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# Assessing Rates of Return to Public and Private Agricultural Research

Jet Yee

**Abstract.** Previous work on the rate of return to public agricultural research for the United States has neglected private agricultural research expenditures. This study, which factors in production variables like weather and the shifting health of the national economy over a 70-year period (1915-85), does include private research. When private research is omitted, the rate of return to public research rises by almost 20 percent. This finding supports the extension of Federal and State funding for agricultural research, especially if it can be coordinated with efforts in the private sector.

**Keywords.** Public agricultural research, private agricultural research, agricultural productivity, rate of return

Public investment in agricultural research has significantly boosted U.S. farm productivity. Is it worthwhile then for society to invest public funds in research and development (R&D)? A large number of studies have estimated the rate of return to public agricultural research (Ruttan, 1980, 1982, Echeverria, 1990).<sup>1</sup> Most of them found rather high rates of return. However, the only costs usually considered have been direct public research expenditures.

The omission of other production variables, however, may bias estimates of the rate of return to public agricultural research. Extension variables (Griliches, 1964, Huffman, 1978) or weather variables (White and Havlicek, 1982, Thistle and Bottomley, 1988) have been featured in some studies. No previous work explicitly considers private agricultural research expenditures, which this article does in estimating the rates of return to public and private agricultural research. Huffman and Evenson (1989) take private research into account in their model. However, they use the number of patents in agricultural technology fields rather than private research expenditures. In addition, I introduce a new weather index, factor in the state of the general economy, and employ a much longer time series on research expenditures than most previous studies.

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<sup>1</sup>Sources are listed in the references section at the end of this article.

## Model Specification and Data

R&D expenditures introduce a time lag that may affect productivity. First, a particular R&D project may take several years to complete. Second, when completed and if successful, it may take some time to decide whether to use it. Third, once a decision has been made to use it, it will affect productivity with a lag because the production process takes time. Fourth, after a number of years have passed, use of the technology will likely decline or even cease completely because a superior technology appears. These considerations suggest that the lag structure of R&D expenditures on productivity is quite complex. An inverted U-shaped or inverted V-shaped distribution may serve as a rough approximation.

My hypothesized production function is

$$Q_t = A \prod_{i=1}^l X_{it}^{\sigma_i} \prod_{i=0}^m PUB_{t-i}^{\alpha_i} \prod_{i=0}^n PRI_{t-i}^{\beta_i} e^{\gamma_1 Ext_t + \gamma_2 GNP_{t-1} + \gamma_3 W_t + u_t}, \quad (1)$$

where  $Q$  is aggregate output,  $A$  is a constant,  $PUB_t$  ( $PRI_t$ ) is public (private) research expenditures in period  $t$ ,  $Ext$  is extension,  $GNP$  is gross national product (a proxy for the state of the general economy),  $W$  is a weather index, and  $u_t$  is an error term. The  $X_i$ 's are conventional inputs, such as land, labor, capital, and materials. Define total factor productivity by

$$P_t \equiv \frac{Q_t}{\prod_{i=1}^l X_{it}^{\sigma_i}} = A \prod_{i=0}^m PUB_{t-i}^{\alpha_i} \prod_{i=0}^n PRI_{t-i}^{\beta_i} e^{\gamma_1 Ext_t + \gamma_2 GNP_{t-1} + \gamma_3 W_t + u_t}, \quad (2)$$

where  $P$  is agricultural productivity. Estimate the  $\sigma_i$ 's by the observed conventional input cost shares. Data on agricultural productivity and public and private agricultural research expenditures are from Langston (1988). Data for public agricultural research funding include only expenditures on production-related research by the U.S. Department of Agriculture and the State Agricultural Experiment Stations. Private research funding is taken from National Science Foundation data for research dollars spent on agricultural chemicals and farm machinery and is also derived from industry sales information.

Langston's compilation of private R&D expenditures data has several drawbacks. First, private R&D expenditures by food, seed, and veterinary pharmaceutical companies are not included. Data on such expenditures are unavailable for the early years. What data are available indicate that private R&D expenditures on agricultural chemicals and farm machinery far exceed those on seed and veterinary pharmaceuticals. The exclusion of R&D expenditures on new food products is not a major problem since this article considers the productivity of the farm sector, not the productivity of the agribusiness sector. Leaving out private R&D expenditures on seed and veterinary pharmaceuticals may bias the calculated rate of return to public R&D upward, with the actual direction of bias depending on how public and private R&D expenditures are correlated.

Second, private R&D expenditures before 1952 come from sales data and assume that a certain proportion of sales by farm input suppliers is spent on R&D. The proportion of sales spent on R&D is calculated for the years known and extrapolated to the unknown years. Langston gives several references to justify this assumption about research spending behavior.

The R&D expenditures data in Langston are in current dollars. I deflated R&D expenditures by a price deflator for agricultural R&D (Paidey, Craig, and Hallaway, 1989). The period of my estimation is 1931-85. However, data on public and private research expenditures covered 1915-85 to account for the lag structure of R&D expenditures on productivity (fig. 1).

Fluctuations in agricultural productivity growth are caused largely by changes in weather. I include a weather index in my model, first regressing crop production per acre on a constant and the first, second, third, and fourth powers of time for the period 1910-86. The data for crop production per acre are from the *Economic Report of the President*. I then used the estimated coefficients to obtain a fitted trend curve for crop production per acre. Any deviation of actual crop production per acre from trend is interpreted as a deviation in weather from "normal" conditions.

There is almost no change in trend crop production per acre from 1910 to the late 1920's (fig. 2). A long period of rapid growth in trend crop production per acre starts in the late 1920's. This growth slows after the early 1970's. The rise in trend crop produc-

Figure 1  
Real public and private research expenditures

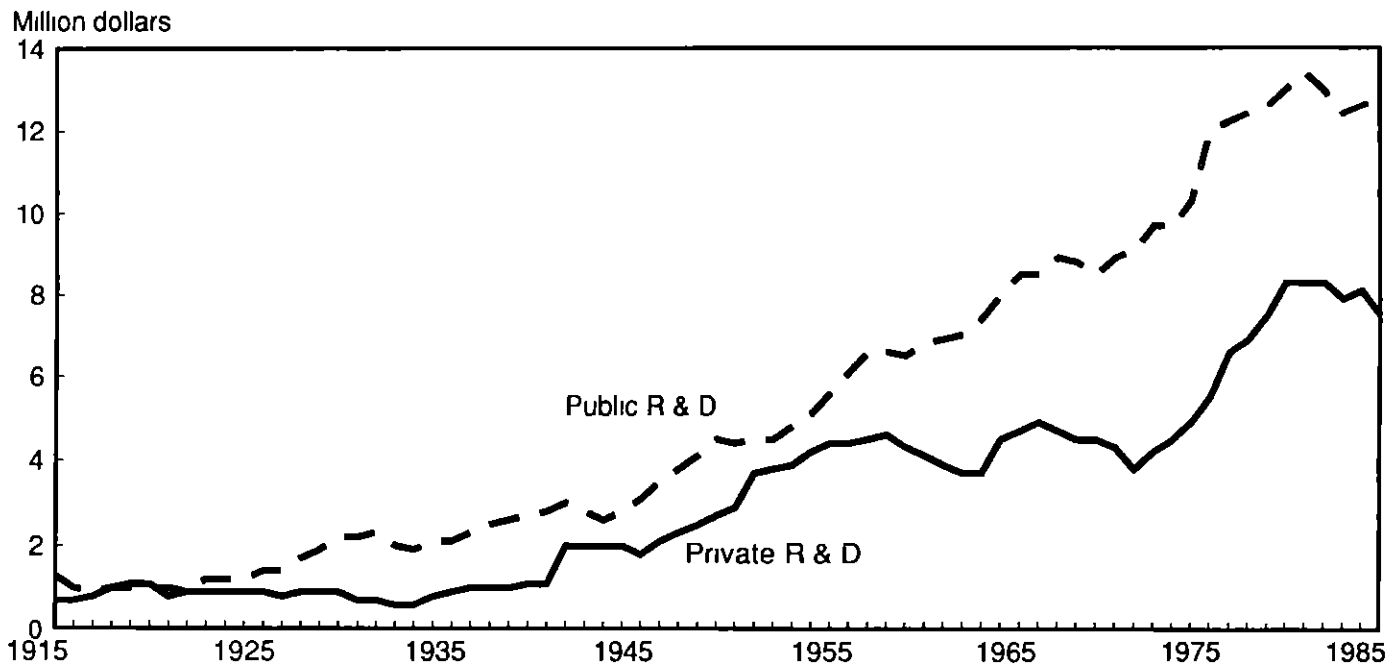
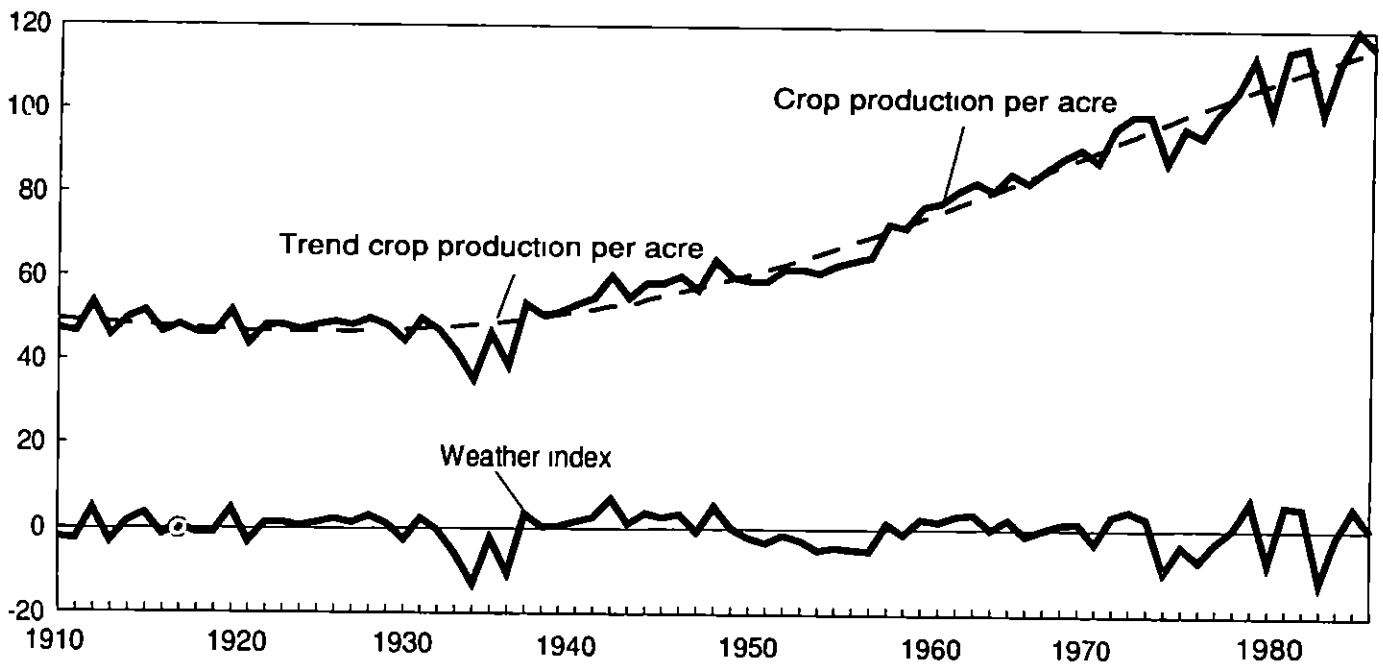


Figure 2  
**Crop production per acre and weather index, 1910-86**



tion per acre can be attributed to increases in the quality and quantities of nonland inputs (for example, farm machinery and chemicals) and to the development of improved plant varieties like hybrid corn

My weather index can show negative, zero, or positive values. The higher the value of the weather index, the better the weather for agricultural productivity. Extended periods of good weather, as indicated by the weather index, occurred in the 1940's, 1960's, and to a lesser extent, in the 1920's. The weather index is especially low for the mid-1930's, most of the 1950's, and the mid-1970's. Volatility in the weather index increased after the late 1970's. This increase in volatility suggests that the weather index includes nonweather factors, especially economic and policy. For example, the sharp drop in my weather index in 1983 may be traced to the influence of drought as well as the Payment-in-Kind (PIK) program.

The correlation coefficient of the weather index (W) and the ratio of annual total precipitation to annual mean temperature (P/T) were computed for several climatological stations from 1915-70 (using data from *Historical Statistics of the United States*). The correlation coefficients of W and P/T for climatologi-

cal stations in California, Illinois, Montana, Nebraska, North Dakota, New York, Oklahoma, and South Dakota were 0.27, 0.24, 0.50, 0.34, 0.39, 0.27, 0.50, and 0.25. This suggests the weather index is a reasonable reflection of actual weather conditions. Note that the weather index was based on crop production per acre while the productivity measure was based on all outputs and all inputs.

I assumed that agricultural productivity also benefits from public extension. Extension is different from R&D in two respects. First, extension affects productivity mostly in the current period. Second, extension does not have the public-good nature of R&D. It is for these two reasons that I use extension stock (constructed using a depreciation rate of 50 percent) per farm as my extension variable. That is,

$$Ext_t = \frac{1}{n_t} \sum_{i=0}^{\infty} (1 - \delta)^i E_{t-i},$$

where  $Ext_t$  is real extension expenditures in period  $t$ ,  $n_t$  is the number of farms in period  $t$ , and  $\delta = 0.5$ . Data on public extension expenditures are from Huffman and Evenson (1987). The data on the number of farms are from *Agricultural Statistics*.

Economic conditions have a hypothetical effect on incentives for innovation, with resulting economic and productivity growth in the agricultural sector. The notion that inventive activity is largely driven by demand has been most strongly advocated by Schmookler (1966), who showed that inventive activity (as measured by patents) was related to earlier movements in investment and output of the relevant industries. Real gross national product (GNP) acts as a proxy for the economic conditions facing the agricultural sector. (A reviewer suggested that a variable more directly related to the economic health of the agricultural sector may be preferable to real GNP. Real gross farm income performed slightly worse and real net farm income performed much worse than real GNP (in terms of t-statistics).) Since producers will likely respond to changing economic conditions with a lag, I used GNP lagged one period in estimation. The data for GNP are from the *Economic Report of the President*.

To estimate the parameters of equation 2, take the log of both sides to get a distributed lag model

$$\ln P_t = \ln A + \sum_{i=0}^m \alpha_i \ln PUB_{t-i} + \sum_{i=0}^n \beta_i \ln PRI_{t-i} + \gamma_1 Ext_t + \gamma_2 GNP_{t-1} + \gamma_3 W_t + u_t \quad (3)$$

However, the large number of lagged variables are likely to be highly correlated, using up a large number of degrees of freedom. Thus, to estimate equation 3, I use the Almon (1965) distributed lag procedure, a method employed in previous studies, including Cline (1975), White and Havlicek (1982), and Thirtle and Bottomley (1988). I assumed that current public R&D (private R&D) has no effect on productivity for the first 3 (2) years, but, thereafter, effects last for the next 15 years. Private R&D expenditures are assumed to have a shorter time lag before having an effect on productivity to reflect the applied research (that is, short-term) orientation of much private research. The assumption that R&D expenditures have an effect on productivity over 15 years is consistent with previous studies, including Evenson (1968) and Cline (1975). I also assumed that the nonzero weights  $\alpha_i$ 's ( $\beta_i$ 's) follow a second-degree polynomial  $\alpha_i = a_0 + a_1 i + a_2 i^2$  ( $\beta_i = b_0 + b_1 i + b_2 i^2$ ) as per my earlier discussion suggesting an inverted U-shaped distribution. Imposing the endpoint restrictions  $\alpha_3 = \alpha_{18} = 0$  and  $\beta_2 = \beta_{17} = 0$  produces the equation to estimate

$$\ln P_t = \ln A + a_2 \ln S_{1t} + b_2 \ln S_{2t} + \gamma_1 Ext_t + \gamma_2 GNP_{t-1} + \gamma_3 W_t + u_t \quad (4)$$

$$\text{where } \ln S_{1t} \equiv \sum_{i=4}^{17} (54 - 21i + i^2) \ln PUB_{t-i}$$

$$\text{and } \ln S_{2t} \equiv \sum_{i=3}^{16} (34 - 19i + i^2) \ln PRI_{t-i}$$

The nonzero weights can be obtained as  $\alpha_1 = (54 - 21i + i^2) a_2$  and  $\beta_1 = (34 - 19i + i^2) b_2$ . Derivations are given in the appendix.

Using the estimated parameters from the distributed lag model equation 3, one can estimate the rate of return to public agricultural research. The parameter estimates for the distributed lag coefficients in equation 3 are the output elasticities of the R&D variables for each year of the lag, for example

$$\alpha_\tau = \frac{\partial \ln P_t}{\partial \ln PUB_{t-\tau}} \quad (5)$$

That is,  $\alpha_\tau$  gives the effect on current productivity of public R&D expenditures  $\tau$  periods back.

The estimated  $\alpha$ 's can be used to calculate the rate of return to public R&D as follows (from equation 5)

$$\alpha_\tau = \frac{\partial \ln P_t}{\partial \ln PUB_{t-\tau}} = \frac{\partial P_t}{\partial PUB_{t-\tau}} \frac{PUB_{t-\tau}}{P_t} \quad (6)$$

Rearrange equation 6 to get the marginal product of public R&D

$$\frac{\partial P_t}{\partial PUB_{t-\tau}} = \alpha_\tau \frac{P_t}{PUB_{t-\tau}} \quad (7)$$

Multiplying both sides of equation 7 by  $(\partial Y_t / \partial P_t)$ , where  $Y$  is the value of output, yields

$$\frac{\partial P_t}{\partial PUB_{t-\tau}} \cdot \frac{\partial Y_t}{\partial P_t} = \alpha_\tau \frac{P_t}{PUB_{t-\tau}} \frac{\partial Y_t}{\partial P_t} \quad (8)$$

or

$$VMP_{t,t-\tau} \equiv \frac{\partial Y_t}{\partial PUB_{t-\tau}} = \alpha_\tau \frac{P_t}{PUB_{t-\tau}} \frac{\partial Y_t}{\partial P_t} \quad (9)$$

Equation 9 gives the effect of public R&D expenditures in period  $t-\tau$  on the value of output in period  $t$ . The rate of return ( $r$ ) for an additional research expenditure of  $\Delta PUB_{t-\tau}$  in period  $t-\tau$  is the discount rate that results in the following equality

$$\Delta PUB_{t-\tau} = \sum_{i=0}^m \frac{\Delta Y_{t-\tau+i}}{(1+r)^i} \quad (10)$$

or

$$\sum_{i=0}^m \frac{VMP_{t-\tau+i,t-\tau}}{(1+r)^i} - 1 = 0 \quad (11)$$

## Empirical Results

I estimated equation 4 by ordinary least squares (OLS) for 1931-85. Since the Durbin-Watson statistic from the OLS estimation was so low (1.26), I used the iterative Prais and Winsten algorithm implemented in LIMDEP to correct for autocorrelation. Table 1 shows the estimated parameters. The coefficients of extension, GNP, and weather are expected to be positive and the coefficients of  $\ln S_1$  and  $\ln S_2$  (see appendix) to be negative. All the estimated parameters have the expected signs. Public R&D, extension, and weather are significant at the 5-percent level. Private R&D is significant at the 10-percent level, while GNP is significant at the 20-percent level. The derived estimates of the  $\alpha_i$ 's and  $\beta_i$ 's are also presented in table 1.

Using equations 9, 11, and the parameter estimates of equation 4, one can obtain the rate of return to public R&D. Used for  $Y_t$ , value of output, are cash marketing receipts from *Agricultural Statistics* deflated by the GNP deflator from the *Economic Report of the President*. Mean values for  $(P_t/PUB_{t-1})$  and  $(\partial Y_t/\partial P_t)$  are used in equation 9. For the period of estimation (1931-85), the geometric mean of  $P$  is 73.02 and the geometric mean of  $PUB$  is \$466.7 million. The mean value of  $(\partial Y/\partial P)$  of nearly \$1.1 billion follows as the slope coefficient of a regression of  $Y$  on a constant and  $P$ . Using those values yields a calculated rate of return to public agricultural research of 49 percent.<sup>2</sup>

For comparison, I also estimated a model that omits private research to determine the bias that results from its exclusion (see the parameters in table 1). All the estimated parameters have the expected signs. Public R&D, extension, and weather are significant at the 5-percent level. GNP is significant at the 20-percent level. My estimate of the rate of return to public R&D rises to 58 percent when I omit private R&D.

Many studies have estimated the rate of return to public agricultural research. Most of them found rather high rates of return, usually in the 30- to 60-percent range (Ruttan, 1980, 1982, Echeverria, 1990). My estimates of the rate of return to public agricultural research of 58 percent without private research and 49 percent with private research are consistent with estimates presented in the literature.

<sup>2</sup>My procedures are standard in the production function approach to measuring the returns to R&D. A second approach for measuring returns to R&D estimates the consumer and producer surplus associated with R&D. See Norton and Davis (1981) for a review of studies that employ the consumer and producer surplus approach.

Table 1—Contribution of public and private R&D expenditures to U.S. agricultural productivity<sup>1</sup>

Explanatory variables	Dependent variables	
	lnP (with private R&D)	lnP (without private R&D)
Constant	3.86 (133.8)	3.84 (133.3)
$\ln S_1$	-0.0031 (-2.16)	-0.0047 (-3.68)
$\ln S_2$	-0.0012 (-1.94)	
Ext	0.368 (2.47)	0.375 (2.25)
GNP	0.0001 (1.62)	0.0001 (1.39)
W	0.0089 (8.72)	0.0087 (8.66)

	Public R&D lag coefficients, $\alpha_i$ (with private R&D)	Private R&D lag coefficients, $\beta_i$	Public R&D lag coefficients, $\alpha_i$ (without private R&D)
t	0	0	0
t - 1	0	0	0
t - 2	0	0	0
t - 3	0	0.017	0
t - 4	0.0043	0.0031	0.0066
t - 5	0.0081	0.0043	0.012
t - 6	0.0112	0.0053	0.0169
t - 7	0.0136	0.0060	0.0207
t - 8	0.0155	0.0065	0.0235
t - 9	0.0167	0.0067	0.0254
t - 10	0.0174	0.0067	0.0263
t - 11	0.0174	0.0065	0.0263
t - 12	0.0167	0.0060	0.0254
t - 13	0.0155	0.0053	0.0235
t - 14	0.0136	0.0043	0.0207
t - 15	0.0112	0.0031	0.0169
t - 16	0.0081	0.0017	0.0122
t - 17	0.0043	0	0.0066
t - 18	0	0	0
$\rho$		0.384	0.467
DW		2.00	2.08
R <sup>2</sup>		984	982
No. of obs		54	54

<sup>1</sup>Estimates of equation 4 with and without private research.

Notes: T-statistics are shown in parentheses.  $P$  is agricultural productivity. Ext is extension stock per farm. GNP is real gross national product.  $W$  is a weather index.  $S_1$  and  $S_2$  are defined in the text.  $\rho$  is the estimated value of the first-order autoregressive coefficient of the disturbance terms. DW is the Durbin-Watson statistic (after correction for autocorrelation).

Most previous studies, however, explicitly considered only public research, obtaining their rate of return estimates by dividing the marginal product of public research by a factor of 3 to take into account the two omitted variables, private research and public extension. These studies assume that public research expenditures, private research expenditures, and extension expenditures are each about

equal (See, for example, Griliches, 1964, p 968, Bredahl and Peterson, 1976, p 688, and White and Havlicek, 1982, p 52) Data suggest this may not be a good assumption. By contrast, my model explicitly takes into account private research expenditures and public extension expenditures as well as weather and the state of the general economy

My estimate of the rate of return to private R&D based on the geometric mean of PRI (private research expenditures) of \$302 million is 38 percent, almost 25 percent lower than the rate of return to public R&D. There are several possible explanations for the lower rate of return to private R&D. First, the main purpose of the private sector is to make profits and only indirectly to increase agricultural productivity. By contrast, one of the main goals of public agricultural research is to increase agricultural productivity. Second, a public extension system facilitates the adoption of public research results.

Third, private agricultural R&D grows from firms that think they will be able to appropriate all the returns to their R&D by increasing the prices of their outputs. These price increases should be reflected as quality changes in quality-adjusted price indexes and thus should already be taken into account in a constructed measure of total factor productivity for the agricultural sector. Private agricultural R&D may have little additional influence on agricultural productivity once the higher quality of the inputs has been taken into account. However, some private research spending must be accounted for if firms are not able to appropriate the full returns to their R&D investment, and the price indexes have not been quality-adjusted. Mansfield and others (1977) determined that the private rate of return to private research was about half the social rate of return.

While I found a higher rate of return to public R&D than private R&D in agriculture, studies of rates of return to R&D in manufacturing find just the opposite (Lichtenberg and Siegel, 1991). This difference may be because publicly funded R&D in manufacturing is mainly in areas in which it is difficult to measure productivity, such as in defense and space.

## Conclusions

Taking private agricultural research, weather, extension, and the state of the general economy into account, I calculated a lower rate of return to private than public agricultural research. Future Federal and State funding for agricultural research should be continued in light of its high rate of return. However, the rate of return to public agricultural research has been overestimated since

most studies have ignored private research expenditures. Decisions on the allocation of public research funds to various research areas should take into account the type and volume of research being conducted in the private sector.

## References

- Almon, Shirley. 1965. "The Distributed Lag Between Capital Appropriations and Expenditures," *Econometrica* Vol 30, pp 178-96
- Bredahl, Maury, and Willis Peterson. 1976. "The Productivity and Allocation of Research U.S. Agricultural Experiment Stations," *American Journal of Agricultural Economics* Vol 58, pp 684-92
- Cline, Phillip L. 1975. "Sources of Productivity Change in United States Agriculture" Ph.D. dissertation Oklahoma State University, Stillwater
- Echeverria, Ruben G. (ed.) 1990. "Assessing the Impact of Agricultural Research," *Methods for Diagnosing Research System Constraints and Assessing the Impact of Agricultural Research*. International Service for National Agricultural Research, The Hague, The Netherlands
- Evenson, Robert E. 1968. "The Contribution of Agricultural Research and Extension to Agricultural Productivity" Ph.D. dissertation University of Chicago
- Griliches, Zvi. 1964. "Research Expenditures, Education, and the Aggregate Agricultural Production Function," *American Economic Review* Vol 54, pp 961-74
- Huffman, Wallace E. 1978. "Assessing Returns to Agricultural Extension," *American Journal of Agricultural Economics* Vol 60, pp 969-75
- Huffman, Wallace E., and Robert E. Evenson. 1987. "The Development of U.S. Agricultural Research and Education: An Economic Perspective" Staff Paper No. 168 Dept. of Econ., Iowa State University, Ames
- \_\_\_\_\_ 1989. "Supply and Demand Functions for Multiproduct U.S. Cash Grain Farms: Biases Caused by Research and Other Policies," *American Journal of Agricultural Economics* Vol 71, pp 761-73
- Langston, Jackie B. 1988. "Dynamic Strategies for Research Expenditures in Agriculture: Public and Private, Basic and Applied" Ph.D. dissertation University of Georgia, Athens

Lichtenberg, Frank R, and Donald Siegel 1991 "The Impact of R&D Investment on Productivity—New Evidence Using Linked R&D-LRD Data," *Economic Inquiry* Vol 29, pp 203-29

Mansfield, E, J Rapaport, A Romeo, S Wagner, and G Beardsley 1977 "Social and Private Rates of Return from Industrial Innovations," *Quarterly Journal of Economics* Vol 91, pp 221-40

Norton, George W, and Jeffrey S Davis 1981 "Evaluating Returns to Agricultural Research A Review," *American Journal of Agricultural Economics* Vol 63, pp 685-99

Pardey, Philip G, Barbara Craig, and Michelle L Hallaway 1989 "U S Agricultural Research Deflators 1890-1985," *Research Policy* Vol 18, pp 289-96

Ruttan, Vernon W 1980 "Bureaucratic Productivity The Case of Agricultural Research," *Public Choice* Vol 35, pp 529-47

\_\_\_\_\_ 1982 *Agricultural Research Policy* Minneapolis University of Minnesota Press

Schmookler, Jacob 1966 *Invention and Economic Growth* Cambridge, MA Harvard University Press

Thirtle, C, and P Bottomley 1988 "Is Publicly Funded Agricultural Research Excessive?" *Journal of Agricultural Economics* Vol 39, pp 99-111

U S Department of Agriculture *Agricultural Statistics*

U S Department of Commerce *Historical Statistics of the United States*

U S Government Printing Office *Economic Report of the President*

White, Fred C, and Joseph Havlicek 1982 "Optimal Expenditures for Agricultural Research and

Extension Implications of Underfunding," *American Journal of Agricultural Economics* Vol 64, pp 47-55

## Appendix

Since  $\alpha_i = 0$ , for  $i = 0, 1, 2, 3$ , and  $i \geq 18$ ,

$$\sum_{i=4}^{17} \alpha_i \ln PUB_{t-i} = \sum_{i=4}^{17} (a_0 + a_1 i + a_2 i^2) \ln PUB_{t-i},$$

on substituting for  $\alpha_i$ . The endpoint restrictions,  $\alpha_3 = \alpha_{18} = 0$ , give two equations

$$\alpha_3 = a_0 + 3a_1 + 9a_2 = 0, \text{ and}$$

$$\alpha_{18} = a_0 + 18a_1 + (18)^2 a_2 = 0,$$

from which I can solve for  $a_0$  and  $a_1$  in terms of  $a_2$

$$a_0 = 54a_2 \text{ and } a_1 = -21a_2$$

Substituting for  $a_0$  and  $a_1$  gives

$$\sum_{i=4}^{17} (54a_2 - 21a_2 i + a_2 i^2) \ln PUB_{t-i}$$

$$= a_2 \sum_{i=4}^{17} (54 - 21i + i^2) \ln PUB_{t-i}$$

$$= a_2 \ln S_{1t},$$

and

$$\alpha_i = a_0 + a_1 i + a_2 i^2$$

$$= 54a_2 - 21a_2 i + a_2 i^2$$

$$= (54 - 21i + i^2)a_2$$

The sign of  $a_2$  is expected to be negative (for  $\alpha_i$  to have a maximum) A similar procedure can be employed to obtain  $\ln S_{2t}$  and  $\beta_1$