Staff Papers Series

Staff Paper P88-45 December 1988

ESTIMATION OF A DIRECT UTILITY FUNCTION FOR FOOD EXPENDITURES

by

Nathan Young
The Urban Institute
2100 M St., N.W.
Washington, D.C. 20037

and

Benjamin Senauer
231 Classroom Office Building
1994 Buford Avenue
University of Minnesota
St. Paul, MN 55108

Department of Agricultural and Applied Economics

University of Minnesota
Institute of Agriculture, Forestry and Home Economics
St. Paul, Minnesota 55108

Staff Paper P88-45 December 1988

ESTIMATION OF A DIRECT UTILITY FUNCTION FOR FOOD EXPENDITURES

by

Nathan Young
The Urban Institute
2100 M St., N.W.
Washington, D.C. 20037

and

Benjamin Senauer
231 Classroom Office Building
1994 Buford Avenue
University of Minnesota
St. Paul, MN 55108

HEADNOTE

Only a very limited number of utility function specifications yield demand functions that are expressible in "closed form". Although "closed form" demand functions do not exist for the utility function used in this paper, a methodology is demonstrated which enables both the estimation of the parameters of the utility function and the derivation of the basic demand elasticities. The approach employed here extends the work of Wales and Woodland (1983), and uses the Kuhn-Tucker first-order conditions to define a set of nonlinear implicit simultaneous regressions. The parameters of a characteristics/attributes version of a random constant elasticity of substitution (CES) direct utility function are estimated for a 19 commodity system of U.S. food demand. The data are from the 1980 and 1981 Bureau of Labor Statistics' Continuing Consumer Expenditure Survey. Lancaster's (1966) concept of characteristics/attributes is employed as a source of prior information and as an alternative to the assumption of separability. The estimation procedure had to be consistent with choice-determined zero observations, since many households did not purchase certain commodities during the survey period. An approximate instrumental variables least-absolute-deviations estimator is used to avoid the numerical limitations imposed by multivariate maximum likelihood.

ESTIMATION OF A DIRECT UTILITY FUNCTION FOR FOOD EXPENDITURES

The concept of agents maximizing utility subject to their feasibility set is a fundamental paradigm of neoclassical economics. However, very few utility function specifications lead to demand functions that are expressible in "closed form".[1] Though simple "closed form" demand equations do not exist for the utility function specified in this paper, a methodology is demonstrated which allows both the estimation of the parameters of the utility function and the derivation of the demand elasticities for a system of 19 food commodities.

Wales and Woodland (1983) demonstrated a technique for estimating parameters of a random direct utility function using the Kuhn-Tucker first-order conditions as implicit regressions. Central to their treatment of zero observations is the use of multivariate normal-distribution maximum likelihood. For numerical reasons this limits the size of the system to three variables. In this study, a least-squares approximation to a least-absolute deviation estimator is proposed that makes it possible to estimate inequality constrained optimization systems of arbitrary size.

Lancaster (1966) introduced the concept of commodities as bundles of characteristics/attributes. In this study, the parameters of a characteristics/attributes version of a random constant elasticity of substitution (CES) direct utility function are estimated for a 19 commodity system of U.S. food demand. This linear transform CES (LTCES) utility function fails to yield "closed form" demand functions. In the manner of Wales and Woodland, this necessitates the use of implicit regressions based on the first-order conditions. In contrast to the quadratic utilitiy function employed by Wales and Woodland, the system of LTCES first-order conditions are nonlinear both in variables and parameters.

A primary motivation for this study was to employ the Lancaster attribute model as a basis for prior information in U.S. food demand estimation. Separable utility functions have often been justified by assertions that they preserve necessary degrees of freedom in multivariate demand analysis. Indeed separability assumptions can do this quite effectively. Lancaster models are an alternative structure for relating goods within utility functions, and thus may also serve this purpose effectively. In the particular version of the LTCES model used in this study, six specific characteristics are employed as priors, establishing the structure that in a separable utility function would be established by the division of goods into mutually exclusive or hierarchical categories. The LTCES priors are likewise categorizations, goods either possess or do not possess attributes, but are distinct from separability assumptions in that the categorizations are neither mutually exclusive or hierarchical. For many commodities, including food, overlapping categories may be preferable to separable categories.

Data from the 1980 and 1981 Bureau of Labor Statistics Continuing Consumer Expenditure Survey were used to estimate the free parameters in the LTCES utility function. As with most household expenditure data, many households are observed to not purchase some commodities during the two-week survey period, suggesting the use of an estimation procedure consistent with the presence of choice-determined zero observations. To avoid the numerical limitations imposed by multivariate maximum likelihood, an estimator asymptotically equivalent to instrumental variables least-absolute-deviations (LAD) is employed.

The nondifferentiability of absolute deviations estimators creates difficulty from both a theoretical and empirical perspective. Amemiya (1982) addressed some of the theoretical difficulties by considering a differentiable asymptotic approximation to the absolute value function. In this paper an asymptotic approximation based on Amemiya's function is used to circumvent numerical complexities. Powell (1984) established that LAD estimators can consistently estimate regression parameters. Under stronger distributional assumptions, we use an asymptotic LAD (ALAD). The actual estimator is a nonlinear three-stage least squares estimator, and thus standard econometric packages such as SAS might be employed to compute parameters in simultaneous systems of large size.

The first section of this paper discusses the LTCES utility function model and the resulting first order conditions. The second section briefly describes the data, variables, and the characteristics priors. The third covers the econometric procedures and the fourth presents and discusses the empirical results for the 19 food commodity system, including analysis of simulation derived own-price, cross-price and income (total food expenditure) elasticities. The concluding section focuses on the major contributions of this work and on suggestions for further research in this area.

UTILITY FUNCTION ESTIMATION

Wales and Woodland (1983) proposed the approach of estimating the system of first-order conditions derived from a random direct utility function model. A household was hypothesized to solve the optimization problem:

$$\max \quad u = v(q) + e'q \qquad (1)$$
$$\text{s.t. } \Sigma p_i q_i = m: \qquad q_i \geq 0$$

where q represents quantities, p prices, and m income. If v is twice - continuously differentiable one has the following first order conditions, that must hold for "optimal" q.

$$\partial v/\partial q_i + e_i - \alpha p_i = 0 \qquad \text{for} \quad q_i > 0 \qquad (2)$$
$$\partial v/\partial q_i + e_i - \alpha p_i \leq 0 \qquad \text{for} \quad q_i = 0$$
$$\Sigma p_i q_i = m$$

Let $v_i = \partial v/\partial q_i$, and assuming $q_1 > 0$, then it is possible to solve for the Lagrange multiplier, $\alpha = v_1/p_1 + e_1/p_1$. Substituting this expression for α in the set of equations and dividing by p_i isolates the errors on the right side of the relation:

$$v_i/p_i - v_1/p_1 \leq e_i/p_i - e_1/p_1 \qquad (3)$$
$$\Sigma p_i q_i = m$$

Following Wales & Woodland (1983), these equations may be used to estimate parameters ß, in the direct utility function v(q:ß). In this random utility model, e_i represents a random variable observable to the household, assumed to have a multivariate symmetric distribution with mean and median equal to zero, independent of the price variables. In their paper, Wales and Woodland estimated a quadratic random utility function. With a quadratic utility function it is possible to express the Kuhn-Tucker conditions as linear combinations of the expenditures. This makes the subsequent maximization of the normal likelihood function less difficult. Even so, due to the difficulty of integrating the multivariate normal density function, their technique is limited to handling a system of no greater than three in dimension.

CES and LTCES Utility Functions

The utility model that is employed in this research is an extension of the CES utility function (Barten, 1977). The standard form of a CES utility function (which is called that because of its similarity to the constant elasticity of substitution production function) may be written as:

$$u = (1/r)\sum_{i}^{N} B_i(q_i)^R \tag{4}$$

where the q_i are market products. The CES function was chosen as a basic model to work from because of its wide usage.[2] In the case of R = 0, (4) becomes a Cobb-Douglas utility function.

In the Lancaster (1966) model, the utility function takes as arguments linear combinations of the goods available for purchase, the q_i.

This provides an avenue for incorporating a large amount of prior information into a demand system. Let z represent the vector of linear combinations; $z = Cq$ where C is some fixed matrix. Replacing the direct market products q_i in (4) by the attributes/characteristics z_j, where $z_j = \Sigma c_{ij} q_i$, a new utility function can be constructed.

$$u = (1/r) \sum_{j}^{M} B_j (z_j)^R \tag{5}$$

or

$$u = (1/r) \sum_{j}^{M} B_j \Big(\sum_{i}^{N} C_{ij} q_i\Big)^R \tag{6}$$

Though a "closed" does not exist for the LTCES demand equations, it is possible to invoke an optimization routine to calculate them. For $R < 1$, and $C_{ij} \geq 0$, $B_j > 0$, $q_i > 0$ all i,j, the LTCES has a unique solution subject to a linear budget constraint, and the reduced Hessian is everywhere negative-definite. The LTCES function is undefined for $R \leq 0$ and $z_j \leq 0$. Therefore a further modification to the LTCES function is called for. It is necessary to add a constant to each argument of the power transformation:

$$u = (1/r) \sum_{j}^{M} B_j \Big(\sum_{i}^{N} C_{ij} q_i + k\Big)^R \tag{7}$$

For z_j not near 0, the impact of adding k to the argument is quite small. Therefore away from $z_j = 0$, this function behaves in a manner very similar to the standard LTCES.

The LTCES function will not be identifiable unless prior information about the parameters is utilized. This paper employs a combination of equality and presence/absence information, implementing absolute restrictions. Goods are treated as either having a particular characteristic or not possessing it. Reparameterizing the LTCES function aids interpretation. Let the coefficients of the "z-goods" power function, be denoted by A's. It is assumed that any "A" characteristics are shared by two or more goods. In addition let every good have its own unique characteristic. Associated with the power transformation of individual goods are the "B" coefficients. Finally let the degree of curvature in the utility function be set by the parameter R.

First Order Conditions

For interior solutions, consider a three good random LTCES utility function. Suppose that all households consume positive amounts of q_1, q_2, and q_3, and $p_1q_1+p_2q_2+p_3q_3 = m$.

$$\begin{aligned} u_t(q_{1t},q_{2t},q_{3t}) &= -(A/R)(q_{1t}+q_{2t}+q_{3t}+1)^{-R} + \\ &-(B_1/R)(q_{1t}+1)^{-R} + -(B_2/R)(q_{2t}+1)^{-R} + -(B_3/R)(q_3+1)^{-R} \\ &+ \epsilon_{1t}q_{1t} + \epsilon_{2t}q_{2t} + \epsilon_{3t} \end{aligned} \tag{8}$$

The first-order Kuhn-Tucker conditions pertaining to each household are:

$$\begin{aligned} &A(q_1+q_2+q_3+1)^{-(R+1)} + B_1*(q_1+1)^{-(R+1)} + \epsilon_1 - \alpha p_1 = 0 \\ &A(q_1+q_2+q_3+1)^{-(R+1)} + B_2*(q_2+1)^{-(R+1)} + \epsilon_2 - \alpha p_2 = 0 \\ &A(q_1+q_2+q_3+1)^{-(R+1)} + B_3*(q_3+1)^{-(R+1)} + \epsilon_3 - \alpha p_3 = 0 \\ &p_1q_1 + p_2q_2 + p_3q_3 = m \end{aligned} \tag{9}$$

Dropping the fourth condition, and substituting for α:

$$f_{2t}(q,p:A,B,R) = A(q_1+q_2+q_3+1)^{-(R+1)}[1/p_1-1/p_2] +$$
$$(B_1/p_1)(q_1+1)^{-(R+1)} - (B_2/p_2)*(q_2+1)^{-(R+1)}$$
$$= - \epsilon_1/p_1 + \epsilon_2/p_2 \tag{10}$$

and

$$f_{3t}(q,p:A,B,R) = A(q_1+q_2+q_3 1)^{-(R+1)}[1/p_1-1/p_3] +$$
$$(B_1/p_1)(q_1+1)^{-(R+1)} - (B_3/p_3)*(q_3+1)^{-(R+1)}$$
$$= - \epsilon_1/p_1 + \epsilon_3/p_3 \tag{11}$$

Assuming that the random elements in these nonlinear equations (ϵ) are random preference deviations between the households uncorrelated with a conditioning vector w, then it is possible to solve a set of nonlinear equations for A, B and R.

$$\Sigma w_t' f_t(q,p:A,B,R)=0 \tag{12}$$

Extending this to a censored demand system, suppose good, q_1, has an interior solution, $q_1^* > 0$, then the standard Kuhn-Tucker conditions imply

$$f_{2t}(q,p:A,B,R) < -\epsilon_1/p_1 + \epsilon_2/p_2 \tag{13}$$
$$f_{3t}(q,p:A,B,R) < -\epsilon_1/p_1 + \epsilon_3/p_3$$

Assume that the conditional median of the right-hand sides equals zero, $MED[-\epsilon_1/p_1+\epsilon_i/p_i|w] = E[sgn(-\epsilon_1/p_1+\epsilon_i/p_i)|w]=0$ for all w in the sample.

Solve for A and B by setting the sample average cross products between the signs of the raw residuals and the instruments equal to zero.

$$(1/T)\sum_{t=1}^{T} w_t' \mathrm{sgn}[f_{2t}(A,B)]\ ) = 0 \qquad (15)$$

$$(1/T)\sum_{t=1}^{T} w_t' \mathrm{sgn}[f_{3t}(A,B)]\ ) = 0 \qquad (15)$$

The critical distributional assumption in this estimation procedure is that the conditional median of the price weighted random errors is zero, i.e. that at least 50% of all the sample households have positive expenditures for all commodities.

DATA, VARIABLES AND THE CHARACTERISTICS PRIOR

The data used in this study were from the diary portions of the 1980 and 1981 Bureau of Labor Statistics Continuing Consumer Expenditure Survey. The sampled households kept a diary of their expenditures on a variety of items for a two-week period. The aggregate expenditure section of the public use tape codes the 19 separate food classifications used in this analysis:

1 Baked Goods (bak)
2 Cereals (cer)
3 Beef (bef)
4 Pork (prk)
5 Other Meats (otm)
6 Poultry (pol)
7 Fish and Seafood (sea)
8 Eggs (egg)
9 Fresh Milk & Cream (mlk)
10 Other Dairy Products (che)
11 Fresh Fruits (ffr)
12 Fresh Vegetables (fvg)
13 Processed Fruit (pfr)
14 Processed Vegetables (pvg)
15 Sugar and Other Sweets (sug)
16 Nonalcoholic Beverages (bev)
17 Fats & Oils (fat)
18 Miscellaneous Foods (msc)
19 Food Away form Home (awy)

The price series were drawn from the Bureau of Labor Statistics, <u>CPI Detailed Report</u>, issues January 1980 through December 1981, "Table 3, Consumer Price Index for All Urban Consumers: Unadjusted indexes - food expenditure categories." In this series the food expenditure categories precisely match those on the aggregate section of the public use tape. Because only national data were used, the price variation in the estimation was limited to cyclical and trend components. Prices were re-indexed with a base of 1.0 for January 1980. The number of variables available on households in the BLS survey is large, but in the final estimation only yearly household income and number of members in a household are used.

After excluding households which did not participate for the whole two-week period, 8373 observations were left in the 1980/81 BLS data. Approximately 1/5 of the entire sample available was used to estimate the model. The model formulation used was determined by another 1/20 of the sample. Those observations which had an annualized expenditure of over $5000 for any individual commodity were deleted from the sample. Most of these observations were orders of magnitude higher, indicating errors in data entry. Households were also deleted which had zero annual income.

Six characteristics were introduced into the LTCES model as prior information. The characteristics chosen were based on the researchers' judgement concerning what constituted an interesting subset of the potentially very large characteristics transformation matrix C in equation (3). A particular commodity was considered to either contain or not contain each characteristic. The following list gives the six characteristics and the individual commodities that contain each one.

Characteristics	Individual Commodities
Starches	Bakery, Cereals, Sugar, Beverages, Away from Home
Meat Group	Beef, Pork, Other Meats, Poultry, Seafood, Away From Home
Fatty Foods	Bakery, Beef, Pork, Other Meats, Eggs, Milk, Other Dairy, Oils, Away from Home
Drinks	Milk, Processed Fruit, Processed Vegetables, Beverages, Away from Home
Breakfast Foods	Bakery, Cereals, Pork, Eggs, Milk, Other Dairy, Fresh Fruit, Processed Fruit, Processed Vegetables, Sugar, Beverages, Oils, Away from Home
Fruits & Vegetables	Fresh Fruit, Fresh Vegetables, Processed Fruit, Processed Vegetables, Away from Home

This Lancaster structure was embedded in the LTCES function through a C matrix composed entirely of ones and zeros. The 6 by 13 matrix assumed in the final estimation was,

$$\begin{matrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{matrix}$$

where the characteristics are indicated in the vertical dimension in the order given above and the commodities horizontally in the order previously indicated. The prior distribution used for the C matrix, was dictated in part by prior choice of the researcher, and in part by a model selection process where C was estimated in conjunction with a loose prior. Among the attributes considered and deleted due to near singularity of the Jacobian were lunch, convenience, high nutrient density, and low nutrient density.

ECONOMETRIC PROCEDURES

Asymptotic Least Absolute Deviations Estimation

The primary problem to be tackled when estimating parameters in the presence of limited dependent variables, especially simultaneously limited dependent variables, is to relax the restrictions embodied in continuity assumptions. Powell (1984) introduced in the censored standard linear regression, an interesting solution. For the model:

$$y_t = X_t\beta + u_t \qquad X_t\beta \geq -u_t \tag{16}$$

$$y_t = 0 \qquad \text{otherwise}$$

Powell proposed the least absolute deviation (LAD) estimator, β_p where β_p is the solution to:

$$\text{Minimize } S_T = (1/T)\Sigma\ |y_t - \max(0, x_t\beta)| \tag{17}$$

and found conditions under which it would be consistent and asymptotically normal. The main difficulty in working with the LAD estimator from a theoretical and operational standpoint is that the derivative of the absolute value function is discontinuous at 0, and hence the typical approach of finding solutions and showing asymptotic normality are inadequate.

Amemiya (1982) proposed a functional approximation to the least-absolute deviation estimator that was everywhere differentiable. As the number of observations in the sample became larger the closer the approximation became. Denoting this function $l_T(.)$:

$$\lim_{T\to\infty} l_T(x) = |x| \tag{18}$$

A primary contribution of this paper is to use Amemiya's function, in the place of the absolute value function, in a computational, as opposed to a theoretical framework. The function that Amemiya (1982) used is the integral of a logistic function:

$$l_T(x) = (2/c_T)\log(1 + e^{(-c(T)x)}) + x \tag{19}$$

where

$$c_T = T^d \quad \text{and } 1/3 < d < 1/2 \tag{20}$$

This function $l_T(x)$ is twice continuously differentiable and the important first derivative with respect to its argument (denoted $I_T(.)$) is:

$$I_T(x) = 1 - 2/(1 + e^{(-c(T)x)}) \tag{21}$$

I_T is a useful function, that approximates the "sign" function. It is an even function, $[I_T(x)=I_T(-x)]$. All orders of derivatives of $I_T(x)$ are continuous. The sign function $I(x)$ equals -1 for all values of $x < 0$ and equals +1 for all values of $x>0$. For $x=0$, it may, in different contexts, be equal to different values, for simplicity assume $I(0)=0$.

With this background, consider the standard regression LAD estimator using approximating functions l_T and I_T in the place of the absolute value and sign function. Denote the resulting estimator the ordinary asymptotic least absolute value deviation estimator ALAD. The ALAD estimator b_A solves the minimization problem:

$$\min_b \, (1/T)\Sigma \, l_T(y_t - x_t b) \tag{22}$$

The k dimensional system of first order conditions for this optimization problem are:

$$(1/T)\Sigma x_t i_T(y_t - x_t b_A) = 0 \tag{23}$$

If x_t includes a constant term, the sample median of $(y_t - x_t b) = 0$. It is a natural extension to consider the instrumental variable estimator based on the first order conditions:

$$(1/T)\Sigma z_t I_T(y_t - x_t b_A) = 0 \tag{24}$$

where z_t is of the same dimension as x_t. This estimator would be based on an assumption that the correlation between the instruments and the sign of the error is zero. Amemiya (1982) discusses alternative consistent estimators of b when x_t is not orthogonal to the errors. The smoothness of the objective function in (24) is convenient, as it allows this estimator to be implemented using standard nonlinear least squares packages.

Assuming that I_T is not a function of T, the asymptotic theory is straightforward as long as the standard regression assumptions are satisfied (Gallant 1987). From an applied perspective this is probably sufficient, since the estimator is not affected greatly by how close the approximation is to the actual sign function. Elements in the approximation O(1) do not influence small sample properties substantially. The conditions under which this is a reasonable estimator of b are more stringent than those observed in the scalar regressor case, as it must be assured that the conditional median of y_t given x_t, is everywhere in the sample greater than zero. Otherwise, $\mathrm{med}(u_t|x_t)$ may not equal $\mathrm{med}(y_t|x_t)$.

Estimation of the LTCES Utility Function

The estimated random LTCES direct utility function is made up of the sum of two components: a deterministic linear consumption technology utility function (u') and a random CES utility function (u").

$$u = u'(q{:}A,C,R_1,q) + u''(q,\epsilon{:}B,R_2) \tag{26}$$

$$u' = -(1/R_1)\sum_{j=1}^{6}[A_{j1} + A_{j2}(HS_t+1.0)] * [\sum_{i=1}^{19}C_{ij}(q_i+50)]^{-R}$$

$$u'' = -(1/R_2)\sum_{i=1}^{19}B_i(q_i+50)^{-R} - \sum_{i=1}^{19}q_i\epsilon_i \tag{27}$$

In order to identify some of the parameters, an arbitrary scaling restriction is imposed:

$$\sum_{i=1}^{19}B_i = 750.0 \tag{28}$$

This scaling restriction is necessary to identify a unique member of the LTCES class of utility functions. It is in the identical spirit of restricting the coefficients of a Cobb-Douglas utility function to lie in a simplex. The power transformation coefficients were fixed at 1.5, a value derived from an analysis of historical values of food demand own and cross-price elasticities. As Gallant (1987) mentions, implicit nonlinear regressions are seldom well identified with respect to monotonic transformations.

A stochastic Langrangian $Z(q,\alpha:\epsilon,A,B,C,R,p,m)$ was constructed and random Kuhn-Tucker conditions were found. By this, a household optimization restriction was imposed.

$$Z_t = u'_t + u''_t + \alpha_t(m_t - \Sigma p_{it} q_{it}) \tag{29}$$

and

$$\partial u'/\partial q_{it} + \partial u''/\partial q_{it} - \alpha_t p_{it} \leq 0 \qquad i = 1,\ldots,19 \tag{30}$$

$$\Sigma p_{it} q_{it} = 0$$

Solving (30) for α_t:

$$\alpha_t \geq [\partial u'/\partial q_{it} + \partial u''/\partial q_{it}]/p_{it} \qquad i = 1,\ldots,19 \tag{31}$$

Over 92% of the households who purchased any food items at all during the observation period, purchased bakery items (the first indexed good). Therefore assume,

$$\alpha_t = [\partial u'/\partial q_{1t} + \partial u''/\partial q_{1t}]/p_{1t} \tag{32}$$

Substituting (32) into the $i=2,\ldots,18$ remaining inequalities in (31) implies:

$$[\partial u'/\partial q_{it}+\partial u''/\partial q_{it}]/p_{it} - [\partial u'/\partial q_{1t}+\partial u''/\partial q_{1t}]/p_{1t} \leq 0. \tag{33}$$

Let $f_{it}(q:A,B)$ be defined as the difference between the marginal utility of good i divided by its price, and the marginal utility of good 1 divided by its price, then from (32) and (33), 18 inequalities are derived:

$$f_{it}(q,A,B) \leq \epsilon_{it}/p_{it} - \epsilon_{1t}/p_{1t} \qquad (34)$$

To estimate the parameters A,B from (34), it is assumed that for $t=1,\ldots,T$ and $i=2,\ldots 19$

$$\mathrm{sgn}[\epsilon_{it}/p_{it} - \epsilon_{1t}/p_{1t} | \text{exogeneous variables}] = 0. \qquad (35)$$

Defining w_t as a vector of instruments, (34) and (35) imply that:

$$E[I(f_{it}(q,A,B))| \ w_t] = 0 \qquad i=2,\ldots 19 \qquad (36)$$

This leads to the sample system of equations:

$$\Sigma w_t' I_T(f_{it}(q,A,B)) = 0 \qquad i=2,\ldots,19 \qquad (37)$$

In other words the Kuhn-Tucker conditions allow the definition of a set of nonlinear implicit simultaneous regression equations. The parameter of indicator function approximation d was fixed at (1/3). For the ALAD estimator, $c_t = T^d$ and $I_T(x) = 1.0 - 2.0/(1.0+ e^{(-c(T)x)})$.

The exogenous variables z_t, consist of household income, household size(HS_t), and a constant, as well as transformations of these variables. Other exogenous variables were examined, including reported average grocery expenditures, but their inclusion had little effect on the estimated parameters.

The objective function minimized using MINOS with respect to A_{ij}, B_{ij}, was the Jorgenson and Laffont (1974) expression for nonlinear three-stage least squares (NL3S):

$$(1/T)\Sigma(z_t' \otimes h_t')'(V_T^{-1} \otimes M_{zzT}^{-1})(1/T)\Sigma(z_t' \otimes h_t')/2 \qquad (38)$$

where:

$h_t = (I_T[f_2(.)], I_T[f_2(.)], \ldots, I_T[f_{19}(.)])$

a vector sign function

$V_T = (1/T)(\Sigma h_t h_t')$

covariance matrix of the sign Kuhn-Tucker conditions

$M_{zz} = (1/T)(\Sigma z_t z_t')$

covariance matrix of exogenous variables.

The estimation, while somewhat formidable looking as a series of algebraic expressions, is not very difficult to interpret. Under the assumption that households optimize, the ratio of marginal utilities to their prices $[(\partial u^*/\partial q_i)(1/P_i)]$, is equalized across all commodities. Therefore it is reasonable to believe that the difference between $(\partial u^*/\partial q_i)(1/P_i)$ and $(\partial u^*/\partial q_1)(1/P_1)$ is close to zero, for $i=2,19$. Terming this difference f_i, one expects that f_i is near zero. Moreover this residual, if it is centered around zero, can be expected to have approximately as many occurrences above zero as below. Register the sign of the residual by $h_{ti}(f_{ti})$. Expecting that the sign of the residuals has a median of zero, is equivalent to assuming that the covariance between a constant and the sign of the residuals is zero. Carrying this further, assume that the covariance between the exogenous variables and the sign of the residuals is zero:

$$(1/T)\Sigma(h_t \otimes z_t) = 0 \tag{39}$$

Assuming that the covariance between the 5 exogenous variables and the sign of the 18 residuals is zero, leaves 90 equations to solve. Since the utility function has 31 free parameters, the system is overidentified, (there are not enough parameters to satisfy all the equations simultaneously). Therefore in line with NL3S, the number of equations is reduced to 31, by taking weighted sums of the 90 equations. The weights are given by the variance-covariance matrices of the exogenous variables, M_{zzT}, the residuals, V_T, and the partial derivatives with respect to the parameters. The system was solved by using the optimization package MINOS where the 31 equations are the first-order conditions of the objective function (38), using a quasi-Newton method with supplied gradients. The primary computer used was a 4-processor Cray2. Standard econometric packages such as SAS's nonlinear systems procedures would be capable of the estimation as well.

EMPIRICAL RESULTS

In table 1, the estimated parameter vector for the LTCES model is presented. These estimates were found using NL3S over a sample of 1341 households. The estimated LTCES model does not deviate greatly from the traditional CES formulation, as the additional parameters in the LTCES model are of a relatively small magnitude. While the B parameters average 39.5 in value, the coefficients of the z-good power transformation are much smaller, approximately 1.5 for the a constant and -1.8 for the household size coefficients.

In accordance with our expectations, the effect of household size is to imply economies of scale in the production of all of the derived goods, except for fat ($A_{2j} < 0$ $j \neq 3$). The common incidence of fatty foods in juvenile diets may account for the distinction here. The constant attributes parameters (A_{1j}'s) are all positive except for the parameter associated with the starch group and fat groups, indicating that most characteristics are indeed valued positively.

The standard errors were computed by selecting four subsamples, and estimating the parameters independently. The observed variance in the parameter estimates was used to calculate a sampling distribution. The standard errors thus computed were divided by 2 to estimate the standard errors for the whole sample, which was 4 times the size of the subsamples.

Testing the LTCES Model

A formal hypothesis test:

H_0: CES model is true

H_A: LTCES model is true

would involve testing simultaneously whether the 12 Aij's or group parameters are significantly different from zero.

A test of this hypothesis is based on the NL3S analog to the likelihood ratio test, or the difference in the sum of squared errors test. In this test, discussed at length in Gallant and Jorgenson (1979), the test statistic is computed by multiplying the number of observations by the difference between the objective functions in the NL3S regressions. The values of the objective function for the unrestricted and restricted regressions were: .20199966 and .19529567. The resulting statistic which is asymptotically distributed as chi-square with 12 degrees of freedom for either the unrestricted or restricted regression is:

$T = 8.99$

At a 5% critical value, the hypothesis that the simpler CES model was true, is rejected when $T \geq 21.026$. Therefore the test fails to reject the null hypothesis that the LTCES model does not add anything to the basic CES model.

Note that this result is in contrast to the overwhelming significance of the individual A_{ij}s, the least significant of which has a t-statistic of -1.8. Monte-Carlo results establish that the actual critical region of a likelihood ratio test of nonlinear parameters for small samples is much closer to the asymptotic critical region than is the actual critical region of the Wald test (Gallant, 1987). This and other reasons may explain the discrepancy.

Computed Elasticities

Certain aspects of this model are revealed in the estimated demand elasticities. To save computer time, the elasticities of demand were computed at a particular point, between median and mean total food expenditure, and for a three-person household. In the middle range of total food expenditure, the elasticities are not sensitive to the amount of expenditure, so this may serve as a representative household.

To derive a sampling distribution for the elasticities, the parameters of the utility function were estimated on four randomly selected independent subsamples of about 335 households each. Fixing the food expenditure and household size, the demand for each of the 19 commodity groups was estimated at the base point and for a 5% change in total food expenditures and prices. Each of these 21 simulations was done for the 4 independent utility function parameter estimates. Out of these simulations, 4 independent estimates of the demand elasticity matrices were generated.

The parameters in the elasticity matrix have a variance since the utility function parameters that generate the elasticity are different subsample to subsample. One measure of the sampling distribution of the elasticity estimates is based on the observed deviation among the elasticity estimates sample to sample.[3] This estimate of a sampling distribution is similar to a Bootstrap or Jackknife method, both of which would be superior but considerably more costly to compute.

For own-price elasticities and expenditure elasticities, 3 columns of estimates are reported in tables 2 and 3. In the first column the estimates computed at the parameter values estimated over the 1341 observations are given. In the second column are the means of the elasticities computed for the subsamples (the average of four independent computations on the different, 355 observation subsamples). In the third column are the elasticity estimates computed for the CES model estimated over the 1341 observations. The expenditure elasticies indicate the percent change in purchases of a particular commodity with regard to a percent change in total food expenditures.

In table 4, some information on cross-elasticities is presented. Since there are 342 cross-price elasticities, a summary table was prepared, where the average elasticity response for a good is reported for a change in all other prices, a "row average". To give a representative standard error, the standard errors for the individual elements in a row were estimated, and the row average of these standard errors is reported. The third column is the ratio of the first two, and is useful as a guide to scale.

Several points are apparent in examining the elasticity estimates in tables 2, 3 and 4. Overall, the own-price elasticities are somewhat higher than in such classic studies of U.S. food demand as George and King (1971) and Brandow (1961). The results for the cross-price elasticities seem to be most consistent with previous analyses. Expenditure elasticities, as would be expected with the CES model, tend to a unitary magnitude. The estimated standard errors are small, and therefore the confidence intervals around the elasticity estimates are tight. The LTCES estimates of price elasticities and the CES estimates are very similar.

On average, the own-price elasticities are -.71, deviating around that from -.45 to -.81. The expenditure elasticities average 1.02, (if this was an expenditure weighted elasticity it would be 1.00,) with a range from .78 to 1.19. The average cross-price elasticity is 0.0004, and ranges from a low of -.14, to a high of .094. As the contribution of the linear consumption technology part of the direct utility function was not large, the elasticities show the same constancy over a range that the CES utility function demonstrates.

CONCLUDING REMARKS

The major contribution of this study is the demonstration of a methodological approach rather than the specific empirical results obtained. The parameters of a CES utility function, which covered 19 food commodities and incorporated characteristics as prior information, were estimated treating the Kuhn-Tucker first-order conditions as a simultaneous system of implicit nonlinear regression equations. Approaches similar to the one employed here could be applied to a wide variety of nonstandard optimization models.

The contribution of the characteristics prior was inconclusive. Asymptotic t-statistics indicated significance, whereas a generalized likelihood-ratio test did not. Nevertheless, the Lancaster concept of characteristics/attributes provides a potentially rich source of prior information which deserves further attention in demand analysis, particularly as an alternative to assumptions of separability.

Representative demand elasticities were obtained using a simulation technique. Elasticity estimates appear most in line with previous results for budget proportions near the mean. Repeated sampling techniques and simulation were employed to numerically derive sampling distributions and t-statistics for the demand elasticities. The computation of empirical sampling distributions is an approach worth emphasizing, although it is certainly not unique to this analysis.

The estimation procedure is unique and represents a key contribution. A new type of asymptotic least absolute deviations (ALAD) estimator was successfully demonstrated. In addition, nonlinear three-stage least squares (NL3S) was employed rather than a maximum likelihood technique. This approach both addresses the problem of zero expenditures for some commodities by some households and also does not impose a limitation concerning the size of the system. Wales and Woodland (1983) who used a quadratic random utility function and maximum-likelihood approach were constrained to a system of only three commodities.

Further research is needed to examine different utility functions and characteristics priors. In retrospect, the choice of the CES functional form had some distinct drawbacks because of its restrictive properties, which strongly influenced the empirical results. The effect was perhaps most obvious for the income (total food expenditure) elasticities, which were all close to unity as expected with the CES form. In future work, the specific utility function chosen should ideally be flexible enough so that the functional form has no intrinsic tendency to yield demand elasticities which are equal across commodities.

BIBLIOGRAPHY

Amemiya, Takeshi. "Two Stage Least Absolute Deviations Estimators." Econometrica, 50(1982):689-711.

Barten, Anton P. "The Systems of Consumer Demand Functions Approach: A Review." Econometrica 45(1977): 23-51.

Brandow, G. E. Interrelations Among Demands for Farm Products and Implications for Control of Market Supply, Pennsylvania Agric. Exp. Sta. Bul. 680, 1961.

Gallant, Ronald A. Nonlinear Statistical Models, New York: John Wiley and Sons, 1987.

Gallant, Ronald A., and Dale W. Jorgenson. "Statistical Inference for a System of Simultaneous, Non-linear, Implicit Equations in the Context of Instrumental Variables Estimation." J. of Econometrics 11(1979):275-302.

George P. S., and G. A. King. Consumer Demand for Food Commodities Giannini Foundation Monograph, March 1971.

Hansen, Lars P. "Large Sample Priorities of Generalized Method of Moments Estimators," Econometrica 50(1982):1029-1054.

Jorgenson, D.W. and J.J. Laffont. "Efficient Estimation of Nonlinear Simultaneous Equations with Additive Disturbances". Annals of Economics and Social Measurement, 3(1974): 615-640.

Lancaster, Kevin J. "A New Approach to Consumer Theory." J. Polit. Econ. 74(1976):132-157.

Lee, Lung-Fai, and Mark Pitt. "Microeconometric Demand Systems with Binding Non-negativity Constraints." Econometrica 5(1986):1237-1242.

Powell, James L. "Least Absolute Deviations Estimation for the Censored Regression Model." J. of Econometrics, 25(1984)303-325.

Wales, T.J., and A.D. Woodland. "Estimation of Consumer Demand Systems with Binding Non-negativity Constraints." J. of Econometrics, 21(1983):263-285.

FOOTNOTES

1 An approximate definition of a closed form is that $y = f(x)$ is closed form, if it may be written as a finite recursion of elementary functions $g_1, g_2, \cdots\cdots, g_n$.

2 According to Barten (1977), the demand function for the CES utility function is:

$$q_i = ß_i p_i^{(1/(r-1))} m[\Sigma ß_k p_k^{(r/(r-1))}]^{-1}$$

where $ß_i = B_i^{(1/1-r)}$, and prices and income are denoted p, and m respectively.

3 The reported standard errors are computed by the formula:

$$se(e_{ij}) = (1/2)\surd[(1/3)\sum_t^4(e_{ijt}-(1/4)\sum_t^4 e_{ijt}^2$$

The estimated standard deviation among the parameter estimates for the subsamples are divided by 2, to get the standard deviation for the full sample.

TABLE 1: LTCES DIRECT UTILITY FUNCTION PARAMETER ESTIMATES

BETA	VALUE	STD ERROR	T-STAT	GROUP
A11	-1.27	.69	-1.8	Starches
A12	-2.58	.50	-5.1	Starches
A21	5.00	.21	23.3	Meats
A22	-4.31	.64	-6.6	Meats
A31	-31.62	.19	-164.3	Fatty Foods
A32	10.29	.44	23.3	Fatty Foods
A41	13.28	.78	16.9	Drinks
A42	-2.99	.44	-6.7	Drinks
A51	22.16	.13	171.1	Breakfast
A52	-10.16	.59	-17.2	Breakfast
A61	1.67	.24	6.8	Fruits & Vegs
A62	-1.60	.30	-5.2	Fruits & Vegs

TABLE 1 (continued)

BETA	VALUE	STD. ERROR	T-STAT	COMMODITY	
B1	65.7	1.23	53.0	bak	Baked Goods
B2	34.2	.12	276.4	cer	Cereals
B3	58.0	1.68	34.3	ber	Beef
B4	39.9	.05	705.6	prk	Pork
B5	36.8	.06	558.4	otm	Other Meats
B6	30.4	.14	213.4	pol	Poultry
B7	25.4	.07	365.4	sea	Fish and Seafood
B8	28.3	.17	162.2	egg	Eggs
B9	48.8	.87	55.9	mlk	Fresh Milk & Cream
B10	47.6	.82	58.1	che	Other Dairy Products
B11	35.1	.38	91.4	ffr	Fresh Fruits
B12	39.3	1.07	36.6	fvg	Fresh Vegetables
B13	28.7	.74	38.5	pfr	Processed Fruit
B14	26.9	.85	31.5	pvg	Processed Vegetables
B15	34.4	.10	324.3	sug	Sugar and Other Sweets
B16	48.3	.08	574.7	bev	Nonalcoholic Beverages
B17	31.7	.19	166.8	fat	Fats & Oils
B18	51.8	.75	68.4	msc	Miscellaneous Foods
B19	87.7	.16	547.7	awy	Food Away form Home

TABLE 2 OWN-PRICE ELASTICITIES

	(T=1341:LTCES)	AVG OF(T=335)	(T=1341:CES)	STD DEV
bak	-.646	-.645	-.651	.002
cer	-.773	-.739	-.767	.020
bef	-.543	-.608	-.882	.011
prk	-.724	-.677	-.676	.006
otm	-.771	-.717	-.681	.021
pol	-.800	-.791	-.787	.013
sea	-.801	-.795	-.991	.010
egg	-.804	-.834	-.748	.028
mlk	-.609	-.631	-.628	.007
che	-.617	-.632	-.668	.006
ffr	-.773	-.726	-.744	.016
fvg	-.739	-.674	-.676	.004
pfr	-.787	-.739	-.936	.040
pvg	-.793	-.802	-.974	.053
sug	-.771	-.742	-.762	.021
bev	-.660	-.624	-.588	.006
fat	-.813	-.757	-.744	.013
msc	-.616	-.602	-.575	.004
awy	-.446	-.507	-.501	.018

TABLE 3 EXPENDITURE ELASTICITIES

	(T=1341:LTCES)	AVG OF (T=335)	(T=1341:CES)	STD DEV
bak	.895	.891	.899	.001
cer	1.073	1.069	1.059	.006
bef	.860	.922	1.130	.014
prk	1.056	1.023	.972	.003
otm	1.092	1.040	.967	.005
pol	1.085	1.091	1.045	.012
sea	1.064	1.144	1.199	.010
egg	1.190	1.111	.979	.008
mlk	.948	.960	.946	.008
che	.954	.970	.977	.007
ffr	1.076	1.053	1.040	.006
fvg	1.061	1.018	.984	.012
pfr	1.058	1.099	1.200	.007
pvg	1.049	1.107	1.215	.026
sug	1.073	1.061	1.055	.011
bev	.997	.968	.912	.004
fat	1.105	1.083	1.010	.002
msc	.953	.941	.901	.005
awy	.783	.809	.815	.018

TABLE 4: AVERAGE CROSS-PRICE ELASTICITIES (T = 1341: LTCES)

	ELASTICITY	AVG STD ERROR	COL1/COL2
bak	-.0151	.0100	-1.5118
cer	-.0302	.0096	-3.1490
bef	.0404	.0274	1.4747
prk	-.0563	.0087	-6.4815
otm	-.0715	.0083	-8.6417
pol	-.0072	.0110	-.6507
sea	.0691	.0089	7.7279
egg	-.1069	.0217	-4.9372
mlk	-.0134	.0100	-1.3428
che	-.0104	.0062	-1.6668
ffr	-.0413	.0073	-5.6865
fvg	-.0551	.0074	-7.4831
pfr	.0335	.0224	1.4977
pvg	.0612	.0233	2.6287
sug	-.0322	.0104	-3.0921
bev	-.0551	.0068	-8.0733
fat	-.0470	.0133	-3.5404
msc	-.0377	.0074	-5.1085
awy	.0089	.0054	1.6536