Supply Response and the Theory of Production and Profit Functions

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1. Introduction

The majority of production systems of interest to agricultural economists are those in which there are multiple inputs and outputs. To effectively model such systems it is often useful to take account of the theoretical restrictions provided by production theory. The aim in this paper is to present a short review of that part of the production theory which is relevant to the estimation of supply response elasticities. The theory of production presented here is based on the behaviour of a multiple input, multiple output firm facing given prices and a given production technology. The two fundamental approaches used in studying production decisions, the production function (primal) approach and the profit function (dual) approach are reviewed.

Under appropriate regularity conditions, with the assumption of profit maximisation, the production function and the profit function are dual, that is, both functions contain the same essential information on a production technology. The regularity properties which are required for duality to hold between production and profit functions are discussed. In addition, some common characteristics of production technologies such as homogeneity, homotheticity, separability and jointness are described and their implications for econometric modelling are outlined. Production models are often used to measure price elasticities, the bias of technological change and returns to scale. Mention is made of how these properties can be measured. Essentially, standard production theory is static, risk is ignored and profit maximisation is a maintained hypothesis. There is a brief discussion of how production theory has been extended to include risk and dynamics. In most cases the extensions involve the assumption that producers maximise either the expected present value of profit or the expected utility of profit.

Production theory is documented in several texts, for example, Lau (1978a), McFadden (1978), Varian (1978) and Nadiri (1982). It is these texts which provide the basis for much of the following discussion.

2. Properties of a Production Technology Set

A production technology consists of the alternative methods of transforming factors of production (inputs) into goods and services (outputs). The technological limits of a production technology can be described by a production transformation set which contains all the feasible input-output combinations. The boundary of a production transformation set is not only determined by the state of technological knowledge and physical laws but also by the climate and legal restrictions. The boundary of a production transformation set may be represented by:

(1) $F(Y, X; Z) = 0,$

where

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$Y = Y_1,...,Y_m$ = vector of outputs
$X = X_{m+1},...,X_n$ = vector of variable inputs, and
$Z = Z_{n+1},...,Z_p$ = vector of quasi-fixed inputs.

Variable inputs are those inputs that adjust fully to their profit maximising levels within one sample period. Quasi-fixed inputs are those inputs which do not necessarily adjust fully within one sample period, instead they are allowed to adjust partially.

With the loss of little generality the following three regularity properties can be assumed to hold for a production transformation set, F:

Property (1) F is a non-empty and closed set. This property is required so that there are no thresholds at which discontinuities in inputs or outputs occur. A set is closed if it contains its boundaries. The property of being closed does not rule out the possibility of lumpy commodities. The properties of being closed and non-empty are both weak mathematical regularity properties that cannot be contradicted by empirical data.

Property (2) F is bounded. The property of being bounded ensures that a bounded and attainable solution exists for all feasible input levels. If this property holds then production plans at large scale levels are irreversible in the sense that it is not feasible to swap the role of inputs and outputs and produce the inputs. Non-reversibility will hold if labour cannot be produced and all non-zero production plans require some labour input (McFadden 1978, p.62). Hence, the property of being bounded can be expected to hold for most production technologies.

Property (3) F is strongly continuous. This property is required so that a production technology shrinks and expands regularly without breakthroughs.

There are two further properties which are often assumed to hold for F:

Property (4) F is a convex set. Geometrically, this is equivalent to the condition that the chord between any two points on the boundary of a production transformation set lies on or below the boundary itself. The property of convexity holds if a production technology is subject to diminishing marginal rates of transformation of outputs for inputs, increasing marginal rates of substitution of outputs for outputs and diminishing marginal rates of substitution of inputs for inputs (Diewert 1973, p.286).

Property (5) There is free disposability of inputs. That is, if there is an input bundle x which can produce an output bundle y and a second input bundle $\tilde{x}$ which is at least as large as x in every component, then $\tilde{x}$ can also produce y. This property is sometimes termed monotonicity. The free disposal property will hold if producers can stockpile or refuse delivery of inputs, or if the production technology is such that the application of an additional unit of input always yields some non-negative amount of additional output and outputs can be disposed of freely if necessary (McFadden 1978, p.8).

For rigorous mathematical definitions of the five properties for F discussed above see Takayama (1974).

By assuming that a producer seeks to maximise variable profits, that is, revenue minus variable costs, and that a production technology can be described by equation (1), then the profit maximisation problem can be expressed as:

$$(2) \qquad \pi(P, R; Z) = \max P'Y - R'X$$

$$\text{s.t. } F(Y, X; Z) \leq 0,$$

where

$P = P_1,...,P_m$ = vector of output prices, and

$R = R_{m+1},...,R_n$ = vector of variable input prices.

For a producer who faces a production technology, F, which satisfies only properties 1 and 2, McFadden (1978, p.67) established that the following properties on the profit function, π, will hold:

Property (6) For every positive fixed input, π is a positively linearly homogeneous, convex and closed function in both output and variable input prices.

Property (7) For every positive fixed input, π is a continuous function in both output and variable input prices.

If properties 1, 2 and 3 hold for F, then a further property will hold for π as follows (McFadden 1978, p.73):

Property (8) π will be continuous jointly for all output and input prices and for all fixed inputs.

An additional property that will hold for π is:

Property (9) π will be non-decreasing in output prices and non-increasing in input prices. That is, π is monotonic in prices. This property is obvious, since increasing every input price cannot increase maximum profit (Varian 1978, pp.28-30).

McFadden (1978) described equation (2) as a restricted profit function. He described a profit function as a special case of the restricted profit function with no fixed inputs. Diewert (1974) described equation (2) as a variable profit function. For simplicity throughout this review equation (2) will be described as a profit function rather than a restricted profit function or a variable profit function. Profit functions without fixed inputs are of little interest in agriculture.

Cost and revenue functions can also be used to study production decisions and these functions may be considered as special cases of the profit function. The cost minimisation problem can be expressed as:

$$(3) \qquad C(Y, R; Z) = \min R'X$$

$$\text{s.t. } F(X; Y, Z) \leq 0.$$

The revenue maximisation problem can be expressed as:

$$(4) \qquad REV(X, P; Z) = \max P'Y$$

$$\text{s.t. } F(Y; X, Z) \leq 0.$$

The cost function describes the minimum cost which is required to produce output Y with input prices R and given fixed inputs Z. The revenue function describes the maximum revenue which can be obtained with output prices P and employing variable inputs X and fixed inputs Z.

The convexity of π in prices is a consequence of profit maximisation and does not depend on whether the convexity property holds for F. The importance of properties 4 and 5--that is, convexity and monotonicity--lies in their analytical convenience rather than their economic realism. The main purpose of these two properties is that they provide the groundwork for the application of calculus tools to the profit maximisation problem. Convexity and monotonicity are often assumed to hold for F because of the argument that the economic behaviour implied by profit maximisation would always be consistent with these properties being true for F. This result is explained in greater detail in the next section.

3. Duality

In essence duality means that the profit function or the production transformation set can be used to describe the production technology of a firm equally well, provided that both π and F satisfy certain regularity properties. McFadden (1978, p.81) proved the duality between profit functions and production transformation sets using the mathematical theory of convex conjugate functions. Other duality proofs can be found in Gorman (1968), Diewert (1974), Jorgensen and Lau (1974) and Lau (1978a). Duality has also been proved to hold between cost functions and production transformation sets as well as between revenue functions and production transformation sets provided that each satisfies certain regularity properties (Shephard 1953, Diewert 1974, McFadden 1978). For a review of duality theory and its application to both production and consumer theory see Diewert (1982).

McFadden's duality proof can be used to establish that a production transformation set satisfying properties 1 and 2 will yield a profit function satisfying properties 6 and 7. Furthermore, a profit function satisfying properties 6 and 7 will yield a production transformation set satisfying properties 1, 2, 4 and 5. It follows that the profit function, and the output supply and input demand functions which can be derived from the profit function, can be treated as if they come from a production technology which satisfies the properties of convexity and monotonicity even if these properties fail to hold for the true production technology. Many researchers have assumed that convexity and monotonicity hold for the production transformation set because these properties follow as a result of profit maximisation.

In short, if the original production technology is convex and monotonic, then the production technology constructed from it using duality theory will be identical to the original production technology. Alternatively, if the original production technology is non-convex or non-monotonic then the constructed production technology will be a convex and monotonic version of the original production technology. However, both the original and constructed production technologies will have the same profit function.

4. Output Supply and Input Demand

A major advantage of duality theory is that a system of output supply and input demand equations, which are consistent with the profit maximising behaviour of a firm, can be easily derived. They are derived by differentiating the profit function with respect to prices. This property is usually known as Hotelling's Lemma (Hotelling 1932). However, if the primal approach is used to study production decisions, then the derivation of the output supply and input demand equations is more complex because it involves solving a constrained maximisation problem.

Using Hotelling's Lemma the following will hold:

(5) $Y_i (P, R; Z) = \partial\pi (P, R; Z)/\partial P_i \ , \ \forall \ i = 1,\ldots,m,$

and,

(6) $X_j (P, R; Z) = -\partial\pi (P, R; Z)/\partial R_j \ , \ \forall \ j = m+1,\ldots,n.$

Furthermore, it is easy to derive the following fundamental propositions of neoclassical profit-maximising behaviour. These propositions are a result of assuming profit maximisation and can be obtained without assuming convexity and free disposability for F:

<u>Proposition (1)</u> The output supply functions slope upward because:

(7) $\partial Y_i(P, R; Z)/\partial P_i = \partial/\partial P_i [\partial\pi (P,R;Z)/\partial P_i] = \partial^2\pi(P, R; Z)/\partial P_i^2,$

which is non-negative since it is a convex function.

<u>Proposition (2)</u> The input demand functions slope downward because:

(8) $\partial X_j (P, R; Z)/\partial R_j = \partial/\partial R_j[-\partial\pi (P ,R; Z)/\partial R_j] = -\partial^2\pi (P, R; Z)/\partial R_j^2,$

which is non-positive since π is a convex function.

<u>Proposition (3)</u> The cross price effects are symmetric because:

(9) $\partial Y_j (P, R; Z)/\partial P_i = \partial/\partial P_i [\partial\pi (P, R; Z)/\partial P_j] = \partial/\partial P_j [\partial\pi (P, R; Z)/\partial P_i]$

$= \partial Y_j (P, R; Z)/\partial P_j.$

This property holds because of Young's Theorem which states that cross partial differentials are identical for functions which have continuous second derivatives. The same proof holds for:

(10) $\partial X_i (P, R; Z)/\partial R_j = \partial X_j (P, R; Z)/\partial R_i \ .$

<u>Proposition (4)</u> The output supply and input demand equations are homogeneous of degree zero in prices. If a function is homogeneous of degree k, then its first derivative is homogeneous of degree (k-1). Since π is linearly homogeneous in prices, the output supply and input demand functions must be homogeneous of degree zero in prices.

Equivalent properties will also hold if cost and revenue functions are specified. Using Shephard's Lemma (Shephard 1953), the input demand functions are the first order partial derivatives of the cost function with respect to input prices, and output supply functions are the first order partial derivatives of the revenue function with respect to output prices. However, an input demand function derived from a cost function is a Hicksian or constant-output demand function and, similarly, an output supply function derived from a revenue function is a constant-input supply function. The output supply and input demand functions derived from a profit function are termed Marshallian functions and are not input or output constrained. This difference between the Hicksian and Marshallian measures is of importance when estimating production relationships. For example, a price elasticity derived from a revenue function only reflects movements along the iso-product surface--that is, at constant input levels--whereas a price elasticity derived from a profit function allows input levels as well as output combinations to adjust to price changes. Thus, Hicksian price elasticities may be considered as measures which are applicable only to periods of time or industries where there are output or input constraints to adjustment. Price elasticities are discussed in more detail below.

The observed output supply and input demand functions at the industry level are derived from the decisions of individual producers who are assumed to be maximising profits. However, no producer maximises the industry profit. Hence, the problem is whether the theory of production, which is based on a profit maximising individual producer, can be applied at higher levels of aggregation than the individual producer. This problem, frequently referred to as "the aggregation problem", also arises in demand theory. If perfect competition and profit maximisation are assumed and all inputs are variable then the aggregation condition is automatically satisfied (Bliss 1975, pp.68-9). When these conditions are not met, a representative producer who reflects the average behaviour of the population is assumed to exist. However, the conditions for the behaviour of this representative producer to be exactly identical to the behaviour of the population are very stringent (Blackorby and Schworm 1982, F.M. Fisher 1982, Lopez 1985, Antle 1986). Most researchers simply assume that the theory of production and the restrictions derived from that theory hold at higher levels of aggregation and ignore any aggregation errors (for example, Phlips 1983, p.101).

5. Characteristics of a Production Technology Set

There are several characteristics which are not only useful for describing a production technology but also have implications for the modelling of that technology. The characteristics described in this section are homogeneity, homotheticity, separability and jointness.

5.1 Homogeneity

A production technology which is almost homogeneous of degree k in outputs is defined as:

(11) $$F(\lambda^{\kappa}Y, \lambda X; \lambda Z) = 0,$$

where

λ = positive scalar, and

κ = degree of homogeneity.

In other words, if all the inputs are increased by l and, as a result, all outputs increase by λ^{κ}, then the production technology is almost homogeneous of degree λ^{κ}. Almost homogeneity is a generalisation of the standard homogeneity property to accommodate fixed inputs (Aczel 1966). There is no reason that the scale effects should be uniform between outputs and so a generalisation of the almost homogeneity condition given in equation (11) which allows different scale effects for each output would be:

(12) $$F(\lambda^{\kappa 1}Y_1, \lambda^{\kappa 2}Y_2, \ldots, \lambda^{\kappa m}Y_m, \lambda X; \lambda Z) = 0,$$

where

$\kappa_1, \kappa_2, \ldots, \kappa_m$ = degree of homogeneity for each output.

When a production technology is almost homogeneous of degree λ^{κ} in outputs, the expansion path, which joins the least cost combination points, is a straight line through the origin.

5.2 Homotheticity

Homotheticity was introduced by Shephard (1953) to describe a special type of non-homogeneous shift in isoproduct surfaces. A production technology is almost homothetic if it can be written as:

$$(13) \qquad F[G(Y, X; Z), X; Z)] = 0,$$

where F is monotonic in G, and G is homogeneous degree one in Y.

Almost homogeneity in outputs requires that a proportional change in inputs, at all input levels, results in an identical shift of the isoproduct surface if all outputs are changed proportionately. Homotheticity generalises this condition such that the shift of the isoproduct surface varies with the initial level of inputs. Although the expansion paths are still straight lines through the origin, homotheticity is more general than homogeneity. In fact, every homogeneous function is homothetic, but a homothetic function is not necessarily homogeneous.

A production technology will be homothetic in a subset of its arguments if it has strictly non-zero first partial derivatives and if, and only if, the ratio of each possible pair of partial derivatives with respect to the elements in that subset is a homogeneous function of degree zero in the elements of that subset (Lau 1978a, p.153). Thus, in order for F to be homothetic in a subset N of outputs, using Euler's theorem the following must hold:

$$(14) \qquad \sum_{\forall k \varepsilon N} [\partial(F^i/F^j)/\partial Y_k]Y_k = 0 \quad \forall\ i,j,k\ \varepsilon\ N,$$

where

$F^i = \partial F/\partial Y_i$, and
$F^j = \partial F/\partial Y_j$.

Similar conditions will hold for F to be homothetic in a subset of variable inputs or a subset of fixed inputs.

5.3 Separability

The concept of separability was introduced independently by Leontief (1947) and Sono (1961) and the term is due to Strotz (1957, 1959) and Gorman (1959). If outputs and inputs are partitioned into three subsets $N_1 = (Y_1,...,Y_m)$, $N_2 = (X_{m+1},..., X_n)$ and $N_3 = (Z_{n+1},..., Z_p)$ called partition Q, then a production technology will be weakly separable in partition Q if it can be written as:

$$(15) \qquad F(h_1(Y_1,...,Y_m), h_2(X_{m+1},..., X_n); h_3(Z_{n+1},..., Z_p)) = 0,$$

where h_1, h_2, h_3 are aggregator functions.

A production technology will be strongly separable in partition Q if it can be written as:

$$(16) \qquad F(h_1(Y_1,...,Y_m) + h_2(X_{m+1},..., X_n) + h_3(Z_{n+1},..., Z_p)) = 0.$$

The number of subsets is not restricted to three. The maximum number of subsets is the total number of outputs and inputs in the production technology set minus one. A common assumption in production studies is that inputs and outputs are strongly separable, or in other words, input and output mixes are independent of each other. This restriction is implicitly imposed in all studies that use an aggregate output measure in a multiple output industry. If inputs and outputs are strongly separable then the production technology can be written as:

(17) $h_4(Y) - h_5(X; Z) = 0.$

For a production technology to be weakly separable with respect to any partition Q, the marginal rate of substitution between all pairs of products in any subset N_i must be independent of all other products not in N_i. For example, if a subset of outputs, N_i, are to be weakly separable then:

(18) $\partial(F^i/F^j)/\partial Y_k = 0, \quad \forall\ i, j \in N_i, \quad \forall\ k \notin N_i,$

$\partial(F^i/F^j)/\partial X_k = 0, \quad \forall\ i, j \in N_i, \quad \forall\ k=m+1,\dots,n$ and

$\partial(F^i/F^j)/\partial Z_k = 0, \quad \forall\ i, j \in N_i, \quad \forall\ k=n+1,\dots,p.$

For a production technology to be strongly separable with respect to any partition Q, the marginal rate of substitution between all pairs of products in any two subsets, N_i and N_j, must be independent of all other products not in N_i and N_j. For example, if two subsets of outputs, N_i and N_j, are to be strongly separable then:

(19) $\partial(F^i/F^j)/\partial Y_k = 0, \quad \forall\ i \in N_i, \quad \forall\ j \in N_j, \quad \forall\ k \notin N_i, N_j,$

$\partial(F^i/F^j)/\partial X_k = 0, \quad \forall\ i \in N_i, \quad \forall\ j \in N_j, \quad \forall\ k=m+1,\dots,n$ and,

$\partial(F^i/F^j)/\partial Z_k = 0, \quad \forall\ i \in N_i, \quad \forall\ j \in N_j, \quad \forall\ k=n+1,\dots,p.$

Strong separability implies weak separability, but weak separability only implies strong separability when there are two subsets. Weak separability is a necessary but not sufficient condition for two stage optimisation or consistent aggregation. A sufficient condition is weak homothetic separability. However, when a production technology is assumed to be linearly homogeneous, as is commonly done, the conditions for weak separability and weak homothetic separability are the same. In this case separability can be used as the rationalisation to aggregate outputs or inputs.

5.4 Homothetic separability

In most economic analyses, highly aggregated data are used and hence a certain level of consistent aggregation is always implicitly assumed. Despite its importance, the assumption of consistent aggregation is rarely tested. Consistent aggregation occurs when the quantity and the price of an aggregate index is proportional to the sum of the price-quantity products of the elements in the aggregate index. Consistent aggregation allows simplification in economic analysis because it allows decisions to be made in two stages. For example, a producer may choose the optimal combination of groups of outputs and inputs. Then a producer may choose combinations of outputs and inputs within each group. If consistent aggregation holds then quantities of items within an aggregated group do not depend on the quantities of items outside that aggregated group. Only the quantity of the entire aggregate is a function of quantities outside the group. Consistent aggregation is possible if one of the three following conditions hold: (a) quantity ratios of all items outside the group move in fixed proportion (Leontief aggregation); (b) price ratios of all items outside the group move in fixed proportions (Hicksian aggregation); and (c) quantities of all items within the group are weak homothetic separable from all items outside the group.

Since prices or quantities rarely move in fixed proportions, the possibility of consistent aggregation, and simplification of production decisions, usually depends on whether the weak homothetic separability condition holds. A function is weak homothetic separable in a subset if it is both weakly separable and homothetic in that subset.

The property of consistent aggregation and its sufficiency condition, weak homothetic separability, has implications for model specification. A researcher should specify a large set of items to be included in a model (low degree of aggregation) and then test for the existence of conditions for further aggregation. However, there is a trade-off between econometric feasibility and theoretical appeal when specifying this low level of aggregation. Usually multicollinearity problems and data availability restrict the number of products that can be used in practice. In the absence of evidence concerning the separability properties of the original items a researcher must rely on intuition and previous empirical evidence. Unfortunately, there is little empirical evidence concerning the separability properties of production technologies. There are two main reasons for this. First, many of the commonly used functional forms, such as the Cobb-Douglas, cannot be used to test for separability because separability is a maintained hypothesis. Second, when flexible functional forms are specified there is no global algebraic test which can be applied that does not also impose strong restrictions on the production technology.

5.5 Jointness

When an output is produced by a production technology which is joint in input quantities, decisions about its production are dependent on decisions about other outputs. Alternatively, if a production technology is non-joint in input quantities then the supply of each output can be examined without regard to other outputs. In this case, a production technology may be described by a set of independent production functions, that is:

(20) $$F_i(Y_i, X_{ij}; Z_{ik}) = 0, \quad \forall \; i = 1,\dots,m, \quad \forall \; j=m+1,\dots,n, \quad \forall \; k = n+1,\dots,p,$$

where

X_{ij} = amount of variable input X_j allocated to output Y_i; and

Z_{ik} = amount of fixed input Z_k allocated to output Y_i.

The total use of inputs is given by:

(21) $$X_j = \sum_{i=1}^{m} X_{ij}, \quad \forall \; j = m+1,\dots,n \text{ and}$$

$$Z_k = \sum_{i=1}^{m} Z_{ik}, \quad \forall \; k = n+1,\dots,p.$$

Hall (1973) and Lau (1978a, pp.186-187) showed that, in general, a production technology cannot be both separable and non-joint in input quantities in the same products. This result only fails to hold in the case where each production function is identical except for pure scale effects (Hall 1973, p.891). Usually the "input quantities" ending is dropped and non-jointness in input quantities is called simply non-jointness.

Another type of non-jointness is non-jointness in output quantities. This will arise where there is one input which is used to produce several outputs. This case is not of much interest in agriculture where the use of multiple inputs is virtually the rule. Kohli (1983) introduced a further two types of non-jointness: non-jointness in input prices; and non-jointness in output prices. If a production technology is non-joint in input prices then it can be written as:

(22) $F_i\,(Y_i, X_j; Z_k) = 0, \;\; \forall \;\; i = 1,...,m, \;\; \forall \;\; j = m+1,...,n, \;\; \forall \;\; k = n+1,...,p.$

The difference between non-jointness in input quantities and non-jointness in input prices is that in the first case outputs depend on only a proportion of all inputs, whereas in the second case outputs depend on the entire amount of all inputs that are available.

Shumway, Pope and Nash (1984) showed that, if an allocatable fixed input such as land is the only thing giving rise to joint production, then the dual approach cannot be used to yield allocation equations for the variable inputs or the fixed but allocatable inputs. Only total input demand equations can be identified. However, the primal approach can be used to derive the allocation equations. The implication of this result is that if the allocation equations are of interest to a researcher then the dual approach is not of much use.

6. Duality and the Characteristics of a Production Technology Set

It follows from duality principles that the production characteristics which have been discussed in the previous section will have equivalent implications for the profit function. Lau (1978a) derived some of these implications using the duality relationship between the normalised profit function and the production function. There is a one-to-one correspondence between the normalised profit function and the profit function and a one-to-one correspondence between the production transformation set and the production function. Lau used the normalised profit function and the production function in his proofs because they were more convenient to work with.

Lau's regularity conditions for duality between the normalised profit function and the production function are less general than McFadden's (1978) regularity conditions. Lau assumed that both the production function and the normalised profit function have the following properties: (a) finite and non-negative real values; (b) continuity; (c) smoothness; (d) monotonicity; (e) the production function is concave and the normalised profit function is convex; (f) twice differentiable; and (g) bounded. Under these assumptions Lau proved, using the Legendre transformation, that the production function and the normalised profit function are dual. By extending Lau's results, Weaver (1977, pp.228-231) proved the following properties for a multiple output production technology with fixed inputs:

Property (10) F is uniformly almost homogeneous of degree $k, (k \neq 1)$ in outputs if, and only if, the profit function is homogeneous of degree $1/(1\text{-}k)$ in output prices and fixed factors. Where $k = 1$, the profit function is homogeneous of degree one in fixed inputs.

Property (11) F is homogeneous of degree k in variable inputs if, and only if, the profit function is homogeneous of degree $1/(1\text{-}k)$ in output prices.

Property (12) The profit function is homogeneous of degree $1/(1\text{-}k)$ in output prices, if and only if, it is homogeneous of degree $\text{-}k/(1\text{-}k)$ in input prices.

Property (13) F is homothetically separable in a group of products if, and only if, the associated profit function is homothetically separable in that group's prices.

Property (14) F is non-joint in input quantities if, and only if, all mixed partial derivatives of the profit function with respect to prices are zero. Thus for output Y_i to be non-joint then:

(23) $\partial^2\pi/\partial P_i \partial P_j = 0, \quad \forall \quad j=1,\dots,m, \quad i \neq j.$

7. Applications of Profit Functions

There are a number of structural properties of a production technology including elasticities of substitution, price elasticities of supply and demand, the bias of technological change and returns to scale that are of interest when studying production systems. These structural properties are discussed below.

7.1 Substitutability of inputs and outputs

The ease with which inputs and outputs can be substituted between one another is always of interest when studying production decisions. One measure of the ease with which inputs may be substituted in the production process is the Allen partial elasticity of substitution (Allen 1938). The Allen partial elasticity of substitution provides a measure of the change in a firm's demand for input i given a change in the price of input j, when output is held fixed. This measure is defined as:

(24) $\sigma_{ij} = [\sum_{k=m+1}^{n} X_k F^k][X_i X_j].H^{ij}/H, \quad \forall \quad i,j = m+1,\dots,n,$

where

σ_{ij} = Allen partial elasticity of substitution;

H = bordered Hessian matrix of cross partial derivatives; and

$H^{ij} = (ij)^{th}$ cofactor of H.

Berndt and Christensen (1973a) established that the weak separability conditions given in (18) are equivalent to certain equality restrictions on the Allen partial elasticities of substitution. They showed that if a production technology is weakly separable with respect to any partition Q then:

(25) $\sigma_{ik} = \sigma_{jk}, \quad \forall \quad i, j \in N_i, \quad \forall \quad k \notin N_i$

Blackorby and Russell (1976), using duality theory and results on functional structure, derived the necessary and sufficient conditions for the equality restrictions on the Allen partial elasticity of substitution to hold. The equality condition only implies separability and not homothetic separability. Intuitively this is because the equality condition yields information about the curvature property of each iso-product surface but not about relationships, such as homotheticity, between each iso-product surface (Blackorby, Primont and Russell 1978, p.266). Thus, for the equality condition given in (25) or the equivalent weak separability restriction to be used as a test for consistent aggregation, homotheticity has to be a maintained hypothesis. An example of this can be found in Berndt and Christensen (1974) who specified a constant returns to scale production function (thus maintaining homotheticity) to test for an aggregate index for labour inputs in the United States manufacturing sector. Their test for an aggregate labour input index depended critically on the assumption of constant returns to scale in the production function, that is homotheticity.

Diewert (1974, p.144) generalised Allen's elasticity of substitution measure to the multiple output, multiple input case and defined the measure in terms of the derivatives of the profit function. The elasticity of transformation can be defined as:

(26) $\theta_{ij} = \pi.\pi^{ij}/\pi^{i}.\pi^{j}, \quad \forall \quad i, j=1,\ldots,n,$

where

θ_{ij} = elasticity of transformation;
$\pi^{i} = \partial\pi/\partial P_i$;
$\pi^{j} = \partial\pi/\partial P_j$; and
$\pi^{ij} = \partial^2\pi/\partial P_i \partial P_j$.

The elasticity of transformation provides a measure of the responsiveness of outputs and variable inputs to changes in the prices of outputs and variable inputs. The elasticity is symmetric, that is $\Theta_{ij} = \Theta_{ji}$. Diewert (1974, pp.144-5) also extended the elasticity of substitution measure to provide substitution measures between outputs and fixed inputs, variable inputs and fixed inputs and between different fixed inputs.

An alternative elasticity measure is the price elasticity of supply (or demand). The price elasticity of supply (or demand) measures the change in quantity of product i given a change in price j, allowing all outputs and inputs to adjust optimally. This elasticity can be defined as:

(27) $\eta_{ih} = \partial Y_i/\partial P_h \,.\, P_h/Y_i, \quad \forall \quad i, h=1,\ldots,m$ and

$\eta_{jk} = \partial X_j/\partial R_k \,.\, R_k/X_j\,, \quad \forall \quad j, k=m+1,\ldots,n.$

where

η_{ih} = price elasticity of supply for output i with respect to price h; and
η_{jk} = price elasticity of demand for input j with respect to price k.

It can be shown that the following is true (Kohli 1978):

(28) $\theta_{ij} = \eta_{ij}/S_i, \quad \forall \quad i, j=1,\ldots,n,$

where

S_i = the share of product i in profits that is, P_iY_i/π in the case of outputs and R_iX_i/π in the case of variable inputs.

When there are more than two outputs or inputs the elasticity of substitution measure has no straightforward economic interpretation, whereas the price elasticities of output supply and input demand do. In fact, an elasticity of substitution measure is insufficient as a description of a production technology (Lau 1976).

The homogeneity condition can be used to prove that:

(29) $\sum_{h=1}^{m}\eta_{ih} - \sum_{h=m+1}^{n}\eta_{ih} = 0, \quad \forall \quad i = 1,\ldots,m$ and

$$\sum_{k=1}^{m}\eta_{jk} - \sum_{k=m+1}^{n}\eta_{jk} = 0, \quad \forall \quad j = m+1,\ldots,n.$$

In the multiple output, multiple input case the signs on the price elasticities derived from a profit function do not represent the curvature of the iso-product surface. They represent the change in quantity of an output or input given a particular price change holding everything else constant, but allowing choices of outputs and inputs to adjust optimally. The final change in output supplied or input demanded is a result of two effects; the substitution and expansion effects. Lopez (1984) reported a method by which the price elasticities derived from a profit function can be separated into the substitution and expansion effects. On the basis of the sign of the substitution effect conclusions can be drawn about whether outputs or inputs are substitutes or complements.

7.2 Technical change and returns to scale

Technical change deals with the consequences of the adoption of new techniques for a production technology. New techniques can have either a neutral effect on a production technology or they can change the input-output relationships. Several definitions of technical change have been proposed with the most popular being the Hicks form. Technical change is said to be Hicks neutral if the marginal rate of substitution between each pair of products is independent of technical change. In other words, when technical change homothetically shifts an iso-product surface the change is Hicks neutral. If technical change skews an iso-product surface then technical change will be either biased in favour of, or against, output i relative to output j. Thus, according to the definition of weak separability given previously, the definition of Hicks neutrality is equivalent to requiring that the outputs and inputs form a weakly separable group.

Returns to scale are usually defined as the relative increase in output resulting from a proportional increase in all inputs. Hanoch (1975) demonstrated that it is more relevant to measure returns to scale along the expansion path of a firm rather than along a ray through the origin. These two paths will only coincide if the production technology is homothetic.

The assumption of constant returns to scale has been imposed widely in many studies of production decisions without much justification. A standard argument in favour of the constant returns to scale assumption is that a producer can always duplicate what is presently being done. In other words, if a producer duplicates all the inputs presently being used, total output should double. This argument is not very convincing because the reverse must be true as well if constant returns to scale hold--that is, a reduction in the size of a firm by half has to halve the output as well. Furthermore, in practice there will always be some sort of advantage or disadvantage in areas such as transport or management which will result in either an economy or diseconomy of scale when production is duplicated. If constant returns to scale holds then under the conventional production theory there is no determinate solution to the profit maximisation problem because a producer can always increase profits by increasing output. However, it has been shown that the introduction of risk into the standard theory can lead to a determinate solution for the perfectly competitive firm under constant returns to scale (Arrow 1978; Quiggin 1982).

8. Extensions to the Theory of Production

In the preceding discussion a production theory has been described where the hypotheses that producers know all prices and all production responses with certainty and that all adjustments of outputs and variable inputs to price changes occur within one time period have been maintained. Approaches to modelling production decisions when these assumptions are relaxed are surveyed briefly below. Because this area is complex the connection between the theory and the final empirical model is usually not as close as is the case for the static and riskless production theory.

8.1 Imperfect adjustment

There have been two common *ad hoc* approaches to including a dynamic element in production theory, both of which are often termed the Nerlovian model. The first approach, used frequently when modelling investment, is to postulate that there is a desired level of output or input which is not achieved for a number of time periods. Usually this leads to a specification for output supply and input demand where a lagged endogenous term is included, such as:

(30) $Y_t = g_1(Y_{t-1}, P, R; Z)$ and

$$X_t = g_2(X_{t-1}, P, R; Z).$$

The second *ad hoc* approach is to postulate that producers respond to an expected price which is based on past prices. This can either lead to the same specification for supply and demand as the first approach, that is equation (30), or a specification with lagged prices instead of actual prices. Single equation examples of this type of model for Australian agriculture can be found in Duncan (1972), Anderson (1974), Fisher (1975), Griffiths and Anderson (1978) and Pandy, Piggott and MacAulay (1982). Askari and Cummings (1977) and Nerlove (1979) provided a more detailed description and review of the theory and application of the Nerlovian model of supply dynamics. The main shortfall of the Nerlovian model is that it introduces only a simplistic dynamic element into what is basically a static concept (Lau 1978, p.213). The Nerlovian model is termed *ad hoc* because there is no formal description of how outputs or inputs are adjusted toward optimal or desired levels. Rather, the simple assumption is made that a portion of the difference between actual and desired levels is eliminated in each time period.

An approach adopted by Eckstein (1984, 1985) was to apply the linear rational expectations model which is used in the macroeconomic literature. Eckstein assumed that producers maximised the expected present value of profit by choosing an output mix subject to a dynamic and stochastic technology as well as uncertain price movements. For given assumptions he showed that the Nerlovian model and the linear rational expectations model are observationally equivalent in the sense that they both have similar reduced forms where current supply is a function of supply in previous periods, expected prices and exogenous factors. However, both models do not lead to the same production behaviour or policy conclusions. For example, if previous price changes are serially uncorrelated then rational producers do not interpret previous price changes as a signal for a permanent change in prices, whereas Nerlovian producers would. Furthermore, any change in taxes or any other exogenous variable alters the structural form of the supply equations in the rational expectations model, but not in the Nerlovian model. For an example of how changes in exogenous factors change the structural form of rational expectations models see B.S. Fisher (1982).

Other recent attempts to improve on the Nerlovian model and create what can be termed a dynamic theory of production has been carried out by Pindyck and Rotemburg (1983), Epstein and Yatchew (1985), Chetty and Heckman (1986), Morrison (1986) and Prucha and Nadiri (1986).

The profit function framework described in the previous sections where profits are a function of both variable and quasi-fixed inputs can be considered a dynamic model to the extent that the quasi-fixed inputs are not necessarily at their long run profit maximising levels. However, the profit function framework does not explicitly treat the adjustment path of the quasi-fixed factors. As a result there is no information about the direction of the adjustment path or the length of time of adjustment of the quasi-fixed inputs. In the dynamic models mentioned above, strong and *ad hoc* assumptions about the direction and length of time of the adjustment path and the adjustment criteria are made (Kulatilaka 1985, p.258).

Researchers have shown that some of the results which hold for static models do not apply in the dynamic case. Taylor (1984) showed that when price expectations have a Markovian structure (where the expected price is conditional on previous prices) and where producers maximise the expected present value of profit, Hotelling's Lemma does not apply. In other words, when these conditions hold, dynamic output supply and input demand equations cannot be derived by simply differentiating the profit function. McLaren and Cooper (1981) derived an intertemporal analogue to Hotelling's Lemma which allows the derivation of optimal input demand equations by differentiating an optimal value function rather than a profit function.

8.2 Uncertainty

As is well known, standard production theory leads to output being chosen where the output price equals marginal cost and inputs are employed up to the point where the marginal product equals the product price ratio. However, once known prices and production responses are replaced by random prices and stochastic production responses the standard production theory no longer holds. Sandmo (1971) showed that for the risk averse firm facing uncertain output prices, output is less than output under certainty. In addition, output is more for risk loving firms than output under certainty. In standard production theory the output supply curve of the perfectly competitive firm is upward sloping but when the price is stochastic is is possible for the supply curve to be downward sloping (Baron 1970). Further results about how the introduction of price risk changes the standard production theory have been presented by Batra and Ullah (1974), Hartman (1975), Ratti and Ullah (1976) , Ishii (1977), Chambers (1983), Daughety (1983), Hoel and Vislie (1983), MacMinn and Holtman (1983) and Just and Zilberman (1986).

The issue of how the degree of risk aversion should be measured is still not settled (Brennan 1982, Katz 1983, Briys and Eeckhoudt 1985). Furthermore, while some research into the attitudes of producers to risk appears to suggest that risk aversion is dominant (Lin, Dean and Moore 1974, Bond and Wonder 1980, Quiggin 1981), others have found the opposite (Francisco and Anderson 1972). It is more likely that there is not one typical attitude to risk but a range of attitudes depending on the profit level. Young (1979) noted that the conclusion that farmers are risk averse is at best only tentative because research results were often based on small sample sizes and that other explanations aside from risk averseness may explain observed producer behaviour.

Researchers studying the effects of risk have usually postulated that producers are maximising the expected utility of profit. Pope (1980) showed that output supply and input demand can no longer be derived using Hotelling's Lemma if some types of risk aversion hold and producers maximise the expected utility of profit. Pope (1982) presented a method for estimating the amount by which input demand and output supply are biased by ignoring risk aversion. His approach is only applicable to the single output firm and requires a measure of the aversion or love of risk. When risk aversion is near zero or output is inelastic to price changes, the bias is small.

Although the derivatives of an expected utility function no longer yield input demand and output supply functions Hallam *et al.* (1982) showed that input demand and output supply functions can be determined by taking the ratio of the derivatives of the expected utility function with respect to own price and fixed inputs. Their method cannot be used to determine output supply functions when both production and price are stochastic because of the correlation between the marginal utility of profit and random output. In this case Hallam *et al.* suggested an alternative method for deriving output supply and input demand functions by changing the objective function from maximising the expected utility of profit to maximising the expected utility of profits per unit of input (*e.g.* returns per hectare). At present, researchers have not developed the appropriate regularity conditions which ensure that an arbitrarily specified expected utility function relates to some plausible properties and a well behaved technology, although Hallam *et al.* (1982, pp.195-8) presented some preliminary analysis in this area.

Other research on stochastic production responses has been carried out by Pope and Just (1977), Just and Pope (1978, 1979), Anderson and Griffiths (1981, 1982), Griffiths and Anderson (1982), Easter and Paris (1983), Antle and Goodger (1984) and Griffiths (1986). In addition, Pindyck (1982) examined the effects of future uncertainty of demand and costs on the production behaviour of the firm. For reviews on the effects of risk on production decisions see Hey (1979), Newbery and Stiglitz (1981), Antle (1983) and Scandizzo, Hazell and Anderson (1984).

9. Concluding Remarks

Although it is widely recognised that most agricultural production systems have a multiple output, multiple input nature, there are few supply response studies in the literature in which explicit account has been taken during parameter estimation of all of the information available from the theory. In the past, economic theory has generally been used in deciding which variables to include in models of supply response but little advantage has been taken of the parameter restrictions that arise from the theory.

If the prime concern in an empirical study is to estimate supply response elasticities in a multiple output industry it will be usually more convenient to adopt a profit function approach rather than to use a production function model. The output supply functions can be easily obtained as the first order partial derivatives of the profit function. The profit function specification is less restrictive than the production function model in the sense that non-jointness or output separability do not have to be maintained. In addition, in a profit function model, prices are specified as exogenous variables rather than input quantities as is the case when a production function approach is adopted. In the case where farm level data are employed it is reasonable to assume that individual producers have no influence on output prices and it is also likely that accurate data on prices will be more readily and cheaply attainable than accurate data on input quantities.

There are several advantages in having a theory on which to base econometric models of supply response. First, the theory of production provides a framework which is useful for describing and interpreting the observed behaviour of producers. In addition, the theory provides a set of parameter restrictions that can be imposed during estimation. This additional information, if it is correct, increases the efficiency of the final parameter estimates and helps to ensure that forecasts made using supply response models of multiple output industries are consistent across products.

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