises from the use of unit value data. To do so, we analyze two scenarios. Both scenarios involve a monopolist located in the home country producing a low- and high-quality variety of a good. In the first scenario, arbitrage prevails and the monopolist charges the same price for each variety in both markets. In the second scenario, arbitrage is not possible and the monopolist price discriminates between the two markets. Pseudo PTM is found in both scenarios. Findings of PTM when the law of one price (LOP) holds are purely spurious, whereas they represent a combination of real and pseudo PTM when markets are segmented. Moreover, pseudo PTM is found even under perfect competition. Pseudo PTM occurs because movements in the exchange rate alter the product-quality mix sold to each market thus affecting the unit values, even when the prices to the two markets are identical by variety.

For both scenarios, we determined that product differentiation increases the extent to which results are biased by pseudo PTM, thus increasing the likelihood of false detection of PTM in empirical work. Our simulation results show that for sufficiently differentiated products, a statistical finding of PTM occurs when the LOP holds. Moreover, the PTM elasticity increases with product differentiation in both scenarios.

While other potential reasons have been raised for bias in PTM findings (e.g., currency invoicing and menu costs), our results suggest that the prevalence of PTM findings in the literature could also be attributed to the use of unit values aggregating differentiated products. PTM findings have been interpreted as evidence of price discrimination and market power, without explaining the source of market segmentation or market power (see Goldberg and Knetter, 1997 for a discussion). Sexton and Lavoie (2001) also observe the general lack of justification for the examination of imperfect competition and price discrimination among PTM studies focusing on food and agricultural products. Thus, our research emphasizes the need for future PTM studies to

\(^{21}\)We can verify numerically using our theoretical results that \( \frac{\partial \ln X}{\partial \ln e} \) increases with \( q_h \).
1) investigate the plausibility of market power in international trade of the product of interest, 2) evaluate the level of differentiation present in the export unit value data for the product category chosen, and 3) interpret the results accordingly. Alternatively, more confidence can be placed on results obtained using disaggregated data for which there are good reasons to believe exporters have market power in the international market (i.e., they produce a differentiated product relative to other countries' products, exports are conducted by a large entity, such as a state-trading firm, the exporter has a large world market share, etc.).22 Such caution is especially important when results are used for policy purposes.

We have examined the bias in PTM results when unit values aggregate vertically differentiated products. Future research includes finding ways to mitigate pseudo PTM (e.g., by controlling for quality changes) and analyzing pseudo PTM in other settings, such as horizontal differentiation.

22See for example Gil-Pareja (2002), and Glauben and Loy (2003) where such care is taken.
APPENDIX

1. Derivations of equilibrium prices and quantities

Scenario 1

In country 1, the consumer indifferent between buying the low-quality product or buying nothing is defined by the value of \( \theta \) solving \( \theta q_l - p_l = 0 \), i.e., \( \theta_{1l} = \frac{p_l}{q_l} \). Similarly, the consumer indifferent between the low- and high-quality products is defined by the value of \( \theta \) solving equation \( \theta q_h - p_h = \theta q_l - p_l \), i.e., \( \theta_{1h} = \frac{p_h - p_l}{q_h - q_l} \).

Thus the low-quality product is purchased by consumers with \( \theta \in [\theta_{1l}, \theta_{1h}] \) and the demand for the low-quality product is,

\[
d_{1l} = \frac{\theta_{1h} - \theta_{1l}}{\theta_1} = \frac{q_hp_l - p_l q_h}{(q_h - q_l)q_l \theta_1}.
\]

The high-quality product is purchased by consumers with \( \theta \in (\theta_{1h}, \theta_1] \) and the demand for the high-quality product is,

\[
d_{1h} = \frac{\theta_1 - \theta_{1h}}{\theta_1} = 1 - \frac{p_h - p_l}{(q_h - q_l)q_l \theta_1}.
\]

The demands for the low- and high-quality products in country 2 can be obtained in a similar manner. Note however that the demands of consumers in country 2 depend on the price of the product expressed in local currency, i.e., \( p_l \cdot e \) and \( p_h \cdot e \), where \( e \) is the exchange rate expressed in units of country 2’s currency per unit of country 1’s currency.

The demands in country 2 can be represented as,

\[
d_{2l} = \frac{\theta_{2h} - \theta_{2l}}{\theta_2} = e \frac{q_hp_l - p_l q_h}{(q_h - q_l)q_l \theta_2}, \quad \text{and}
\]

\[
d_{2h} = \frac{\theta_2 - \theta_{2h}}{\theta_2} = 1 - e \frac{p_h - p_l}{(q_h - q_l)q_l \theta_2}.
\]

The firm’s profit is

\[
\pi = (p_l - \frac{1}{2} q_l^2) \frac{q_hp_l - p_l q_h (1 + \frac{e}{\theta_2})}{(q_h - q_l)q_l} + (p_h - \frac{1}{2} q_h^2) \left[ 2 - \frac{p_h - p_l}{(q_h - q_l)} \left( \frac{1}{\theta_1} + \frac{e}{\theta_2} \right) \right]
\]

with first-order conditions:

\[
\frac{\partial \pi}{\partial p_l} = \frac{1}{2} \left( \theta_2 + e \theta_1 \right) \left[ 4 (p_h q_l - p_l q_h) + q_h q_l (q_l - q_h) \right] = 0, \quad \text{and}
\]

\[
\frac{\partial \pi}{\partial p_h} = \frac{1}{2} \left( \theta_2 + e \theta_1 \right) (4 p_h q_l - 4 p_l q_h + q_l^2 - q_h^2) - 4 \theta_1 \theta_2 (q_h - q_l) = 0.
\]
Similarly, the firm’s problem in country 2 is, \( \frac{[4\theta_1\theta_2 + q_h(\theta_2 + e\theta_1)]q_h}{\theta_2 + e\theta_1} \), \( \frac{[4\theta_1\theta_2 + q_l(\theta_2 + e\theta_1)]q_l}{\theta_2 + e\theta_1} \).

The equilibrium quantities are

\[
\begin{align*}
d^*_1 &= \frac{q_h}{4\theta_1}, & d^*_1 &= \frac{4e\theta_1^2 - (q_l + q_h)(\theta_2 + e\theta_1)}{4\theta_1(\theta_2 + e\theta_1)} \\
d^*_2 &= \frac{q_l}{4\theta_2}, & d^*_2 &= \frac{4\theta_1^2 - e(q_l + q_h)(\theta_2 + e\theta_1)}{4\theta_2(\theta_2 + e\theta_1)}
\end{align*}
\]

For \( d^*_1 > 0 \) and \( d^*_2 > 0 \), \( q_h + q_l < \min\left[ \frac{4e\theta_1^2}{\theta_2 + e\theta_1}, \frac{4\theta_1^2}{e(\theta_2 + e\theta_1)} \right] \) must hold. We assume that this is the case throughout the article.

**Scenario 2**

The monopolist treats each market independently due to market segmentation and constant marginal cost. The firm’s problem in country 1 is,

\[
\max_{p_{1l}, p_{1h}} (p_{1l} - \frac{1}{2}q_l^2)d_{1l} + (p_{1h} - \frac{1}{2}q_h^2)d_{1h},
\]

Similarly, the firm’s problem in country 2 is,

\[
\max_{p_{2l}, p_{2h}} (p_{2l} - \frac{1}{2}q_l^2)d_{2l} + (p_{2h} - \frac{1}{2}q_h^2)d_{2h}.
\]

We solve the firm’s problem in the market 1 first. The marginal consumers are,

\[
\theta_{1l} = \frac{p_{1l}}{q_l}, \quad \theta_{1h} = \frac{p_{1h} - p_{1l}}{q_h - q_l},
\]

and thus the demands can be represented by,

\[
d_{1l} = \frac{\theta_{1h} - \theta_{1l}}{\theta_1} = \frac{q_l p_{1h} - p_{1l}q_h}{(q_h - q_l)q_l \theta_1},
\]

\[
d_{1h} = \frac{\theta_{1} - \theta_{1h}}{\theta_1} = 1 - \frac{p_{1h} - p_{1l}}{(q_h - q_l)\theta_1}.
\]

Firm’s profit is,

\[
\pi_{1} = (p_{1l} - \frac{1}{2}q_l^2)d_{1l} + (p_{1h} - \frac{1}{2}q_h^2)d_{1h} = (p_{1l} - \frac{1}{2}q_l^2)\frac{q_l p_{1h} - p_{1l}q_h}{(q_h - q_l)q_l \theta_1} + (p_{1h} - \frac{1}{2}q_h^2)(1 - \frac{p_{1h} - p_{1l}}{(q_h - q_l)\theta_1})
\]

The first order conditions are,

\[
\frac{\partial \pi_{1}}{\partial p_{1l}} = \frac{4(p_{1h}q_l - p_{1l}q_h) + q_l q_h (q_l - q_h)}{2(q_h - q_l)q_l \theta_1},
\]
\[
\frac{\partial \pi_1}{\partial p_{1h}} = \frac{4(p_{1h} - p_{1l}) - (q_h - q_l)(q_l + q_h + 2\theta_1)}{2(-q_h + q_l)\theta_1}.
\]

Solving these two equations simultaneously for \( p_{1l} \) and \( p_{1h} \), we have,

\[
p_{1l}^* = \frac{1}{4} q_h (2\theta_1 + q_h), \quad p_{1h}^* = \frac{1}{4} q_l (2\theta_1 + q_l).
\]

Thus the equilibrium quantities are,

\[
d_{1l}^* = \frac{q_h}{4\theta_1}, \quad d_{1h}^* = \frac{2\theta_1 - q_l - q_h}{4\theta_1}.
\]

Similarly, by solving the maximization problem of the monopolist in country 2, we can obtain the following equilibrium prices and quantities,

\[
p_{2h}^* = \frac{1}{4} q_h (2\theta_2 + e q_h)/e, \quad p_{2l}^* = \frac{1}{4} q_l (2\theta_2 + e q_l)/e
\]

\[
d_{2l}^* = \frac{e q_h}{4\theta_2}, \quad d_{2h}^* = \frac{2\theta_2 - e(q_l + q_h)}{4\theta_2}.
\]

For \( d_{1h}^* > 0 \) and \( d_{2h}^* > 0, q_h + q_l < \min \{2\theta_1, 2\theta_2/e\} \) must hold. Note that this condition is less restrictive than \( q_h + q_l < \min \left[ \frac{4e \theta_2^2}{\theta_2 + e \theta_1}, \frac{4\theta_2^2}{e(\theta_2 + e \theta_1)} \right] \) established in scenario 1 for the quantities in market 2 to be positive. Thus, \( d_{1h}^* > 0 \) and \( d_{2h}^* > 0 \) in scenario 2 holds.

### 2. Derivations of equations associated with corollary 3 and 4.

#### Corollary 3

First, we determine the sign of \( X - 1 \). Using equation (3), the sign of \( X - 1 \) corresponds to the sign of \( (p_1^* - p_h') (\sigma_1 - \sigma_2) \). It can be easily shown that \( (p_1^* - p_h') < 0 \) because \( q_l < q_h \). Moreover,

\[
\sigma_1 - \sigma_2 = \frac{4q_h (\theta_2 + e \theta_1)^2 (\theta_2 - e \theta_1)}{[4e \theta_1^2 - q_l (\theta_2 + e \theta_1)][4 \theta_2^2 - e q_l (\theta_2 + e \theta_1)]}
\]

and the two elements of the denominator are positive given the assumption we made for all quantities to be positive in equilibrium (see scenario 1 of this appendix). Thus, the sign of \( \sigma_1 - \sigma_2 \) depends on the sign of \( \theta_2 - e \theta_1 \). When \( \theta_2 < e \theta_1, \sigma_1 - \sigma_2 < 0 \), and \( X - 1 > 0 \). When \( \theta_2 > e \theta_1, \sigma_1 - \sigma_2 > 0 \), and \( X - 1 < 0 \).

Second, we determine the sign of \( \frac{\partial X}{\partial q_h} \). Because \( X = \frac{P_1}{P_2}, \frac{\partial X}{\partial q_h} = \frac{\partial P_1}{\partial q_h} \frac{P_2 - P_1}{P_2^2} \). Note that \( P_1 = \sigma_1 (p_1^* - p_h') + p_h, P_2 = \sigma_2 (p_1^* - p_h') + p_h, \) and \( q_h \) does not enter \( p_1^* \). Thus,

\[
\frac{\partial X}{\partial q_h} = \frac{\frac{\partial \sigma_1}{\partial q_h} (p_1^* - p_h') + \frac{\partial p_h}{\partial q_h} (1 - \sigma_1)}{P_2} \frac{P_2 - P_1}{P_2^2} \left[ \frac{\frac{\partial \sigma_2}{\partial q_h} (p_1^* - p_h') + \frac{\partial p_h}{\partial q_h} (1 - \sigma_2)}{P_2} \right] - P_1 \left[ \frac{\partial \sigma_2}{\partial q_h} (p_1^* - p_h') + \frac{\partial p_h}{\partial q_h} (1 - \sigma_2) \right].
\]
Rearranging we obtain:

\[
\frac{\partial X}{\partial q_h} = \frac{(p_1^* - p_h^*)}{P_2^2} \left( \frac{\partial \sigma_1}{\partial q_h} P_2 - \frac{\partial \sigma_2}{\partial q_h} P_1 \right) - \frac{\partial p_h^*}{\partial q_h} (P_2 \sigma_1 - P_1 \sigma_2 - P_2 + P_1)
\]  \hspace{1cm} (A.1)

where \( \sigma_1 = \frac{q_h(\theta_2 + e\theta_1)}{4e \theta_2^2 - q_h(\theta_2 + e\theta_1)} \), and \( \sigma_2 = \frac{q_h(e\theta_2 + \theta_1)}{4e \theta_2^2 - q_h(e\theta_2 + e\theta_1)} \).

Given the expressions for \( \sigma_1 \) and \( \sigma_2 \), \( \frac{\partial \sigma_1}{\partial q_h} = \frac{\sigma_1}{q_h}, \frac{\partial \sigma_2}{\partial q_h} = \frac{\sigma_2}{q_h} \). Substituting for \( \frac{\partial \sigma_1}{\partial q_h}, \frac{\partial \sigma_2}{\partial q_h}, P_1 \), and \( P_2 \), equation (A.1) can be re-written as

\[
\frac{\partial X}{\partial q_h} = \frac{1}{P_2^2} \left[ (\sigma_1 - \sigma_2) \left( (p_1^* - p_h^*) \frac{P_h^* q_h}{q_h} - \frac{\partial p_h^*}{\partial q_h} p_h^* \right) \right]
\]

where \( (p_1^* - p_h^*) \frac{P_h^* q_h}{q_h} < 0 \) because \( (p_1^* - p_h^*) < 0 \) and \( \frac{\partial p_h^*}{\partial q_h} = \frac{4q_h(\theta_2 + e\theta_1)q_h(\theta_2 + e\theta_1)}{4(\theta_2 + e\theta_1)} > 0 \).

The above shows that the sign of \( \frac{\partial X}{\partial q_h} \) also depends on the sign of \( \sigma_1 - \sigma_2 \), which we have already determined depends on the sign of \( \theta_2 - e\theta_1 \).

Thus, when \( \theta_2 < e\theta_1, \sigma_1 - \sigma_2 < 0, X - 1 > 0, \) and \( \frac{\partial X}{\partial q_h} > 0 \). When \( \theta_2 > e\theta_1, \sigma_1 - \sigma_2 > 0, X - 1 < 0, \) and \( \frac{\partial X}{\partial q_h} < 0 \).

**Corollary 4**

Because

\[ X_I = \frac{p_{1I}^*}{p_{2I}^*} = \frac{(2\theta_1 + q_h)e}{2\theta_2 + e q_h}, \]

\[ X_h = \frac{p_{1h}^*}{p_{2h}^*} = \frac{(2\theta_1 + q_h)e}{2\theta_2 + e q_h}, \]

\[ X = \frac{p_{1I}^* \sigma_1 + p_{1h}^* (1 - \sigma_1)}{p_{2I}^* \sigma_2 + p_{2h}^* (1 - \sigma_2)} = \frac{e(q_h^2 + 4\theta_1^2 - q_h q_l - q_h^2)(2\theta_2 - e q_l)}{(e^2 q_h^2 + 4\theta_2^2 - e^2 q_h q_l - e^2 q_h^2)(2\theta_1 - q_l)}, \]

then, when \( q_l = q_h = q, X_I = X_h = X = \frac{(2q_h + q)e}{2\theta_2 + e q_h} \).

In what follows, the equations allowing us to sign \( \frac{\partial X}{\partial q_h} \) are derived. Rewrite \( X = \frac{p_{1I}^* \sigma_1}{p_{2I}^* \sigma_2} A \) where \( A = 1 + \frac{p_{1I}^*}{p_{2I}^*} \left( \frac{1 - \sigma_1}{\sigma_1} \right) \) and \( B = 1 + \frac{P_{2h}^*}{p_{2I}^*} \left( \frac{1 - \sigma_2}{\sigma_2} \right) \). Therefore,

\[
\frac{\partial X}{\partial q_h} = \frac{p_{1I}^* \sigma_1}{p_{2I}^* \sigma_2} \frac{1}{B^2} \left[ B \left( \frac{\partial}{\partial q_h} \left( \frac{p_{1I}^*}{p_{2I}^*} \left( \frac{1 - \sigma_1}{\sigma_1} \right) + \frac{p_{1h}^*}{p_{1I}^*} \frac{\partial}{\partial q_h} \left( \frac{1 - \sigma_1}{\sigma_1} \right) \right) \right) - A \left( \frac{\partial}{\partial q_h} \left( \frac{p_{2h}^*}{p_{2I}^*} \left( \frac{1 - \sigma_2}{\sigma_2} \right) + \frac{p_{2h}^*}{p_{2I}^*} \frac{\partial}{\partial q_h} \left( \frac{1 - \sigma_2}{\sigma_2} \right) \right) \right) \right]
\]  \hspace{1cm} (A.2)

where \( \frac{p_{1I}^*}{p_{2I}^*} = \frac{q_h(2\theta_1 + q_h)}{q_l(2\theta_1 + q_l)}, \frac{1 - \sigma_1}{\sigma_1} = \frac{2\theta_1 - q_h - q_l}{q_h}, \frac{p_{1h}^*}{p_{1I}^*} = \frac{q_h(2\theta_2 + e q_h)}{q_l(2\theta_2 + e q_l)}, \) and \( \frac{1 - \sigma_2}{\sigma_2} = \frac{2q_h - e(q_h + q_l)}{e q_h} \). The derivative of these expressions with respect to \( q_h \) can be written as:

\[
\frac{\partial}{\partial q_h} \left( \frac{1 - \sigma_1}{\sigma_1} \right) = -\frac{1}{\sigma_1 q_h}, \quad \frac{\partial}{\partial q_h} \left( \frac{p_{2h}^*}{p_{2I}^*} \right) = \frac{p_{2h}^*}{p_{2I}^* q_h} + \frac{q_h}{4p_{2I}^*}, \quad \text{and} \quad \frac{\partial}{\partial q_h} \left( \frac{1 - \sigma_2}{\sigma_2} \right) = -\frac{1}{\sigma_2 q_h}.
\]
Substituting these last four expressions into \((A.2)\), we obtain

\[
\frac{\partial X}{\partial q_h} = p_{1l}^* \sigma_1 \frac{1}{p_{2l}^* \sigma_2 B^2} \left\{ B \left[ -\frac{p_{1h}}{p_{1l}^* q_h} + q_h \frac{1 - \sigma_1}{4p_{1l}^*} \right] - A \left[ -\frac{p_{2h}}{p_{2l}^* q_h} + q_h \frac{1 - \sigma_2}{4p_{2l}^*} \right] \right\},
\]

which can be rewritten as

\[
\frac{\partial X}{\partial q_h} = p_{1l}^* \sigma_1 \frac{1}{p_{2l}^* \sigma_2 B^2} \left\{ \frac{1}{q_h} \left( \frac{p_{2h}}{p_{2l}^*} A - \frac{p_{1h}}{p_{1l}^*} B \right) + \frac{q_h}{4} \left[ \frac{B}{p_{1l}^*} \left( \frac{1 - \sigma_1}{\sigma_1} \right) - \frac{A}{p_{2l}^*} \left( \frac{1 - \sigma_2}{\sigma_2} \right) \right] \right\}.
\]

After substituting for \(A\) and \(B\) within the curly brackets and rearranging we obtain:

\[
\frac{\partial X}{\partial q_h} = p_{1l}^* \sigma_1 \frac{1}{p_{2l}^* \sigma_2 B^2} \left\{ \frac{1}{q_h} \left[ \left( \frac{p_{2h}}{p_{2l}^*} \right) - \frac{p_{1h}}{p_{1l}^*} \right] + \frac{q_h}{4} \left[ \left( \frac{1 - \sigma_1}{\sigma_1} \right) - \frac{1 - \sigma_2}{\sigma_2} \right] + \frac{p_{2h} - p_{1l}^*}{p_{1l}^* p_{2l}^*} \left( \frac{1 - \sigma_1}{\sigma_1} \right) \left( \frac{1 - \sigma_2}{\sigma_2} \right) \right\}.
\]

Then, we substitute for the equilibrium prices and market shares and simplify to obtain:

\[
\frac{\partial X}{\partial q_h} = \frac{1}{p_{2l}^* \sigma_2 B^2} \frac{\sigma_1}{\sigma_2} \frac{(e\theta_1 - \theta_2)(e\theta_1 + \theta_2)(2q_h + q_l)}{eq(2\theta_2 + eq)}.
\]

The sign of \(\frac{\partial X}{\partial q_h}\) is the sign of \(e\theta_1 - \theta_2\).
References


