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INVENTORIES AND COMMODITY PRICE VOLATILITY:

A TEST OF THE THEORY OF STORAGE[*]

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Abstract: The theory of storage implies that commodity price volatility is inversely related to inventories, and that as inventories decline, spot prices become relatively more volatile than futures prices, and vice versa. These implications are directly tested using inventory and price data for six non-ferrous metals traded on the London Metal Exchange over the period 1989 to 2000. The conditional variances are specified as multiplicative heteroskedasticity models. For four of the metals, the observed relationships between the inventories and the variance of spot and futures prices support the implications of the theory of storage. For the other two metals contracts, the results do not support the theory. The findings thus lend qualified support to the notion that market fundamentals, rather than 'animal spirits', drive commodity price volatility.

Keywords: Commodity Prices; Inventories; Theory of storage.

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1. Introduction

This paper tests the implications of the theory of storage using London Metals Exchange (LME) prices for aluminium, copper, lead, nickel, tin and zinc. The theory of storage implies that the difference between commodity spot and futures prices (the ‘basis’) decreases as inventories decline relative to demand, and that spot and futures prices become more volatile as inventories fall.

Drawing on Samuelson’s (1965) hypothesis that spot prices are more variable than futures prices, several implications of the theory of storage regarding the volatility of — and the correlation between — spot and futures prices were drawn out by Fama and French (1988). In the absence of inventory data, they used the sign of the interest-adjusted basis as a proxy for inventory levels to test these implications.

Ng and Pirrong (1994) built on the Fama and French work by testing the implications of the theory of storage using the lagged basis as an explanatory variable. Ng and Pirrong first employ an error-correction model to specify the conditional means of the (differenced) spot and futures prices, and then specify the variances and covariances of the prices as an augmented bivariate GARCH model.

This paper builds on the Ng and Pirrong study by explicitly examining the effect of inventory levels on the volatility of spot and futures prices using recently compiled LME inventory data. It is found that the variances of spot and futures prices ~~are best~~may be specified as a multiplicative heteroskedasticity model — as opposed to a GARCH model employed by Ng and Pirrong (1994) amongst others — with lagged inventory levels used as an explanator. The results suggest that for the LME aluminium, lead, copper and zinc contracts, spot and futures price volatility is inversely related to inventory levels, and that as inventory levels decline spot prices become relatively more volatile than futures prices. These results do not hold for the tin and nickel contracts, indicating some other factor is driving their volatility.

These results add further support to the notion that price volatility is strongly influenced by market fundamentals, rather than the “animal spirits” hypothesised by Keynes (1936).

The remainder of this paper is structured as follows: Part 2 discusses the theory of storage and its implications for the volatility and correlation of spot and futures prices. Part 3 outlines the data used in the analysis. Part 4 lays out the structural models used to define the conditional means and variances of the spot and futures prices for each commodity. Part 5 outlines and discusses the main results. And Part 6 offers some conclusions.

2. The theoretical relationship between forward and spot prices

There are two major schools of thought on the relation between commodity spot and futures prices. One view is that a futures price is the sum of a future spot price, and an expected risk premium. The alternative view is the theory of storage, which explains the difference between contemporaneous spot and futures prices (the 'basis') as a function of the opportunity cost of storing a commodity, the costs of physically storing a commodity and a convenience yield of holding the commodity. The theory of storage, developed by Kaldor (1939), Working (1948), Brennan (1958) and Telser (1958), is relatively uncontroversial, while there is little agreement on whether futures prices contain expected premiums, or have power to forecast spot prices (Fama and French 1987). Although the two theories are theoretically equivalent, for the purpose of this study, the theory of storage is used to examine the relationship between contemporaneous spot and futures prices.

The theory of storage is predicated on the observation that at any given time, a purchaser can either take out a futures contract, or purchase the physical commodity now and store it.

With arbitrage therefore, the futures price of a commodity at time t for delivery at *T F(t,T)* is equal to the spot price *S(t)*, plus the marginal warehousing cost of holding the commodity for *T-t W(t,T)*, plus the opportunity cost (usually interest rate is used as a proxy) of holding the commodity for *T-t R(t,T)*, minus the marginal convenience yield of the holding the commodity for *T-t C(t, T)*:

$$F(t,T) = S(t) + W(t,T) + R(t,T) - C(t,T) \qquad \textbf{(1)}$$

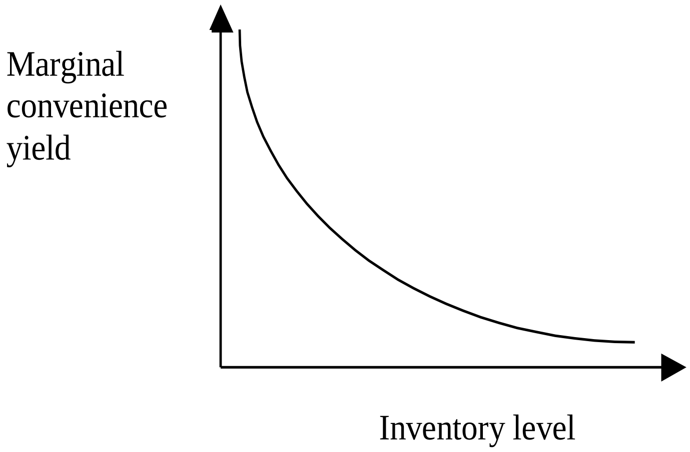
The intuition of a convenience yield stems from the position that an uncompensated carrying cost —where a futures price does not exceed the spot price by enough to offset warehousing and opportunity costs — implies that there must be some other return from holding physical inventories of a commodity (Fama and French 1988). Firms that hold inventories reap a convenience yield because inventories on hand allow them to respond more flexibly and efficiently to unexpected supply and demand shocks (Ng and Pirrong 1994). The marginal convenience yield is assumed to decreasing in inventories *I(t)*, but at a decreasing rate:

$$\frac{\partial C(t,T)}{\partial I(t)} < 0, \quad \frac{\partial^2 C(t,T)}{\partial I(t)^2} > 0$$

The shape of the marginal convenience yield curve (Figure 1) is relatively intuitive. At higher inventory levels, the convenience yield of holding an additional unit of inventory is low, while at low inventory levels, the yield of holding an additional unit of inventory is high.

Throughout the analysis, the marginal warehousing cost *W(t,T)* is assumed to be constant in the relevant range of inventory levels, an approach common in the literature (Fama and French 1988; Heaney 1998).

Figure 1: Marginal convenience yield



2.1 The relation between basis and inventory level

Reworking **(1)**, the interest-adjusted basis can be defined as:

$$F(t,T) - S(t) - R(t,T) = W(t,T) + C(t,T) \qquad \textbf{(2)}$$

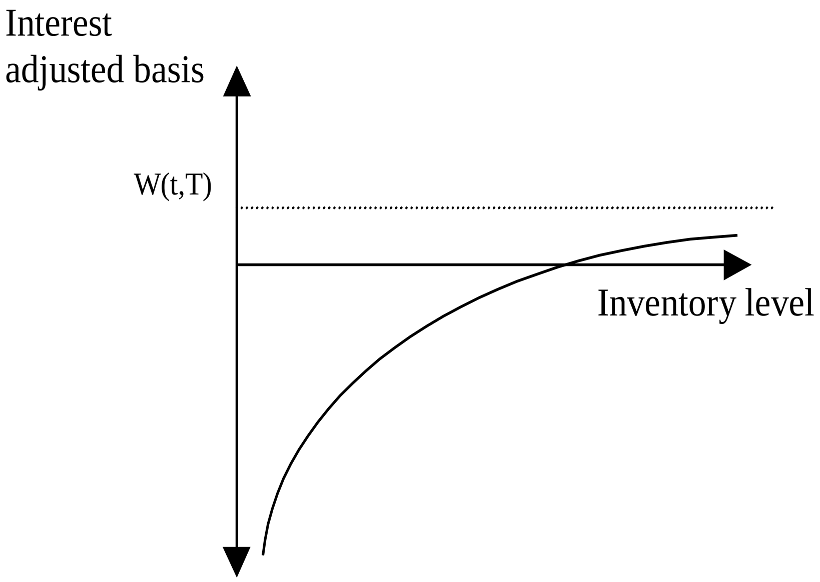
where, due to the assumption that *W(t,T)* is constant, the variation in the interest-adjusted basis is due to variation in the marginal convenience yield. The interest-adjusted basis, expressed as a function of stocks, is illustrated in Figure 2.

The relationship between convenience yield and basis — as summarised in the theory of storage — allows the effects that changes in supply and demand have on spot and futures prices to be identified. To illustrate, assume that there is a permanent, unexpected increase in demand for lead. *Ceteris paribus*, this would lead to an increase in both the spot price for lead, and in the expected price for lead at time *T* (that is, *F(t,T)*). However, because at least some of the increase in lead demand will be offset by increases in lead supply by time *T*, the futures price would rise by less than the spot price. Thus, in general, futures prices are less variable than spot prices, and the greater the time to maturity (that is, the greater *T*-*t*), the less variable the futures price (Samuelson 1965).

The relative impact of an exogenous shock on spot and futures prices is also a function of the level of stocks. Fama and French (1988) identified three key implications of the theory of storage on commodity prices: firstly, if current and permanent shocks dominate[1], both spot and future price volatility is inversely related to the inventory level; secondly, if current and permanent shocks dominate, as inventories fall, spot prices become relatively more volatile than futures prices; and thirdly, when inventories are high, the correlation between spot and futures prices approaches unity. The first two implications are tested in this paper.

Returning to the previous example, when stocks are low, convenience yield rises rapidly when lead is drawn down from stocks to meet the increase in demand. Therefore, there is only a small inventory response, and spot and futures prices increase markedly. Again, the variation in the spot price is larger than futures price variation at low inventory levels because the expected lead supply response moderates the increase in expected spot prices.

Figure 2: Interest adjusted basis as a function of stocks



Conversely, when inventories are high, the marginal convenience yield is low and stable. As a result, the inventory response to the increase in demand can be quite large without affecting the interest-adjusted basis to a great extent. The inventory response dampens the

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[1] Current shocks affect demand and supply at *t* only, while permanent shocks affect demand and supply at *t* and *T* in the same fashion (Ng and Pirrong 1994).

effect of the demand shock on the current lead spot price, and its change is not much greater than the change in the futures price — most of the change in the current lead price is expected to be permanent (Fama and French 1988). When inventories are high, therefore, the correlation between spot and futures prices approaches unity.

3. The data

To test the relationship between market fundamentals and price volatility, this study uses spot and 3 month futures prices for aluminium, copper, lead, nickel, tin and zinc from the LME. Price data from 13 January 1989 to 27 July 2000 are used for the aluminium, copper, lead, nickel, and zinc contracts, while the period 12 January 1990 to 27 July 2000 is used for the tin contract. As is noted in Fama and French (1988), Ng and Pirrong (1994) and Heaney (1998), using LME contracts to examine price dynamics has a number of advantages. The LME has no limits on spot and future price changes — therefore each price recorded represents a market-clearing price. LME spot and futures prices are determined simultaneously in an open outcry setting, and the fact that the contracts examined are metals prices means that they do not exhibit seasonality, unlike agricultural products can — as a result the assumption that current period and permanent demand shocks predominate is appropriate.

The inventory data used is also LME data, and is the amount of each commodity available in LME-approved warehouses. The period of this analysis, 1989 to 2000, was characterised by large inventory build-up as several economies — particularly Japan and, towards the end of the 1990s, other Asian countries — demanded less metal than was expected.

Because each contract is denominated in US dollar terms, the Eurodollar 3-month middle rate — sourced from Datastream — is used as an estimate for the risk-free cost of financing for each commodity.

The LME reports commodity spot and futures prices for each business day. Datastream reports the Eurodollar 3-month interest rate on a daily basis. The periodicity of the inventory data reported by the LME varies over the period examined.

Prior to 26 April 1990, inventory levels are reported on a weekly (Friday) basis, between 30 April 1990 and 30 March 1997 they are reported on a semi-weekly (Tuesday and Friday) basis, and from 1 April 1997 on a daily basis. For continuity, weekly observations — reported for each Friday — were collected for each of the price, inventory and interest rate variables. Where data for a particular Friday was unavailable (usually due to public holidays) data for all variables were collected for the nearest business day.

The data set contains 603 observations for the aluminium, copper, lead, nickel, zinc, and interest rate variables, and 551 observations for the tin variables. Descriptive statistics for the data set are displayed in Table 1.

The descriptive statistics confirm Samuelson's (1965) oft-proven hypothesis that spot prices are more variable than futures prices. For each of the six commodities examined, the standard deviation of the spot price is higher than that of the futures contract.

Table 2 displays the autocorrelations for each of the variables. The autocorrelations are highly persistent for each of the series examined, indicating the possibility of the presence of unit root processes. The presence of a unit root has major implications for further analysis, so unit root test are carried out on the series. The augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root test statistics are also calculated for the series.[2]

Table 1: Descriptive statistics for levels [a]

	Mean	SD	Minimum	Maximum
Aluminium				
Natural log of spot price	7.2950	0.1737	6.9354	7.8120
Natural log of 3-month futures price	7.3074	0.1645	6.9556	7.7832
Natural log of inventory level	13.3803	0.9488	10.6537	14.7939
Copper				
Natural log of spot price	7.6994	0.2170	7.2156	8.1383
Natural log of 3-month futures price	7.6912	0.1989	7.2345	8.0746
Natural log of inventory level	12.5149	0.6600	10.7547	13.6366
Lead				
Natural log of spot price	6.3619	0.2134	5.8958	7.1873
Natural log of 3-month futures price	6.3711	0.1945	5.9309	6.8240
Natural log of inventory level	11.6939	0.7301	9.4650	12.8260
Nickel				
Natural log of spot price	8.8859	0.2975	8.2375	9.8574
Natural log of 3-month futures price	8.8893	0.2849	8.2532	9.8309
Natural log of inventory level	10.3528	1.2167	7.3251	11.9267
Zinc				
Natural log of spot price	7.0561	0.1936	6.7645	7.6544
Natural log of 3-month futures price	7.0590	0.1692	6.7805	7.5934
Natural log of inventory level	12.5868	1.0487	10.2054	14.0295
Tin				
Natural log of spot price	8.6514	0.0849	8.3894	8.8860
Natural log of 3-month futures price	8.6576	0.0847	8.4007	8.8804
Natural log of inventory level	9.4843	0.3975	8.4305	10.3861
Euro Dollar 3-month interest rate				
Interest rate	0.0144	0.0042	0.0078	0.0261

a. N=603 for aluminium, copper, lead, nickel, zinc, and interest rate variables, and N=551 for tin variables.

Table 2: Autocorrelations over levels — lags 1-8 [a]

	ρ1	ρ2	ρ3	ρ4	ρ5	ρ6	ρ7	ρ8	Q(12) **b**
Aluminium									
Log(Inventory)	1.00	0.99	0.99	0.98	0.98	0.97	0.97	0.96	6869.22
Log (Spot price)	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	5611.02
Log (Futures price)	0.98	0.96	0.95	0.93	0.91	0.89	0.87	0.85	5755.11
Lead									
Log(Inventory)	1.00	0.99	0.99	0.99	0.98	0.98	0.97	0.97	6943.14
Log (Spot price)	0.98	0.97	0.96	0.94	0.92	0.91	0.90	0.89	6044.01
Log (Futures price)	0.99	0.98	0.97	0.95	0.94	0.93	0.91	0.90	6208.46
Copper									
Log(Inventory)	0.99	0.98	0.97	0.96	0.94	0.93	0.91	0.90	6240.06
Log (Spot price)	0.99	0.97	0.95	0.94	0.92	0.91	0.89	0.88	5971.94
Log (Futures price)	0.99	0.97	0.96	0.94	0.93	0.92	0.90	0.89	6062.66
Nickel									
Log(Inventory)	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.96	6858.81
Log (Spot price)	0.98	0.97	0.95	0.93	0.91	0.89	0.87	0.85	5706.90
Log (Futures price)	0.99	0.97	0.95	0.93	0.91	0.89	0.87	0.85	5676.35
Zinc									
Log(Inventory)	1.00	0.99	0.99	0.98	0.98	0.97	0.96	0.96	6869.69
Log (Spot price)	0.98	0.97	0.94	0.92	0.90	0.88	0.86	0.84	5606.76
Log (Futures price)	0.99	0.97	0.95	0.93	0.91	0.89	0.87	0.85	5674.42
Tin									
Log(Inventory)	0.99	0.99	0.98	0.97	0.95	0.94	0.93	0.92	5873.96

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[2] SHAZAM version 8.0 (White 1997) has been used for all of the data analysis and model estimation in this study.

Log (Spot price)	0.96	0.92	0.88	0.85	0.82	0.80	0.77	0.75	4217.30
Log (Futures price)	0.97	0.93	0.89	0.86	0.84	0.81	0.79	0.77	4363.63
Euro Dollar 3-month interest rates									
Interest rate	0.99	0.99	0.98	0.97	0.97	0.96	0.95	0.94	6622.89

a. All values are significant at the 5% level. **b**. The column labelled "Q(12) reports the Ljung-Box statistic for twelfth order serial correlation which is distributed on χ^2_{12}.

The ADF and PP statistics are used to test the null hypothesis that there is a single unit root. The results of the tests are displayed in Table 3. For five of the commodities (aluminium, lead, copper, nickel and zinc) there is insufficient evidence to reject the null hypothesis, indicating the presence of unit root processes. For tin, however, the results vary with the type of test and the lag length chosen. Both the ADF and PP tests reject null hypothesis. The rejection of the null hypothesis of a unit root in a commodity price series is rather rare, although Heaney (1998) found similar results for LME lead prices. However, the persistence of the autocorrelations for tin described in Table 2 are indicative of at least near unit root processes.

The evidence of a unit root in the interest rate series — although counter-intuitive — is consistent with the literature (Hall, Anderson and Granger 1992; Heaney 1998; MacDonald and Murphy 1989; Shea 1992).

Differences are also tested for a unit root. There is sufficient evidence that each of the time series is stationary in first differences.

Table 3: Unit root tests — Augmented Dickey-Fuller test statistic and Phillips-Perron test statistic [a]

	ADF $Z(t_\alpha)$			Phillips-Perron $Z(t_\alpha)$		
	5 lag	13 lags	26 lags	5 lag	13 lags	26 lags
Aluminium						
Log(Inventory)	-1.276	-1.610	-1.508	-0.816	-0.982	-1.074
Log (Spot price)	-2.754	-2.615	-2.420	-2.921	-2.932	-2.910
Log (Futures price)	-2.6713	-2.610	-2.369	-2.843	-2.886	-2.900
Lead						
Log(Inventory)	-1.180	-1.572	-1.555	-0.979	-1.081	-1.217
Log (Spot price)	-2.107	-1.971	-2.199	-2.145	-2.123	-2.165
Log (Futures price)	-2.094	-2.080	-2.258	-1.852	-2.005	-2.079
Copper						
Log(Inventory)	-2.404	-2.180	-2.535	-2.206	-2.350	-2.278
Log (Spot price)	-2.490	-2.406	-2.752	-2.280	-2.600	-2.596
Log (Futures price)	-2.431	-2.374	-2.941	-2.447	-2.509	-2.540
Nickel						
Log(Inventory)	-0.134	-0.402	-1.373	-0.373	-0.305	-0.541
Log (Spot price)	-2.890	-2.847	-2.461	-2.403	-2.449	-2.433
Log (Futures price)	-3.039	-2.668	-2.471	-2.425	-2.497	-2.469
Zinc						
Log(Inventory)	-0.660	-0.589	-1.247	-0.351	-0.395	-0.407
Log (Spot price)	-3.081	-2.489	-2.800	-2.569	-2.564	-2.564
Log (Futures price)	-3.071	-2.555	-2.783	-2.435	-2.509	-2.480
Tin						
Log(Inventory)	-2.548	-2.347	-2.159	-2.270	-2.349	-2.286
Log (Spot price)	-2.849	-3.349 [b]	-2.329	-3.350 [b]	-3.465 [b]	-3.333 [b]
Log (Futures price)	-2.182	-3.290 [b]	-2.331	-3.259 [b]	-3.384 [b]	-3.256 [b]
Euro Dollar 3-month interest rates						
Interest rate	-1.725	-2.113	-1.630	-1.2640	-1.320	-1.357

a. Model includes an intercept and a time trend. The critical value for the 10% level of significance is -3.13; **b**. Significant at 10% level

4. Model

This paper aims to examine two of Fama and French's (1988) implications of the theory of storage — namely that both spot and futures price volatility are inversely related to the inventory level, and that as inventories fall, spot prices become relatively more variable than futures prices (and vice versa). To test these hypotheses, the conditional means and variances of the spot and futures prices for each commodity must be determined.

Following a method described by Engle and Granger (1987), the long run relationship between price, inventory and interest rate variables is estimated. If the theory of storage is valid in an empirical sense, then the relationship between the variables should hold in the long term. From the theory of storage, the relationship between the variables is:

$$\ln[F(t,T)] = \alpha + \beta_1 \ln[S(t)] + \beta_2 R(t,T) + \beta_3 \ln[I(t)] + e_t \qquad \textbf{(4)}$$

where α includes warehousing costs (assumed constant), and the β terms are the cointegrating terms. The β_3 term, in estimating the effect of inventory level on the basis, summarises the convenience yield effect. If the error process e_t has a stationary mean, then **(4)** is said to be a cointegrating relationship between the variables. This model can be estimated using ordinary least squares (OLS) — if the variables are cointegrated, an OLS regression yields a 'super-consistent' estimator of the cointegrating β parameters (Enders 1995).

Cointegration is a feature of variables that exhibit an equilibrium relationship. If one or more of the variable changes due to some exogenous shock, then market agents will react quickly to this arbitrage opportunity, and the market will return to its equilibrium (Heaney 1998). So, although each individual variable is a non-stationary process, the residuals of the equilibrium relationship — in this case that described by the theory of storage — are stationary.

The conditional means of the spot and futures price for each commodity are specified as error correction models (ECM) of the form:

$$\begin{aligned}\Delta\ln[F_t] = {} & \alpha_{11} + \alpha_F \hat{e}_{t-1} + \sum_{i=1}^{N}\alpha_{11}(i)\Delta\ln[F_{t-1}] + \sum_{i=1}^{N}\alpha_{12}(i)\Delta\ln[S_{t-1}] \\ & + \sum_{i=1}^{N}\alpha_{13}(i)\Delta\ln[R_{t-1}] + \sum_{i=1}^{N}\alpha_{14}(i)\Delta\ln[I_{t-1}] + \upsilon_{Ft}\end{aligned} \qquad \textbf{(5)}$$

and:

$$\begin{aligned}\Delta\ln[S_t] = {} & \alpha_{21} + \alpha_S \hat{e}_{t-1} + \sum_{i=1}^{N}\alpha_{21}(i)\Delta\ln[F_{t-1}] + \sum_{i=1}^{N}\alpha_{22}(i)\Delta\ln[S_{t-1}] \\ & + \sum_{i=1}^{N}\alpha_{23}(i)\Delta\ln[R_{t-1}] + \sum_{i=1}^{N}\alpha_{24}(i)\Delta\ln[I_{t-1}] + \upsilon_{St}\end{aligned} \qquad \textbf{(6)}$$

where the α's are estimated parameters, and the υ's are residuals. The models are estimated by OLS. The inclusion of the lagged error terms from the long run relationship estimated in **(4)** allows the price in question to be affected by deviations from long run equilibrium conditions. Thus if there is a departure from equilibrium in time *t-1*, then this will be reflected in the change in prices in period *t*, bringing the relationship back towards equilibrium.

The conditional variances are specified as multiplicative heteroskedasticity models of the form:

$$h_{f,t} = \exp\left(\alpha_f + \phi I_{t-1}\right) \quad \textbf{(7)}$$

and:

$$h_{s,t} = \exp\left(\alpha_s + \delta I_{t-1}\right) \quad \textbf{(8)}$$

where $h_{f,t}$ is the conditional variance of the futures price, and $h_{s,t}$ is the conditional variance of the spot price. The inclusion of the lagged inventory term models the variance as a function of inventory levels — as implied by the theory of storage. Theory suggests that $\phi<\delta<0$. The first inequality states that as inventories fall, spot prices become relatively more volatile than futures prices, and vice versa. The second inequality formalises the position that both spot and futures are more volatile when inventories are low, and vice versa. The models are estimated by maximum likelihood estimation.

5. Results and Interpretation

Table 4 presents the results of unit root tests conducted on the residuals of the long run cointegrating relationship in **(4)**. For each commodity the ADF test provides evidence supporting the null hypothesis of a unit root (at 10 per cent level of significance) for tests specified with longer lags. This is the case for all commodities except zinc. However, the theory supporting the ADF test assumes that errors are independent and homogeneous (Enders 1995). In this case, considering the probable heteroskedastic nature of the variance, a more appropriate unit root test may be the PP test, which allows for fairly mild assumptions about the distribution of the errors (Enders 1995).

From Table 4 the PP test finds sufficient evidence to reject the null hypothesis of a unit root in the error process of the long run relationship in all commodities. Thus it can be concluded that the variables are cointegrated in a relationship predicted by the theory of storage.

Table 4: Unit root tests on residuals of cointegrating regressions — Augmented Dickey-Fuller test statistic and Phillips-Perron test statistic a

	ADF $Z(t_\alpha)$			Phillips-Perron $Z(t_\alpha)$		
	5 lag	13 lags	26 lags	5 lag	13 lags	26 lags
Aluminium	-6.877 **b**	-4.957 **b**	-4.000	-7.252 **b**	-6.936 **b**	-6.671 **b**
Lead	-6.921 **b**	-5.163 **b**	-3.720	-7.449 **b**	-6.539 **b**	-6.273 **b**
Copper	-5.807 **b**	-4.734 **b**	-3.190	-7.090 **b**	-7.315 **b**	-7.510 **b**
Nickel	-4.855 **b**	-3.530	-2.182	-8.340 **b**	-9.172 **b**	-10.347 **b**
Zinc	-8.706 **b**	-7.136 **b**	-4.948 **b**	-8.575 **b**	-7.370 **b**	-6.347 **b**
Tin	-4.576 **b**	-3.851	-2.830	-5.590 **b**	-5.859 **b**	-6.114 **b**

a. Cointegration tests include an intercept and a time trend. The critical value for the 10% level of significance is 4.15; **b**. Significant at 10% level

The ECMs, specified as in **(5)** and **(6)**, were estimated to determine the conditional means of the spot and futures prices. The choice of lag length for the differenced variables is important in capturing the full dynamic relationship. ECMs with varying lag lengths — from 2 to 12 lags — were estimated for the spot and futures prices in each commodity, with the parameters proving quite robust to the lag length specified. Without any prior assumptions regarding the appropriate lag structure, a lag length of 5 for each of the differenced explanatory variables was chosen.

The stability of the ECMs was tested by recursive estimation. This involves running a series of regressions by adding one observation per regression, and is useful for testing for structural change (White 1997). All ECMs were found to be structurally stable at the 5 per cent level of significance. The autocorrelation functions of the residuals for each ECM were examined, and all residual processes were found to be stationary.

A battery of heteroskedasticity tests were performed on the ECM residuals, and the results for two of these tests — the ‘Harvey’ test and the ARCH test— are reported in Table 5. A significant result for the Harvey test indicates that the variance is best modelled using a multiplicative heteroskedasticity model, while as it names suggests a significant result for the ARCH test indicates the variance may be an ARCH process.

For all ECMs, the residuals showed significant results for the Harvey test at the 1 per cent level, while only the tin and lead ECMs showed significant results for the ARCH test at the 1 per cent level. This is an interesting finding, as the literature generally finds — or assumes — that a bivariate GARCH specification is appropriate for modelling commodity price variance (Baillie and Myers 1991; Ng and Pirrong 1994).

Table 5: Error correction models— Heteroskedasticity tests on residuals

	Harvey test (χ^2_{21})	ARCH test (χ^2_1)
Aluminium		
Futures ECM	220.7 **c**	0.2
Spot ECM	212.3 **c**	2.6
Lead		
Futures ECM	273.9 **c**	10.4 **c**
Spot ECM	297.9 **c**	109.5 **c**
Copper		
Futures ECM	213.0 **c**	4.8 **d**
Spot ECM	222.8 **c**	19.6 **c**
Nickel		
Futures ECM	259.9 **c**	1.0
Spot ECM	310.0 **c**	1.2
Zinc		
Futures ECM	235.9 **c**	1.5
Spot ECM	214.3 **c**	3.4 **d**
Tin		
Futures ECM	238.2 **c**	9.1 **c**
Spot ECM	239.1 **c**	7.2 **c**

a. Tests for errors specified as $\log(e^2_t)=\mathbf{x}'\boldsymbol{\beta}$; **b.** Tests for errors specified as $e^2_t=\beta e^2_{t-1}$**;** **c.** Significant at 1 per cent level; **d.** Significant at 10 per cent level

Table 6 reports the results from the estimation of the spot and futures price variances (equations **(7)** and **(8)**)[3]. Starting with the results for aluminium, lead, copper and zinc, the values for ϕ and δ are negative. This is consistent with the theory of storage as it implies that volatility is inversely related to inventory levels. However, the estimates of ϕ are only significant for aluminium and zinc (at the 10 and 5 per cent levels of significance, respectively). This result suggests that for copper and lead, futures prices are invariant in inventories.

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[3] As the emphasis in this paper is on the variance of the spot and futures prices, the estimates coefficients from the ECMs are not reported.

Table 6: Multiplicative heteroskedasticity models — estimated constant terms, inventory coefficients and likelihood ratio statistics

	Aluminium	Lead	Copper	Nickel	Zinc	Tin
α_f	-6.230	-6.869	-6.486	-7.768	-5.993	-15.587
	(-7.45) **b**	(-7.38) **b**	(-5.81) **b**	(-15.57) **b**	(-8.45) **b**	(-10.80) **b**
ϕ	-0.104	-0.030	-0.062	0.099	-0.117	0.811
	(-1.69) **d**	(-0.38)	(-0.70)	(2.08) **c**	(-2.10) **c**	(5.34) **b**
α_s	-3.244	-2.501	-2.474	-6.656	-2.638	-14.859
	(-5.04) **b**	(-4.71) **b**	(-4.04) **b**	(-13.34) **b**	(-6.28) **b**	(4.92) **b**
δ	-0.310	-0.374	-0.359	0.003	-0.352	0.748
	(-3.93) **b**	(-2.69) **b**	(-2.22) **c**	(0.06)	(-3.72) **b**	(-10.30) **b**
Futures LR a	56.394 **b**	53.582 **b**	42.704 **b**	39.632 **b**	23.606 **b**	23.914 **b**
Spot LR a	63.994 **b**	89.564 **b**	36.626 **b**	44.748 **b**	26.800 **b**	32.654 **b**

t-statistics in parenthesis. **a**. Testing model with 1 exclusion restriction. LR has (χ^2_1) distribution. **b**. Significant at 1 per cent; **c**. Significant at 5 per cent; **d.** Significant at 10 per cent

For each of the four commodities $\delta > \phi$, again consistent with the theory of storage as it implies that spot prices become relatively more volatile than futures prices when inventories fall, and vice versa. The likelihood ratios for the variance models — testing the significance of the unrestricted model versus a model specified without the inventory term as an explanatory variable — is significant at the 1 per cent level for each commodity.

Turning to the results for nickel and tin, the estimates for δ and ϕ are positive, a finding counter to the theory of storage as it implies that spot and futures prices become more volatile when inventories increase, and vice versa.

With the exception of the estimate of δ for nickel, these findings are significant at at least the 5 per cent level. It is likely that some factor other than inventory levels is driving the price volatility in these commodities — it must be remembered that the Harvey test is specified with all of the variables, not just inventories. Interestingly, Ng and Pirrong (1994) do not estimate the variance for nickel and tin in their paper, despite the availability of price data (for nickel at least) for their period of interest.

6. Conclusions

This paper tests two of the major implications of the theory of storage for commodity spot and futures prices — namely that spot and futures prices vary inversely with inventory, and that as inventories increase, spot prices become relatively more volatile than futures prices. The implications of the theory of storage for price volatility have not been previously tested directly using inventory data, nor have the variances been specified as multiplicative heteroskedasticity models.

The findings give qualified support to the theory, and hence are consistent with the notion that fundamental supply and demand conditions drive commodity price volatility. For the LME aluminium, copper, lead, and zinc contracts, the findings indicate that price volatility is indeed inversely related to inventory levels, and spot prices are estimated to become relatively more volatile than futures prices as inventories decline (and vice versa).

For the tin and nickel contracts, it is likely that some factor other than lagged inventory levels drive price volatility, this is an area for further research.

References

Baillie, R., and Myers, R., 1991, 'Bivariate GARCH estimation of the optimal futures hedge', *Journal of Applied Econometrics*, 6(April-June), pp. 109-124.

Brennan, M.J., 1958, 'The Supply of Storage', *American Economic Review,* 48(1), pp. 50-72.

Enders, W., 1995, *Applied Econometric Time Series*, John Wiley & Sons, USA.

Engle, R.E., and Granger, C.W., 1987, 'Cointegration and Error Correction: Representation, Estimation and Testing', *Econometrica*, 55(March), pp. 251-276.

Fama, E.F., and French, K,R., 1987, 'Commodity Futures Prices: Some evidence on forecast power, premiums and the theory of storage', *Journal of Business*, 60(1), pp. 55-73.

Fama, E.F., and French, K,R., 1988, 'Business Cycles and the Behaviour of Metals Prices', *Journal of Finance*, 43(5), pp. 1075-93.

Hall, A.D., Anderson, H.M., and Granger, C.W.J., 1992, 'A Cointegration Analysis of Treasury Bill Yields', *The Review of Economics and Statistics,* 74, pp. 116-126.

Heaney, R., 1998, 'A Test of the Cost-of-Carry Relationship Using the London Metal Exchange Lead Contract', *The Journal of Futures Markets*, 18(2), pp. 177-200.

Kaldor, N., 1939, 'Speculation and Economic Stability', *Review of Economic Studies*, 7(1), pp. 1-27.

Keynes, J.M., 1936, *The General Theory of Employment*, Interest and Money, Harcourt Brace New York.

MacDonald, R., and Murphy, P.D., 1989, 'Testing for the Long Run Relationship Between Nominal Interest Rates and Inflation using Cointegration Techniques', *Applied Economics,* 21, pp. 439-447

Ng, V.K., and Pirrong, S.C., 1994, 'Fundamentals and Volatility: Storage, Spreads and the Dynamics of Metals Prices', *Journal of Business*, 67(2), pp. 203-230.

Samuelson, P.A., 1965, 'Proof that Properly Anticipated Prices Fluctuate Randomly', *Industrial Management Review,* 6(Spring), pp. 41-49.

Shea, G.S., 1992, 'Benchmarking the Expectations Hypothesis of the Interest Rate Term Structure: An Analysis of Cointegration Vectors', *Journal of Business and Economic Statistics*, 10, pp. 347-366.

Telser, L., 1958, ' Futures Trading and the Storage of Cotton and Wheat', *Journal of Political Economy,* 66(June), pp. 233-255.

White, K.J., 1997, *Shazam, the Econometrics Computer Program: User's Reference Manual,* version 8.0, McGraw-Hill, New York.

Working, H., 1949, 'The Theory of the Price of Storage', *American Economic Review,* 1949(December), pp. 1254-62.