Another Note on the Use of a Logarithmic Time Trend

Luan Ngoc Ho-Trieu and Jim Tucker*

In Watts and Quiggin (1984) (WQ hereafter) the invariance condition for time trend transformations in a linear regression model is satisfied when the coefficients for non-trend variables remain unchanged after a change in start date. The invariance condition is given as:

(2) $f(t+\delta) = vf(t) + w$ (WQ's numbering)

and the general solution given by WQ for the difference equation (2) is:

(3) $f(t) = \exp(t(\log a - \log \delta)) + tb/\delta + c$

Equation (3) can be expressed more compactly as:

(3.1) $f(t) = (a/\delta)^t + (b/\delta)t + c$

If the time origin is changed by δ then (3.1) becomes:

(3.2) $f(t+\delta) = (a/\delta)^{\delta} (a/\delta)^{t} + (b/\delta)t + b + c$

Substituting (3.2) into (2) the following conditions hold for identity over all values of t:

(C1) $(a/\delta)^{\delta} = v$

(C2) $(b/\delta) = v(b/\delta)$

(C3) $b+c = vc + w$

Conditions (C1) to (C3) lead to two possible solutions: (i) $v=1$, $b=w$ and $a=\delta\neq 0$ while c can take any value and (ii) $v\neq 1$, $b=0$ and $c=w/(1-v)$. The first implies that the transformation f(t) given by (3) in WQ is reduced to the following linear transformation:

(3.3) $f(t) = (b/\delta)\, t + c + 1$

While the second gives the exponential transformation:

(3.4) $f(t) = (a/\delta)^t + w/(1-v)$

The mixtures of linear and exponential transformations implicit in (3) are not solutions of (2). Hence, (3) is in one sense too general. It may lead modellers to the use of tests designed for nested hypotheses while the hypotheses are, in fact, not nested. Both the competing time trends satisfy condition (2) and tests for non-nested hypotheses such as those proposed by Davidson and MacKinnon (1981) can be used to discriminate them. In another respect, however, solution (3) is not general enough.

When δ is the order of the linear difference equation (2), the general solution is in fact:

* Australian Bureau of Agricultural and Resource Economics, Canberra

$$v^{t/\delta} \sum_{k=1}^{\delta} (A_k \cos \frac{2k\pi}{\delta} t + B_k \sin \frac{2k\pi}{\delta} t) + \frac{w}{1-v}$$

for $v > 0$ but $v \neq 1$

$$(4) \qquad f(t) = \quad |v|^{t/\delta} \sum_{k=1}^{\delta} (A_k \cos \frac{(2k+1)\pi}{\delta} t + B_k \sin \frac{(2k+1)\pi}{\delta} t) + \frac{w}{1-v}$$

for $v < 0$

$$(w/\delta)\, t + \sum_{k=1}^{\delta} (A_k \cos \frac{2k\pi}{\delta} t + B_k \sin \frac{2k\pi}{\delta} t)$$

for $v = 1$

Again, functions which take the form (4) are consistent with the invariance condition (2). Importantly, equation (4) is exponential when δ=1 and $v \neq 1$, linear when δ=1 and v=1, and cannot apparently be represented as a linear combination of linear and exponential functions.

Although WQ's time trend (3.1) can take an arbitrary change in the start date without changing the coefficients for non-trend variables in a regression model, the above analysis shows that it cannot be considered as a general solution of the invariance condition (2) which is a non-homogenous linear difference equation having the analytical solution (4). Only under the very restrictive condition that δ=1, is equation (3.1) found to be a linear combination of two solutions of equation (2). These two solutions are obtained from two separate cases, depending on whether v is hypothesised to be equal to unity or not. This result has an important implication on the choice of testing procedure to select the correct time trend (either linear or exponential in this case) under the invariance condition (2) and the very restrictive condition that δ=1.

References

DAVIDSON, R. and MACKINNON, J.G. (1981), 'Several tests for model specification in the presence of alternative hypothesis', *Econometrica* 49(3), 781-93

WATTS, G. and QUIGGIN, J. (1984), 'A note on the use of a logarithmic time trend', *Review of Marketing and Agricultural Economics* 52(2), 91-99.