Fully calibrating regional models of agricultural supply under multiple constraints

Cloé Garnache and Pierre R. Mérel
Department of Agricultural and Resource Economics
University of California, Davis

Poster prepared for presentation at the Agricultural & Applied Economics Association's
2012 AAEA Annual Meeting
Seattle, Washington, August 12-14, 2012

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Motivation

- Positive mathematical programming (PMP) allows the exact calibration of agricultural production models against available economic information without the use of artificial flexibility constraints, while requiring minimal data.
- Calibration against the reference allocation (first-order calibration) is always feasible and can be achieved in many ways.
- Calibration against supply elasticity priors (second-order calibration) is not always feasible and, when it is feasible, is achieved in a unique way.

Model

The optimisation program for the fixed-proportion technology and linear cost model for I crops and K constraints can be written as

$$\max_{\mathbf{x}\geq 0} \quad \sum_i p_i \alpha_i x_i^{\delta_i} - (C_i + \lambda_{2i})\, x_i \quad \text{subject to} \quad \mathbf{Ax} = \mathbf{v} \quad [\boldsymbol{\lambda}].$$

- x_i: acreage of crop i
- p_i: price of crop i
- $\alpha_i \geq 0$: scale parameter of crop i
- $\delta_i \in (0,1)$: returns to scale of crop i
- λ_{2i}: crop-specific adjustment cost
- C_i: per acre cost of activity i
- λ_k: shadow value of constraint k
- $a_{k,i}$: constraint coefficient
- v_k: regional constraint k

The calibration system

The model can be calibrated in two steps. The analyst chooses

1. $\boldsymbol{\delta}$ to replicate the set of prior elasticities $\bar{\boldsymbol{\eta}}$ at the reference allocation,
2. $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}_2$ to replicate the reference allocation $(\bar{\mathbf{q}}, \bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})$.

Applying the implicit function theorem, the calibration system for the supply elasticities is

$$\bar{\boldsymbol{\eta}} = \text{VecDiag}\left[\mathbf{D}(\mathbf{I}_I - \mathbf{A}^T(\mathbf{A}\boldsymbol{\Delta}\mathbf{A}^T)^{-1}\mathbf{A}\boldsymbol{\Delta})\right]$$

where VecDiag creates an I-vector out of the diagonal elements of an $I \times I$ matrix, $\boldsymbol{\Delta}$ and $\mathbf{D}$ are the $I \times I$ diagonal matrices with typical element $\frac{b_i}{\delta_i(1-\delta_i)}$ and $\frac{\delta_i}{1-\delta_i}$, respectively, with $b_i \equiv \frac{\bar{x}_i^2}{p_i \bar{q}_i}$.

The calibration conditions

- We show that the calibration system has at most one solution in $(0,1)^I$.
- We derive the necessary and sufficient conditions under which an *acceptable* solution exists. The calibration criterion $\mathcal{C}^K$ for a model subject to K constraints delineates the set of supply elasticities $\bar{\boldsymbol{\eta}}$ that are compatible with the reference allocation, i.e., such that the model exactly reproduces the prior $\bar{\boldsymbol{\eta}}$ and the reference allocation $(\bar{\mathbf{q}}, \bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})$.
- Calibration criterion $\mathcal{C}^1$:

$$\forall i \quad B_i \bar{\eta}_i < \sum_{j \in I_{-i}} B_j \bar{\eta}_j \left(1 + \frac{1}{\bar{\eta}_j}\right)^2$$

with $B_i \equiv a_{1,i}^2 b_i$.

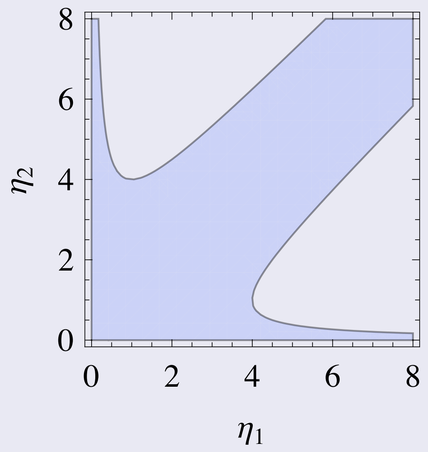

Figure: Calibration region for $I = 2$ and $K = 1$

- Calibration criterion $\mathcal{C}^2$:

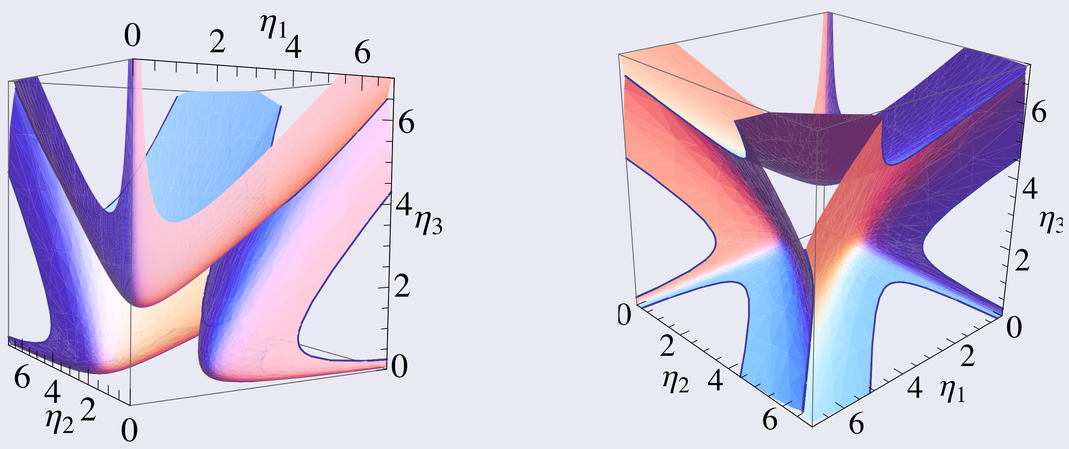

Figure: Calibration region for $I = 3$ and $K = 2$

Using the calibration criterion to allow for calibration

When the prior $\bar{\boldsymbol{\eta}}$ lies outside the calibration region, the analyst can modify the elasticity vector to satisfy the calibration criterion, while minimising the departure from $\bar{\boldsymbol{\eta}}$, e.g., using generalised maximum entropy (GME).

Application to the data published in Howitt (1995): California region with $I = 3$ crops, $K = 2$ constraints and $\bar{\boldsymbol{\eta}} = (\bar{\eta}_{Cotton}, \bar{\eta}_{Wheat}, \bar{\eta}_{Rice}) = (0.47, 0.40, 0.80)$.

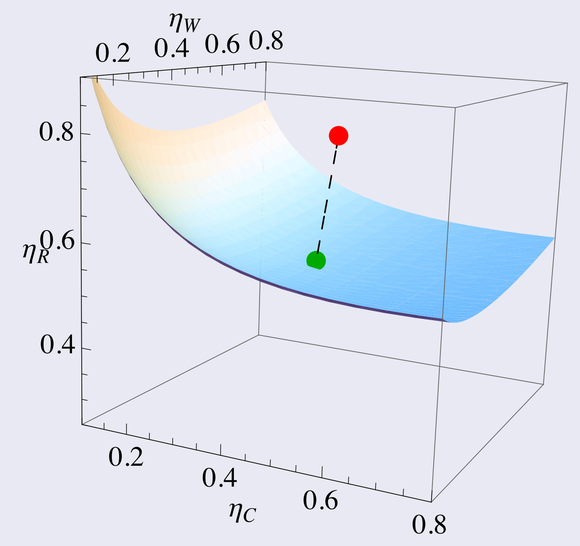

Figure: GME projection of $\bar{\boldsymbol{\eta}}$ into the calibration region

- The criterion $\mathcal{C}^2$ is violated for rice, the prior lying outside the calibration region (red dot).
- Solving a GME program with equidistant supports for each crop, we find $\boldsymbol{\eta}^{GME} = (0.43, 0.42, 0.55)$, lying within the calibration region (green dot).

A tool to disaggregate $\bar{\boldsymbol{\eta}}$ to the regional level

Even when the prior is compatible with the reference allocation, we can use the calibration criterion $\mathcal{C}^1$ to generate regional variation in supply elasticities by maximizing the minimum distance of the regional elasticity to the calibration surface across regions, with the weighted sum of the regional elasticities replicating the prior $\bar{\boldsymbol{\eta}}$.

Application to 3 regions of the California SWAP model: $I = 16$ crops and $K = 1$ constraint.

	Regional output shares			Prior	Regional elasticities		
Crop i	w_1	w_2	w_3	$\bar{\eta}$	$\hat{\eta}_1$	$\hat{\eta}_2$	$\hat{\eta}_3$
Almond, pistachio	0.37	0.22	0.41	0.19	0.14	0.10	0.29
Alfalfa	0.32	0.45	0.22	0.44	0.66	0.22	0.57
Corn	0.32	0.31	0.38	0.21	0.19	0.11	0.32
Cotton	0	0.74	0.26	0.50	-	0.41	0.75
Cucurbitaceae	0.11	0.89	0	0.05	0.08	0.05	-
Dry bean	0.40	0.48	0.13	0.13	0.20	0.07	0.17
Onion, garlic	0	1	0	0.11	-	0.11	-
Other dec. tree cr.	0.86	0.09	0.05	0.19	0.20	0.10	0.29
Other field cr.	0.10	0.58	0.33	0.63	0.95	0.40	0.95
Other truck cr.	0.22	0.52	0.26	0.11	0.17	0.06	0.17
Pasture	0.81	0.14	0.05	0.24	0.25	0.12	0.36
Process. tomato	0	0.67	0.33	0.55	-	0.42	0.83
Rice	0.02	0.93	0.06	0.48	0.72	0.46	0.72
Safflower	0	0.79	0.21	0.34	-	0.30	0.51
Subtrop. cr.	0.93	0.03	0.04	0.03	0.03	0.02	0.05
Vine	0	0.49	0.51	0.05	-	0.03	0.07

The regional variation in economic information can be exploited to generate regional variation in supply responses, even in the absence of region-specific elasticity priors.

Conclusion

- Calibration against supply elasticity priors yields a unique solution, offering a clear advantage to simply using first-order calibration.
- The calibration conditions allow the analyst to *ex ante* determine whether a prior $\bar{\boldsymbol{\eta}}$ is compatible with the reference allocation $(\bar{\mathbf{q}}, \bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})$.
- When the prior lies outside the calibration region, the analyst can use the calibration criterion to modify the elasticity vector in order to allow calibration, while minimising the departure from the prior.
- The calibration criterion provides an information-based tool to disaggregate supply elasticities to the regional level—whether the prior satisfies the criterion or not.