THE ECONOMICS OF SHADOW PRICING: MARKET DISTORTIONS AND PUBLIC INVESTMENT

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1. The Function of Shadow Pricing

1.1 Outline to the Paper

This paper explores the background to an important issue in applied welfare economics--how the commodities used and produced by public projects should be valued in choosing among alternative modes of production. These valuations, however arrived at, have come to be referred to as shadow prices. We review first the reasons why these valuations present a problem, and second the various approaches that have been taken in the literature for obtaining them. Section 1 defines the role of shadow pricing, as the term is used here--a more general role than is normally attributed to it--and catalogues the various sources of market distortions. Shadow prices can potentially serve as substitutes for distorted market prices in some cases, but not in others. Section 2 provides a critical review of the various approaches to shadow pricing that have been advocated in the literature. Most are found either to be wanting in their theoretical foundations or to be internally inconsistent.

*This paper owes a heavy debt to the author's former teacher, Amartya K. Sen, and to the comments and suggestions of Martin E. Abel, Walter P. Falcon and C. Peter Timmer. The author is responsible for all views and any errors it contains.
1.2. **The Role of Shadow Prices**

The economic evaluation of a proposed course of action (henceforth "project") involves three distinct operations: (a) determination of the physical characteristics of the project, (b) translation of these physical quantities into value terms, and (c) application of a decision-making criterion. This paper is concerned with step (b). Step (a) is the most difficult and crucial in practice, and most of the serious errors in benefit-cost analysis probably occur at this stage. Nevertheless these problems are ignored in the present paper, and the information required for step (a) is assumed to be known. Until now step (c) has received the most attention from economists. The criteria that have been proposed include the choice of that project with:

- the highest net present value,
- the highest internal rate of return,
- the lowest domestic resource cost of foreign exchange,
- the highest ratio of discounted benefits to discounted costs.

The choice of criterion is an important, but perhaps not overwhelmingly important, matter. It is assumed in this paper that the net present value criterion has been adopted. The problem of valuation is common to them all, however, and the ultimate choice of projects is typically a good deal more sensitive to the way the valuation problem is handled than to the particular choice of investment criterion.

The most obvious solution to the valuation problem is to value all inputs and outputs of the project at their domestic market prices, and indeed this procedure is not lacking in advocates. Yet there are
good reasons for thinking that market prices frequently do not adequately reflect social valuations, especially in less-developed economies. Of course, it is possible to argue that if market prices are badly distorted the appropriate policy response is to attempt to eliminate these distortions. This is certainly correct, but the project evaluator is ordinarily unable to ensure that these reforms will be adopted. He may well express his professional judgment on how market distortions should be removed, or at least reduced; but having done so he must then assume, for the purposes of project evaluation, either that these distortions will indeed be eliminated or that they will persist. Frequently, the realistic assumption is the latter, and this is the basis for the recent interest in accounting or shadow prices for benefit-cost analysis.

Prices, as the term is used in economics, have two properties. Firstly they describe the rates at which commodities can be exchanged in a market, and secondly they are signals that decision-makers use in deciding which course of action to pursue. Shadow prices have the second characteristic but not necessarily the first. Shadow prices for use in planning and evaluating public projects are intended to serve as the basis for decisions on the design, adoption, and ultimate operation of these projects, even though they are not necessarily the prices the government actually pays or receives for inputs used or outputs produced.

The economic framework assumed in this paper is that of a mixed economy in which the size of the public sector is exogenously given, and market prices are distorted to varying degrees. Insofar as they differ from market prices, the shadow prices used in the public sector are
to be used only in the public sector. Not only are these shadow prices ignored by private sector decision-makers, who base their decisions entirely on market prices, but the determination of these shadow prices has no effect on market prices themselves via the government's tax-cum-subsidy policies.

Benefit-cost analysis, and hence shadow pricing, is sometimes said to be of minor practical importance in less-developed economies. So few projects are designed and reach the "evaluation stage", it is pointed out, that few alternatives are actually rejected. This view is certainly not without merit, but it rests on the assumption that the role of shadow pricing begins with the evaluation of a number of fully-designed mutually exclusive alternatives (one of which is doing nothing) and ends with the acceptance of one of them. This paper takes exception to that assumption. The three operations of (a) project design, (b) economic evaluation, and (c) operation of the adopted project, are typically thought of as being independent and discrete activities. This is reinforced by the fact that three entirely different groups -- engineers, economists, and managers -- are normally responsible for these operations.

Logically, however, precisely the same kinds of decisions are being made at all three stages. In project design some technical possibilities are being selected and others rejected. The operation of projects also involves selection of some technically possible modes of operation and the rejection of others; seldom is the project's technology so rigid that no ex-post substitution possibilities exist. The economic principles that are relevant to the design and operation of projects are thus exactly the same as those normally thought of as being involved in the more formal stage of project evaluation. It is
thus highly desirable that the same shadow prices as are to be used in project evaluation be given to the engineers involved in designing projects and the managers involved in operating projects, and that (at a minimum) they be instructed to keep those prices in mind in carrying out their functions. This is not commonly done in practice.

1.3. Market Distortions and Their Implications

One of the most characteristic features of a less-developed economy is that markets are not well developed and that market prices are distorted. These distortions are due in part to the failure of the markets themselves to function efficiently, and in part to the effects of government policy. Their implications are that market prices are potentially misleading indicators of social valuations. The sources of these distortions include the following:

(a) Non-competitive behavior. This includes any reason why decision-makers do not take prices as given such as monopoly, oligopoly, etc.

(b) Production non-convexities. In general this implies that a competitive equilibrium does not exist. If market failure occurs for this reason shadow prices will normally fail to achieve an optimal solution for exactly the same reason.

(c) Externalities. This includes cases where markets do not exist for things which affect individuals' welfare. The extreme case is that of a pure public good. For example, the emphasis on the importance
of sub-optimal savings in recent years has rested largely on the analogy between savings and a public good in the Samuelsonian sense.¹

(d) Informational failure. Markets may fail to operate efficiently because individuals do not know all the alternatives that are in fact open to them.

(e) Income distribution. It is not hard to see that the distribution of income affects market prices. If that distribution is judged to be non-optimal then the market prices which arise from it may also be judged non-optimal, even if perfectly competitive.

(f) Effects of government policy. The following market-distorting policies are extremely common in less-developed economies:

- over-valued domestic currency,
- tariffs and import quotas,
- interest rate ceilings,
- rental ceilings on land,
- wage rate floors,
- domestic tax-cum-subsidy policies,
- commodity price supports.

Reforms required to stimulate development such as the provision of necessary public and semi-public goods (including roads, marketing facilities, education, research on local production problems, etc.) take time to yield a visible return and are often unpopular with the most powerful groups. For similar reasons reforms required for a more equitable distribution of assets are politically difficult to achieve. All too often it is politically expedient to attempt to defuse political pressures for genuine reforms by imposing distortions on the price

¹ However, this is not the only basis for a belief that savings are typically sub-optimal, especially in less-developed economies. For a brief summary of several strong arguments see Sen (1972), and for a detailed analysis of the public goods argument see Sen (1967).
system. The ultimate beneficiaries, however, tend not to be the poor, but those who can best manipulate the bureaucracies to their own advantage.

If we catalogue the kinds of market configurations that may arise as consisting of:

(a) those in which prices work in principle and market prices are not distorted,

(b) those in which prices do not work in principle, due to the existence of, say, production non-convexities, and

(c) those in which prices work in principle but market prices are distorted,

then this paper is concerned only with case (c). Case (a) is not of interest here since there is no reason not to rely on private markets in this case. Case (b) is not of interest because non-convexities generate the same kinds of problems for shadow pricing as they do for the operation of competitive markets. Furthermore we are concerned only with that subset of distortions of the case (c) variety where the project evaluator cannot ensure that the policy changes necessary to correct the distortion will be carried out.

A distortion of the case (c) kind is depicted in Figure 1. Aggregate production and consumption possibilities for the two commodities 1 and 2 are described by the convex set Z (vertical lines). Suppose that both the public and private sectors are involved in the production of these commodities and that the two sectors are basing their production decisions on the same distorted market prices. Since all producers in the economy are using the same prices, production efficiency is attained and the
economy will be producing on the boundary of $Z$, at say point $A$. However, since these prices are non-optimal the marginal rate of transformation

![Diagram](image)

Figure 1: Implications of a market distortion in production at $A$ is not equal to the social marginal rate of substitution in consumption at $A$. The latter is given by the slope of the social indifference curve $W^o$ at $A$.

Optimality is achieved at $B$, where all producers set their marginal rates of transformation equal to the slope of $W^{**}$ at the point of tangency with $Z$. Yet if private producers cannot be induced to do this, point $B$ is unattainable. If the public sector uses different prices from those used in the private sector, we know that some production efficiency must be sacrificed. The economy will then be producing in the interior of $Z$, denoted $I(Z)$. Representing the upper preference sets of $W^o$ as $W^+_+$ (diagonal lines in Fig. 1), then the question arises as to whether we can find a set of shadow prices for use in the public sector such that the economy will be producing and consuming in the open set given by the intersection of $I(Z)$ with $W^o$ (cross-hatched in Fig. 1). If so, then use of these shadow prices is a good thing; it increases welfare. On the other hand, if the use of shadow prices causes the
economy to locate in the intersection of I(Z) with \( W^0 \), where \( W^0_\_ \) denotes the lower preference sets of \( W^0 \), then their use is undesirable.
2. Approaches to Shadow Pricing

2.1. Introduction

Numerous approaches to the calculation of welfare-increasing shadow prices have been proposed, but the economic literature on the subject is, in general, quite confused. The underlying assumptions made by different authors about the objectives of shadow pricing, the areas in which shadow prices are to be used, and the conditions that hold in the remainder of the economy, are frequently inconsistent. Worse still, these assumptions are seldom spelled out in detail. The reader is forced to try to infer the economic models that different authors have in mind from the way they attempt to defend their particular approaches.\(^1\) In part this is due to the practical orientation of the majority of the literature on shadow pricing. It is assumed that readers are mainly interested in being told "how to do it", and would simply be turned off by abstract theorizing. Such studies certainly have a role to play, but unless the underlying assumptions of the various approaches are made explicit, the basic areas of dispute are not likely to be clarified.

One issue is of critical importance. The existence of non-optimalities in the operation of markets in the private sector is almost invariably the justification for considering the inclusion of a particular investment in the public sector. It is also the reason

\(^1\)Regarding the usage of the term "shadow price", Rudra has written, somewhat sarcastically: "It has become a commonplace notion which everybody, even journalists, seem to understand. One encounters the phrase "shadow price" in every nook and corner of economic policy discussions without any reference to any particular feasibility constraints or any particular objective function." Rudra (1972, p. 13).
for our interest in shadow pricing. These non-optimalities may derive from the nature of the markets themselves, the effects of government policy, or both. To evaluate the investment it is necessary to make assumptions about what will happen to those distortions if the investment is undertaken. For example, suppose the domestic production of rice is protected by a prohibitive tariff. Rice would be imported if it were not for the tariff, but with the tariff no trade in rice occurs at all. Suppose a public sector investment is being considered which would use rice as an input. To evaluate it, we must assume either that rice will continue to be non-traded after the investment is made, or that the investment will cause rice to be traded, because, say, it induces a reduction in the tariff.\(^2\)

In particular circumstances, either assumption may be appropriate -- but it is particularly important which assumption is made. Much of the confusion in the shadow pricing literature stems from the differing assumptions that are made about exactly this issue. In this study it is assumed that project evaluators have control only over the shadow prices used in the public sector, and hence the public projects that are undertaken, and that distortions in the private sector have to be taken as given. Any non-optimalities in the economy outside the control of the project evaluator concerned -- that is, outside the set of public projects he is evaluating -- have to be considered as constraints on his planning exercise. In the case of the above example, we assume that undertaking the investment would not cause the tariff to be reduced and the good to be imported.

\(^2\)For a discussion of this and related cases, see Sen (1972).
2.2. **Arbitrary Adjustment of Market Prices**

The approach to shadow pricing most commonly used in practice is that of arbitrarily adjusting market prices in a direction that seems appropriate. For example, if the market wage is thought to be "too high", the shadow price of labor might be set at half the market wage, or even zero. Examples of exactly this procedure are frequently encountered in the benefit-cost analyses conducted by international aid and lending agencies, as well as by national and regional planning agencies. Clearly, this procedure is open to attack. There is little basis for a belief that arbitrary adjustments of market prices will increase welfare, even if the appropriate direction of adjustment is known.

Suppose, for simplicity, that the relationship between social welfare and the shadow price of, say, labor takes the strictly concave, single-peaked form described in Figure 2. Social welfare is maximized at $W^*$ by a shadow price of labor equal to $s^*$. The market wage, $s^0$, leads to a value of social welfare of $W^0$. If project planners correctly

![Figure 2: Arbitrary adjustment of market prices](image-url)
perceive that \( s^* < s^0 \), but have no way of estimating the magnitude of the difference, they might set the shadow price at \( s^0/2 \). But this implies a further reduction of social welfare to \( W^{oo} \). Obviously, the diagram need not have been drawn in precisely this way. Nevertheless, it is clear that, unless project planners are equipped with a procedure for estimating the quantitative divergence between market prices and optimal shadow prices, there is no theoretical basis for a presumption that the adjustment of market prices in "an appropriate direction" will increase welfare. It is of course true that "small" adjustments are the most likely to increase welfare; but small changes produce small improvements and "small" improvements may not be of interest.

2.3. The Equilibrium Approach

Non-optimal government policies are frequently the main source of market distortions. It is tempting, therefore, to attach normative significance to the price that would hold if these price-distorting policies were discontinued. For example, if minimum wage laws are the source of a distortion in the labor market, usury laws are the source of a distortion in the capital market, or official overvaluation of the domestic currency is the source of a distortion in the foreign exchange market, the shadow price of each might be set at an estimate of the "equilibrium price" that would result from suspension of the distorting policy. Advocates of this procedure include Tinbergen (1958) and, in the case of foreign exchange, Bacha and Taylor (1971).
There are serious conceptual difficulties with this approach. Firstly, the procedure is applied only when the "distortion" is due to the existence of non-optimal government policies. If the distortion arises from, say, the non-competitive behavior of economic agents, the existence of externalities, informational failures, or income distributional reasons, it is the absence of first-best corrective policies (assuming that such policies exist in principle) that is the essence of the problem. It therefore seems arbitrary and inconsistent to define the optimal shadow price to be the equilibrium price that results when government policy is "optimal" in the limited sense above. Policy is assumed to be optimal in the sense that non-optimal interventions currently being practiced are discontinued, but not optimal in the sense that optimal interventions not currently being practiced are in fact instituted. There appears to be no theoretical justification for assuming optimality in the first sense and not in the second.

Secondly, the procedure is applied piecemeal, with the distortions in the various markets analysed separately. "The equilibrium price" in the labor market, "the equilibrium price" in the capital market, and "the equilibrium price" in the foreign exchange market, are each analysed independently in a partial equilibrium framework which assumes that the distorting policy is removed. However none of these markets can logically be separated from the others, and if each market is analysed on the assumption that all the others continue to behave in their present distorted manner the results will be meaningless; each "equilibrium price" will be based on assumptions inconsistent with those underlying every other "equilibrium price". There is no logical
alternative to a general equilibrium treatment. Furthermore, there is no logical reason for separating the analysis of the various markets involved from the distribution of income. If the government can "in principle" pursue an optimal policy with respect to, say, the foreign exchange market, why can it not also "in principle" effect an optimal distribution of income? Since the demand for foreign exchange clearly depends on the distribution of income, it is inconsistent to analyse the foreign exchange market without also inquiring into the implications of optimizing the distribution of income. This applies equally to the other markets mentioned above and, unless the income distribution is thought to be optimal already, failure to consider this could be a serious deficiency.

Thirdly, it is taken for granted by the advocates of this approach that "the equilibrium price", appropriately defined, is what should guide production decisions in the public sector. Assuming that the above problems have been appropriately handled, this is the price that would hold in a particular market if optimal policies were pursued. However, either these policies will in fact be pursued during the life of the proposed public investment, or they will not. If the project evaluator believes that they will, the task of shadow pricing reduces to one of predicting the market prices that will result. If he believes they will not, and this appears to be the more relevant case, it seems rather foolish for him to assume otherwise. If present market distortions are expected to continue, it seems odd indeed to recommend that the government's investment policy be based on the assumption that the economy is at a full-employment, first-best, optimum. In this case the
relevant question is not "What would equilibrium prices be in Utopia?", but "How can resources best be allocated in the public sector given those distortions that cannot, for the time being, be removed?" There is no reason why the answers to these questions should be the same. Thus, even leaving aside the empirical problems of estimating "equilibrium prices", it is clear that this approach is not satisfactory.

2.4. The Programming Approach

In principle, programming models can be used to overcome all of the difficulties raised above. In practice, however, the very problems that cause us to be interested in shadow pricing in the first instance also create serious problems for computable programming models. Firstly, the market distortions listed in Section 2.2 above are extremely difficult to model in a computable program. For the reasons presented above, we are interested here in optimizing only within the public sector; economy-wide programming solutions are clearly irrelevant. But the distortions

3 Tinbergen defines the "intrinsic value" or "accounting price" of labor, capital, or foreign exchange to be that price "that would prevail if (i) the investment pattern under discussion were actually carried out, and (ii) equilibrium existed on the markets just mentioned." Tinbergen (1958, p. 39).

4 In practice the empirical problems will frequently be severe, and the approach will often be more or less comparable to the arbitrary adjustment of market prices considered above. Thus Tinbergen writes: "It may be quite sufficient to make a rough guess as to the consequences of the ... fundamental disequilibrium." Tinbergen (1958, p. 41).

5 For a statement of the desirability of using economy-wide programming models to compute shadow prices for project evaluation, see Chenery (1964). Examples of empirical studies include Adelman (1966), Tendulkar (1971), and Weisskopf (1971). Most studies use economy-wide optimization models, and ignore market distortions. The results are of little relevance to the problems considered here.
in the private sector cannot be ignored since they are the source of our interest in shadow prices. They must be incorporated into the model as constraints. However, the functional relationships involved are virtually all non-linear, and this raises severe computational problems. Non-linear programming models can be computed, but only when they have a small number of variables and constraints. This necessitates a high level of aggregation -- so high, in fact, that the results are of little practical interest. Piece-wise linear models can be constructed, but since they increase the size of the program substantially, their use has, in practice, to be limited to those relationships that are thought to be most important.

Secondly, there is something unscientific about the way computable programs must be constructed in practice. After gathering the inadequate information that is available and "debugging" the model, the programmer will make a trial run. Almost invariably, this produces an absurd solution. Suppose that all nitrogenous fertilizer (N) and all phosphatic fertilizer (P) used in the economy is produced domestically in the public sector. Then the dual solution to this initial run might indicate that the shadow price of N is $1 per ton and that of P is $100 per ton. This is unrealistic, and something has to be done about it. But there are many things that could be done: the production function of N could be made piece-wise linear, so that less could be produced from, say, a given amount of electrical energy; some of the constraints on the supply of inputs into P production could be relaxed, so that more P could be produced; the amount of rice produced from an additional ton of N could be increased; and so on. It is somewhat
arbitrary how these problems are corrected, and it is unclear in the end whether the shadow prices obtained reflect social scarcities at the optimal solution, or merely the particular adjustments that, given his inadequate data base, the programmer has chosen to make to the model.

Thirdly, and this is partly the reason for the second problem, the basic functional relationships involved simply are not known. Most importantly, the production functions in the public sector are not known by any one agency. This is the essence of the motivation for decentralized planning. Project planners may "know" the characteristics of the various alternative projects at their disposal in the sense that, given a set of relative prices, they could determine the profit-maximizing set of projects to adopt, and how best to operate them; but they may not "know" them in the sense that they could list the functional relationships involved, and convey this to the central planners. Even if they could, however, the cost of collecting and processing this information would be huge.

The programming approach to shadow pricing for the public sector would appear to be well worth pursuing as a research topic; but at present the problems of applying it meaningfully to actual planning situations seem to be prohibitive.

2.5 Decentralized Planning Theory

The theory of decentralized planning has been developed in response to the obvious fact that no one agency in the economy has all the information necessary to plan for the entire economy, or even the entire public sector. Central planners may know the government's
objective function and a good deal about the overall structure of the economy, but the production possibilities of individual projects are known only to the individual project managers. The problem of decentralized planning, then, is to find an efficient way of exchanging information between central planners and project managers so that an optimal plan is achieved, or at least acceptably approximated. As yet the theory is not well developed, and it does not seem likely that any country will institute a program of decentralized planning based on this literature in the immediate future. The theory is particularly weak in the quality of the convergence properties that have so far been demonstrated for the various iterative procedures proposed in the literature. Little is known about rates of convergence toward optimal solutions after a finite number of iterations, or about the number of iterations necessary to approximate the optimal solution within a specified neighborhood.

The best known iterative procedure\(^6\) is initiated by central planners sending a vector of prices to project managers. The project managers compute their optimal production programs at those prices and send this information back to the central planners. These production programs for the various projects are then used to compute a new vector of prices to send to project managers, and so on. This

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\(^6\) The discussion here is based on Malinvaud (1967). For models where the direction of signals discussed here is reversed, see Marglin (1969) and Weitzman (1970). A less technical presentation and an extensive bibliography can be found in Hurwicz (1975). See also Heal (1973).
continues until an acceptable and feasible production plan for the economy (or public sector) is attained. The plan is then implemented by means of a final price vector which project managers are instructed to act upon.

While the theory of decentralized planning appears to be a potentially fruitful research area, it has yet to produce much of practical usefulness. Nevertheless, it provides a helpful perspective for the theory of shadow pricing. Shadow pricing truncates the above process at the end of the first step. In decentralized planning procedures, no economic decisions are actually made until numerous iterations of the above process have given central planners a considerable amount of information about production possibilities at the project level. But this exchange of information does not occur in the case of shadow pricing, as discussed here.\footnote{Since actual economic decisions are based on the initial price vector in the above schema, these prices must be set without detailed information about the production possibility sets of individual projects. Clearly, if the optimal shadow price vector is highly sensitive to this information, shadow pricing is in deep trouble.}

However, this may not be the case. For example, if a country produced and consumed only commodities which were traded on perfectly competitive international markets, the dual solution to any production-

\footnote{Of course, as experience of the results of using shadow pricing over time accumulates, feedback of information on production possibility sets can occur. This information may indeed be very useful, but it is slow to accumulate, and little information of this kind is available when a program of shadow pricing is first instituted.}
consumption program would be the international prices. The dual solution is completely insensitive to the details of the production functions involved, or to the form of the social welfare function. However, the details of the optimal production program depend heavily on the form of the production functions involved. It is sometimes claimed that if the optimal set of shadow prices (dual solution) is known by planners, the optimal production plan for the various projects (primal solution) must be known by them also. If central planners do not have sufficient information to compute the optimal production program, it is claimed, they could not know the optimal set of shadow prices either. But this is false. It is logically possible to be in total ignorance about one, while being completely informed about the other.

Nevertheless, since there always exist non-traded commodities, the matter is less straightforward than this. As Sen has put it:

> It is certainly not an all-or-none question, and much depends on the relative sensitivity of optimal quantities and associated prices. If prices respond a little and the quantities a great deal to some variable, the exact value of which the central planners do not know and which the firms know precisely, the stage-wise procedure of choosing prices first and quantities later may make much sense.\(^8\)

As an instrument of planning, shadow pricing has the very economical characteristic that a single set of prices is sent to

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\(^8\) For example of attacks on shadow pricing based on the assumption of a one-to-one correspondence between primal and dual solutions see Rudra (1972) and Weckstein (1972). For a further discussion of this set of issues see Sen (1970).

project managers for use in determining the projects' optimal production plans, but that these prices are set without detailed foreknowledge of exactly what those production plans will be. Yet, to the extent that the valuations implicit in these shadow prices are themselves altered by the production decisions made by projects, this procedure will lead to errors. It is, of course, precisely this simultaneous relationship between valuations (shadow prices) and production decisions that the iterative procedures of decentralized planning theory attempt to cope with. This problem is relevant to all approaches to shadow pricing, including the use of market prices, and we will have occasion to return to it in later sections.

2.6 The Welfare Accounting Approach

A substantial literature has developed in recent years on the application of an approach, the basic proposition of which is that the shadow price of an input into a public project should be set at the cost in social welfare (in terms of some convenient numeraire) of using it, while the shadow price of an output should be set at the gain in social welfare of producing it. This proposition has obvious intuitive appeal -- so obvious, in fact, that its advocates have not thought it necessary to demonstrate its validity, or precise meaning,

\[\text{10}\] The two most influential studies to date are Little and Mirrlees (1969) and Dasgupta, Marglin and Sen (1972). The former is not well written, and clearer sources of Little and Mirrlees' views are Little and Mirrlees (1972) and (1974).
within a formal economic model. The outcome of this is that it has been unclear how the welfare accounting approach relates to optimization theory, and whether the existence of distortions in the private sector is adequately taken account of by this approach. Furthermore, there are serious anomalies and inconsistencies in the way that Little and Mirrlees (1969), the best known of the studies advocating the welfare accounting approach, recommends that the approach be applied. The assumptions underlying Little and Mirrlees' specific recommendations are not always made explicit, and their economic analysis seems arbitrary and ad hoc.

Little and Mirrlees divide commodities into what we will call "tradeables" and "non-tradeables". The former includes all commodities that would be traded if the government pursued an optimal trade policy and thus may include many commodities not currently traded. In the case of our above example, where rice is subject to a prohibitive tariff, Little and Mirrlees would consider rice to be a "tradeable". Tradeables are to be valued at their international prices, since these "measure reasonably well their true social costs and benefits".

Little and Mirrlees advise that evaluators should lean over backwards to deem commodities to belong to the "tradeables" category. The criterion that the good would be traded if trade policy were optimal is explained as follows:

11 Little and Mirrlees use the terms "traded goods" and "non-traded goods". The terms "tradeables" and "non-tradeables" are used above to draw attention to the peculiar meaning that Little and Mirrlees attach to these categories and to distinguish this from the usage of the terms "traded" and "non-traded" adopted here.

12 Little and Mirrlees (1972, p. 260).

13 ibid., p. 263.
"Sometimes, our guess about whether a commodity will be imported or not may be almost a value judgment: we think that a sensible government would plan to import some, so we assume that it will do so. Of course, if one of our assumptions required government action in order to be fulfilled, this should be drawn to the attention of the appropriate authorities".  

This reflects an optimistic faith in the ability and willingness of governments to do what Little and Mirrlees think is sensible; but it is not a faith that Little and Mirrlees apply consistently. The entire structure of domestic prices is assumed by them to remain distorted, so that domestic prices should be used as shadow prices only as a last resort. Yet, if the government can pursue an optimal trade policy, why can it not pursue an optimal domestic policy as well? If this approach were applied consistently all market distortions would have to be assumed away, and the welfare accounting approach would reduce to the utopian form of the equilibrium approach discussed above. This is evidently not what Little and Mirrlees wish to do.

The relevant issue is clearly whether public production or use of a commodity will, or will not, affect trade, given the project evaluator's expectations about future trade policy -- not whether it would affect trade if trade policy were optimal. In this study we assume, for simplicity, that distorting policies not subject to the control of the project evaluator concerned will persist; though, of

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15 Elsewhere, however, Little and Mirrlees state that: "In reality, we do not expect governments to adopt ideal policies...", Little and Mirrlees (1972, p. 263).
course, this is not necessary. Commodities, the use or production of which in the public sector will affect trade, will henceforth be referred to as "traded" and all other commodities will be called "non-traded".

Little and Mirrlees' recommendation that traded commodities be valued at international prices implies that tariffs should be ignored. The rationale for this does not depend on the assumption that trade policy will suddenly become optimal. This is illustrated, in a partial equilibrium framework, in Figure 3. $S_1S_1$ is the domestic supply schedule for the commodity and $D_1D_1$ is the domestic demand schedule, before the proposed public investment is made. The international price for the commodity is $r$, and a tariff is applied at the rate $\bar{t}$. The domestic price is then $r(1+\bar{t})$, since the good is still imported at this price. When a public investment is made which uses the good,

Figure 3: Effects of a non-prohibitive tariff
the demand schedule shifts to $D_2 D_2$, and imports rise from $q_1 - q_0$ to $q_2 - q_0$. The increase in tariff revenues, $\bar{t}(q_2 - q_1)$, is given by the shaded area. Yet this is at most a mere intra-governmental transfer of funds -- from, say, the Treasury to the project to the import agent to the Customs Department, and perhaps back to the Treasury. It has no welfare significance. The social cost of importing the good is $r$, the international price.

This assumes, however, that the tariff was not so high as to preclude any imports of the good at all. A situation where this is so is depicted in Figure 4. The domestic price of the commodity before the shift in the demand schedule is $p_1$. Shifting the demand curve to $D_2 D_2$ causes a rise in the domestic price to $p_2$, and a rise in both domestic production and use of the commodity from $q_1$ to $q_2$. Clearly, trade is unaffected by public use of the good and, by our above definition, the good is non-traded. The international price is not relevant to its valuation.

![Figure 4: Effects of a prohibitive tariff](image-url)
The case of an import quota is depicted in Figure 5. The domestic supply schedule is still $S_1S_1$, but the effect of imposing an import quota of size $Q$ is to make the effective supply schedule the "kinked" line $S_1S_2$. Since the quota is already binding before the shift in the demand schedule, public use of the good causes the domestic price to rise and domestic production and use to rise as in Figure 4. Again, however, trade is unaffected, so from the standpoint of the above definition the good is "non-traded", and the international price is not relevant to its valuation. \(^{16}\) Of course, if the quota is not binding before or after the shift in the demand schedule, it has no effect, and public use affects trade directly as in Figure 3. The international price then measures the good's social cost.

![Figure 5: Effects of a binding import quota](image)

\(^{16}\) Little and Mirrlees (1969, pp. 92-3) are equivocal on the valuation of commodities subject to import quotas. They seem to recommend ignoring the quota, whether it is binding or not, and valuing the good at its international price.
As to the valuation of "non-tradeables", Little and Mirrlees recommend the following:

"Our normal method of valuing such items is to break them down into their inputs with similar iterative treatment of non-traded inputs, until one is left only with traded goods and labour or land (including minerals, etc.)".17/

The tradeables are then valued at their international prices, but special methods are recommended for the valuation of labor. Little and Mirrlees' "shadow wage rate" emphasises the effect that hiring additional workers has on the total amount of savings available for investment. If the rate of savings is suboptimal, and hiring an extra worker transfers resources out of savings and into consumption, then it seems appropriate to take account of this in assessing the social cost of hiring the worker.18/ Since Little and Mirrlees are concerned mainly with evaluating industrial projects, they give little attention to the valuation of land, and assume it to be an unimportant input.

To achieve this breakdown, however, some rate of transformation between "non-tradeables" and their inputs must be assumed. Little and Mirrlees clearly intend that the currently observable rates of transformation should be used. This is equivalent to a procedure recommended more recently by Dasgupta and Stiglitz (1974). It can19/ be

17 Little and Mirrlees (1972, p. 262).
18 The author has attempted, in Warr (1973), to show the relationship between Little and Mirrlees' shadow wage rate and optimization theory, and to draw attention to some particular biases in the Little-Mirrlees formula that this exercise reveals. See also Sen (1972).
19 See Warr (1974), Chapter 3.
shown that this is incorrect. The correct rate of transformation to use is the rate that would obtain when optimal shadow prices are used; but since this rate cannot be determined without knowledge of the optimal shadow prices, it cannot be an input into their calculation. There is thus a fundamental logical error in the way Little and Mirrlees approach the valuation of non-tradeables.

Ideally, however, the welfare accounting approach seems to aim at producing a set of shadow prices whose adoption by all producers in the economy would be socially desirable -- not just public projects. This raises the question of whether these prices are still optimal if applied only partially. Weckstein (1972) has recently criticized this approach, and the programming approach, on these grounds:

"When a price rule is to be applied partially, the objective should be to find prices that signal an optimal accommodation to a given nonoptimal structure of prices and allocations -- a 'second-best' criterion." 20/

Weckstein argues that while

"It is conceivable that an optimization procedure where values not subject to policy control are specified as parameters and constraints might yield such a second-best solution..." 21/

nevertheless, this is not the way shadow prices are normally calculated. He then concludes that:

"If that were the objective, local market prices are probably a more efficient and far more accessible source of such information." 22/

20 Weckstein (1972, p. 478).
21 ibid.
22 ibid.
There are two logical flaws in this argument. Firstly, Weckstein implicitly assumes that "first-best" and "second-best" shadow prices are necessarily different. For example, he criticizes Little and Mirrlees for recommending use of international prices for traded goods in public projects, on the grounds that these are "optimal prices" and are therefore "irrelevant to an economy that does not in fact achieve an optimal allocation of resources." This might be true if there was necessarily a one-to-one correspondence between dual and primal solutions; but as it stands, this part of Weckstein's claim is unproven. Secondly, Weckstein's assertion that "probably" local market prices are the best second-best prices to use is not substantiated, or even explained. Nevertheless, Weckstein's objections are not entirely without merit because it must be said that the advocates of the welfare accounting approach have not demonstrated that this approach satisfies the "second-best" objections that Weckstein raises.

3. Conclusions

The calculation of appropriate shadow prices is an important issue for public investment, especially in economies where market prices are thought to be badly distorted. Shadow prices have relevance not only for project evaluation (benefit-cost analysis), but also for the design of projects and their eventual management as well. Nevertheless there still remain serious unresolved theoretical (not to mention practical) problems concerning the calculation of welfare-increasing shadow prices. (i) Little has been done on the incorporation of income

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23 ibid., p. 479.
distributional objectives into benefit-cost analysis. (ii) The valuation problems implied by the non-existence of markets (e.g., the existence of "externalities" and "public goods") have largely been ignored in the shadow pricing literature.\textsuperscript{23} (iii) The fact that shadow prices are to be applied only partially implies the potential existence of serious "second-best" problems which have been similarly ignored in the shadow pricing literature.\textsuperscript{24} It is important to note that the existence of the above defects in the present techniques for calculating shadow prices do not necessarily imply that the use of distorted domestic market prices is superior to the use of imperfect shadow prices. For example, if the "second-best" problems raised here are presumed to be serious, this implies that we can no longer be certain that the use of "first-best" shadow prices is superior to the use of distorted domestic market prices; but this in itself is not an argument for the use of distorted domestic market prices. The important issue is the quality of approximation involved in each case. What we wish to know is which procedure will, in general, get us to the highest level of social welfare in practice. Future research may help to illuminate this issue. Perhaps the analysis of a numerically computable nonlinear programming model, albeit small and overly simplified, would produce some useful insights.

\textsuperscript{23} The one notable exception to this is the literature on the implications of sub-optimal savings for the valuation of labor. See Sen (1972) and Warr (1973).

\textsuperscript{24} For a rigorous analysis of these issues see Warr (1974).
REFERENCES


Sen, A. K., "Isolation, Assurance and the Social Rate of Discount," Quart. J. Econ. 81 (February 1967), 112-124; also in Layard (1972).


