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Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models

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The parameters of yield response functions can vary by year. Past studies usually assume yield functions are nonstochastic or "limited" stochastic. In this study, we estimate rye–ryegrass yield functions in which all parameters are random. The three functional forms considered are the linear response plateau, the quadratic, and the Spillman-Mitscherlich. Nonstochastic yield models are rejected in favor of stochastic parameter models. Quadratic functional forms fit the data poorly. Optimal nitrogen application recommendations are calculated for the linear response plateau and Spillman-Mitscherlich. The stochastic models lead to smaller recommended levels of nitrogen, but the economic benefits of using fully stochastic crop yield functions are small because expected profit functions are relatively flat for the stochastic yield functions. Stochastic crop yield functions provide a way of incorporating production, uncertainty into input decisions.

Key Words: cereal rye–ryegrass, Monte Carlo, nitrogen, random parameters, stochastic plateau

JEL Classifications: Q10, C12, D24

Optimal nitrogen (N) fertilizer recommendations are often obtained by fitting yield response functions to crop yield data (Babcock, 1992; Cerrato and Blackmer, 1990; Lanzer and Paris, 1981; Makowski and Wallach, 2002; Mooney et al., 2008). Unfortunately, model-based N rate recommendations are vulnerable to misspecification of the yield response functions. This misspecification can affect the accuracy of optimal N recommendations, and any errors can reduce the profit of producers who follow the recommendations and potentially have negative environmental effects if excess N is applied. Of particular interest here is the possible misspecification of assuming parameters are nonstochastic when they are stochastic. In this article, we determine the N rate recommendations for a winter cereal rye (*S. cereale*)–ryegrass (*Lolium multiflorum Lam*) forage crop based on yield functions using three different functional forms in which yield functions are estimated assuming both stochastic and nonstochastic parameters.

Previous work on crop response to N fertilizer has usually used either limiting nutrient response functions or polynomial functional forms. Plateau functions tend to best fit data from field studies (Grimm, Paris, and Williams, 1987;

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Heady and Pesek, 1954; Lanzer and Paris, 1981). Past studies have often assumed that the parameters of the yield function are nonstochastic or "limited" stochastic (some parameters are considered stochastic and others are not) and that all model errors are independent. This often leads to the estimation of the mean yield function conditional on fertilizer inputs but neglects the possible interaction between weather events in a given year with the associated fertilizer response. Research suggests, however, that parameters of yield response functions can vary by year (Cerrato and Blackmer, 1990; Makowski and Wallach, 2002; Tembo et al., 2008). Given that the parameters of the yield response function can vary by year, estimating a random parameter model could give a more realistic model of producers' profit expectations.

Random parameter models have been suggested by Berck and Helfand (1990), Makowski and Wallach (2002), Paris (1992), and Tembo et al. (2008). Berck and Helfand (1990) and Paris (1992) estimate linear response plateau functional forms in which the intercept and plateau parameters are random but without random effects. Tembo et al. add uncorrelated random effects to the intercept and plateau but not to the slope. The Tembo et al. approach was successfully used to model wheat forage data (Kaitibie et al., 2003; Taylor et al., 2010) as well as wheat yield data (Biermacher et al., 2009). We follow Brorsen and Richter (2011), Makowski and Wallach (2002), and Roberts et al. (2011) and treat all of the model parameters as having a random effect that varies by year. Makowski and Wallach (2002) find that it pays to consider all parameters as stochastic and there is a need to determine if their findings apply to other situations.

We consider three crop response functions: the simple linear response plateau (LRP), the Spillman-Mitscherlich, and the quadratic; and we make all parameters of the yield response functions random. Our random parameter crop response functions let parameters vary stochastically by year. The data used are annual rye–ryegrass forage data collected from a long-term N fertilization experiment in south–central Oklahoma. We conduct nested likelihood ratio tests to choose between nonstochastic and stochastic models for each of these three crop response functions (Greene, 2008). Next, we evaluate the economic value of using the alternative models by comparing expected profit. The ultimate goal of this study is to evaluate the economic importance of using a random parameter model to make optimal N rate recommendations for cool-season cereal rye–ryegrass forage producers in southern Oklahoma.

Determining the Profit Maximizing Level of Nitrogen Fertilizer

Consider a risk-neutral forage producer whose objective is to maximize expected net returns from winter cereal rye–ryegrass forage. The producer seeks to maximize expected net return above N cost:

$$(1) \quad \max_{N} E(R_t|N) = pE\lceil y_t \rceil - rN \quad \text{s.t. } y_t = F(N),\ N \geq 0,$$

where R_t is the producer's net return at time t, y_t is the forage yield, N is the level of applied N, r is the price of applied N fertilizer, and p is the price of forage. Yield expectations are obtained through the production function $F(N)$. We consider the three functional forms in turn.

Linear Response Plateau

A stochastic linear response plateau function is specified as

$$(2) \quad y_{it} = \min(\beta_0 + (s_t + \beta_1)N_{it}, \mu_p + v_t) + u_t + \varepsilon_{it},$$

where y_{it} is the forage yield of cereal rye–ryegrass from the i^{th} plot in year t, N_{it} is the level of N fertilizer, μ_p is mean plateau yield, s_t is the slope random effect, v_t is the plateau year random effect, u_t is the (intercept) year random effect, and ε_{it} is a random error term that is normally distributed and independent of the three random effects. The intercept random effect is added to the whole equation rather than just to β_0 so that the model of Tembo et al. (2008) is a special case. The variance parameters u_t, s_t, and v_t are correlated and normally distributed. Makowski and Wallach (2002) use a model in which $(\beta_0, \beta_1, \mu_p) \sim N(\beta, \Omega)$. Our model is parameterized differently but is equivalent to Makowski and Wallach (2002).

The random effect u_t shifts the whole function up or down, which could be the result of a variety of weather factors, insects, or disease. The slope random effect s_t may be the result of N losses from leaching, soil or weather characteristics, or weed pressure during critical growth periods. The plateau year random effect v_t shifts the yield potential from applying more N, which mostly varies as a result of rainfall in a given year. For example, when growing conditions are favorable in a given year, the plateau yield increases as does the amount of N that the plants can use. When the model is nonstochastic, the random variables v_t and s_t will be zero, but u_t may still be included.

The function is continuous, but its derivatives do not exist with respect to either its parameters or N at the knot point where the response and the plateau are joined, but the derivatives of expected yield do exist for the stochastic model. Choosing the level of N (N^*) that maximizes equation (1) follows the rule from economic theory that the marginal factor/input cost (MFC) should equal the marginal expected product value (MVP). With a nonstochastic linear response plateau function, equation (2) will exhibit constant positive marginal product when $\mu_p > \beta_0 + \beta_1 N$. If MVP > MFC, then N should be applied until MVP = MFC. Increasing N beyond the level required to reach μ_p will generate negative marginal returns. Therefore, with the nonstochastic LRP, N^* would either be at the level required to reach the plateau (N_p) or zero:

$$(3) \quad N^* = \begin{cases} N_p, & \text{if VMP} > MFC \\ 0, & \text{otherwise} \end{cases}.$$

For the stochastic LRP, the random variable u_t in equation (2) enters linearly and therefore it drops out after taking expectations. Therefore, the expectation of y becomes

$$(4) \quad E(y_t) = E\left[\min\left(\beta_0 + (\beta_1 + s_t)N, \mu_p + v_t\right)\right].$$

Because s_t and v_t are random and correlated, the expectation in equation (4) requires integrating with respect to s_t and v_t, which defines a double integral that must be solved numerically:

$$(5) \quad E(y_t) = \iint\left[\min\left(\beta_0 + (\beta_1 + s_t)N, \mu_p + v_t\right)\right] \times \varphi(s_t, v_t)\partial s_t \partial v_t,$$

where $\varphi\ (s_t, v_t)$ is the multivariate normal probability density function. Tembo et al. (2008) use the approach developed for Tobit models and obtain N^* by evaluating a univariate normal probability density function because they do not allow the slope to be random. Makowski and Wallach (2002) solve the integral using Monte Carlo integration. The integration in equation (5) can also be solved using other numerical approximation methods such as Gaussian cubature (DeVuyst and Preckel, 2007). We use Monte Carlo integration to solve the double integral. The optimal level of N is obtained by direct nonlinear optimization (grid search would also work because there is only one choice variable).

Spillman-Mitscherlich

The Spillman-Mitscherlich yield response function is an exponential function (Spillman, 1923). A univariate stochastic form of this function is

$$(6) \quad y_{it} = a - (b + s_t)\exp((-c + v_t)N_{it}) + u_t + \varepsilon_{it},$$

where a is the maximum or potential yield obtainable by applying N under the conditions of the experiment; b is the increase in yield resulting from applied N; c is the ratio of successive increments in output a to total output y; u_t, s_t, and v_t are correlated random effects; and ε_{it} is the independent error term. When the model is nonstochastic, the random variables s_t and v_t are zero, but u_t is still included.

Equation (6) shows that as the application rate of N increases, the yield increases at a decreasing rate and asymptotically approaches a maximum as the application rate (theoretically) approaches infinity. The function does not strictly adhere to the law of the minimum as in the case of the linear response plateau. The Spillman-Mitscherlich yield response function allows for convex rather than right-angled isoquants associated with the law of the minimum, but unlike the polynomial functions, it exhibits a plateau. The function exhibits sufficient flexibility to accommodate from near-perfect substitution to near zero factor substitution if the data and production process so suggest (Frank, Beattie, and Embleton, 1990).

The optimal level of N is obtained by substituting equation (6) into equation (1) and then solving the optimization problem. For the nonstochastic Spillman-Mitscherlich yield function, the optimal level of N (N^*) is obtained by solving the first-order condition for N, which gives

$$(7) \qquad N^* = \frac{1}{-c}\left[\ln\left(\frac{r/p}{cb}\right)\right].$$

For the stochastic Spillman-Mitscherlich, because the random variables s_t and v_t do not enter linearly in equation (6), the expectation of y is obtained by numerically solving the integral:

$$(8) \qquad E(y_t) = \iint\left[a + (b + s_t)\exp(-c + v_t)N\right] \varphi(s_t, v_t)\partial s_t \partial v_t.$$

The double integral is solved using Monte Carlo integration. Monte Carlo approximates equation (8) with a summation, which is then substituted into equation (1) and the optimal level of N is then obtained by nonlinear optimization.

Quadratic Response

A random parameter quadratic response model is specified as

$$(9) \quad y_{it} = \beta_0 + (\beta_1 + v_t)N_{it} + (\beta_2 + s_t)N_{it}^2 + u_t + \varepsilon_{it}.$$

where β_0 is the intercept parameter whose position (value) can be shifted up or down from year to year by the year random effect u_t, β_1 is the linear response coefficient with random effect parameter v_t, β_2 is the quadratic parameter whose value can be shifted up or down by the random effect s_t, and ε_{it} is the independent error term assumed to be normally distributed. The random effects v_t, s_t, and u_t are correlated and normally distributed. When the model is nonstochastic, the random effects v_t and s_t would be zero, but u_t is still included.

Because equation (9) is continuously twice differentiable and all the random parameters enter in equation (9) linearly, equation (1) gives the same analytical solution for both stochastic and nonstochastic models. Note that for the nonstochastic model, the values of, v_t, s_t, and u_t are all zero. Hence, the problem of calculating N* simplifies to:

$$(10) \qquad N^* = \left(\beta_1 - \frac{r}{p}\right)\Big/ 2\beta_2.$$

Model Fit and Selection Criteria

Likelihood ratio tests are used to choose between stochastic and nonstochastic models (Greene, 2008). The calculated likelihood ratio statistics have a chi-square distribution under the null hypothesis. To choose between competing functional forms, Davidson and Mackinnon (1981) suggest using formal nonnested tests such as the J-test and P-test. These tests, however, cannot be used here because they can only be used when the nonoverlapping parameters are associated with fixed effects.

The literature on nonnested hypothesis tests provides a variety of criteria to select the model that best fits the data. When competing nonnested models are fully parameterized and estimated by maximum likelihood, a popular criterion is the adjusted model log-likelihood such as Akaike information criterion (Akaike, 1974) and Bayesian information criterion (Schwarz, 1978). However, these criteria do not take into account whether the differences in the penalized log-likelihoods are statistically significant. When observations are independent and identically distributed, a test can be done following Vuong (1989). Pollak and Wales (1991) introduced the Likelihood Dominance Criterion (LDC). The LDC provides rationale to compare two models based on the difference in estimated likelihoods with adjustments for differences in the number of parameters and for a given significance level (Grewal, Lilien, and Mallapragada, 2006; Pollak and Wales, 1991). The criterion involves a fictitious experiment in which two competing hypotheses are nested in a composite and the concept of dominance ordering is used to choose among the two. This criterion is the one we use for testing hypotheses to choose between our nonnested models.

Let H_1 and H_2 be two models (hypotheses) with n_1 and n_2 parameters, respectively, and let L_1 and L_2 be the log-likelihoods. Let $C(v)$ denote a critical value of the chi-square distribution with v degrees of freedom at significance level α. According to the LDC:

1. Select H_1 if $L_2 - L_1 < [C(n_2 + 1) - C(n_1 + 1)]/2$.
2. Select H_2 if $L_2 - L_1 > [C(n_2 - n_1 + 1) - C(1)]/2$.
3. Otherwise, model selection is indeterminate.

When the number of parameters is equal, $n_1 = n_2$ (our case), the indeterminate region reduces to zero and the criterion reduces to a simple comparison of estimated maximum likelihood values (Pollak and Wales, 1991).

Data

Forage yield data are cross-sectional, time-series from a long-term experiment conducted by the Agricultural Division of The Samuel Roberts Noble Foundation, Inc. (1997–2008) at Red River demonstration and research station near Burneyville in south–central Oklahoma. The experiment began in 1979 and was aimed at evaluating the effect of N fertilization rate and harvest timing on the annual rye–ryegrass forage production system using a randomized complete block design. Details of the experimental design are described in Altom et al. (1996) who analyzed the data from 1979–1992.

Our data set covers 14 years from Fall 1993 to Spring 2007. Six treatment levels of N (34–0–0) were administered: 0, 100, 150, 200, 300, and 400 pounds per acre per year. Treatments were replicated three times for each level of N. Split applications were used. Ammonium nitrate was broadcast and incorporated before planting in the fall. Spring applications were not incorporated. Fall fertilization was done between September 24 and October 25. Spring fertilization was done between February 20 and March 17. Phosphorous was banded with the seed at a rate of 50 pounds P_2O_5/acre every year. Potassium was broadcast and incorporated before planting at an average rate of 100 pounds K_2O/acre. Lime was applied to the plots used in the study.

Forage yields were determined by clipping individual plots that were 12 × 13 feet. Plots were clipped multiple times to simulate grazing. Yearly dry matter forage yields were the sum of all clippings for that year. Average annual rye–ryegrass yield response to N fertilization is shown in Figure 1.

Estimation

The models are estimated using the nonlinear mixed procedure in SAS (SAS Institute Inc., 2003). The dependent variable is yield, and the independent variable is N. For the quadratic, the nonstochastic LRP, and the nonstochastic Spillman-Mitscherlich yield response functions, the error term and random effects enter the equations linearly. In the stochastic LRP and the stochastic Spillman-Mitscherlich yield response functions, the two nonintercept random effects enter the equations nonlinearly. The random effects are estimated as free correlated parameters, but the error term is independent.

The SAS nonlinear mixed procedure fits nonlinear mixed models by maximizing the

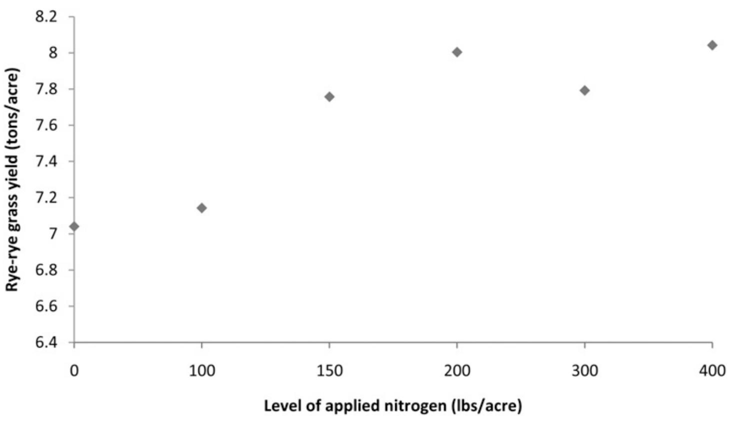

Figure 1. Ryegrass Yield Response to Applied Nitrogen

likelihood function integrated over the random effects. As is common in nonlinear optimization, convergence can be difficult and computing the objective function and its derivatives can lead to arithmetic overflows (SAS Institute Inc., 2003). The models have no closed form and can only be approximated numerically. To achieve convergence, three efforts are used: scaling, varying starting points, and using different optimization techniques.

Pinheiro and Bates (1995) provide evidence that of the several different integrated likelihood approximations methods, adaptive Gaussian quadrature is one of the best. We use this method to approximate the likelihood function integrals and maximize the function by the dual quasi-Newton optimization algorithm. Other optimization techniques that enabled convergence are the Newton-Raphson method with ridging and the Trust-Region Method (SAS Institute Inc., 2003). The quadratic and nonstochastic Spillman-Mitscherlich models converge with less need of scaling and changing starting point values. Estimates obtained are then used to determine the optimal level of N.

For the stochastic linear response plateau and the stochastic Spillman-Mitscherlich crop response functions, the estimated parameters are used in Monte Carlo integration. The random vector $[s_t, v_t]$ is jointly normal distributed, i.e. $[s_t, v_t] \sim N(0, \Omega)$. We use the Cholesky decomposition, $\Omega = P'P$ where P is a lower triangular matrix. Let Z be a 2×1 vector of independent draws, then $PZ \sim N(0, \Omega)$. With sufficient draws, the sample average of the function being integrated provides an approximation to the integral (Greene, 2008, pp. 576–83). We use 10,000 draws for our approximation. To obtain the optimal level of N, we use the SAS PROC NLP procedure and maximize our objective function (equation [1]) using Newton-Raphson with ridging.

Results

Estimated parameters are reported for each of the crop response functions: the quadratic in Table 1, the linear response plateau in Table 2, and the Spillman-Mitscherlich in Table 3. For all models, the mean parameters and variance estimates are statistically significant at the 5% level based on Wald t-tests. Covariance parameters of the stochastic quadratic model are not statistically significant at the 5% level. Covariance parameters of the stochastic Spillman-Mitscherlich and the covariance between the plateau and the slope in the stochastic LRP are statistically significant.

Table 1. Rye–Ryegrass Yield (1000 lbs/acre) Response to Nitrogen (100 lbs/acre) Using the Nonstochastic and Stochastic Quadratic Functional Form

	Stochastic Quadratic		Nonstochastic Quadratic	
Parameter	Estimate	SE	Estimate	SE
Intercept (β_0)	5.74	0.54	5.77	1.15
Slope (β_1)	1.74	0.44	1.64	0.18
Quadratic term (β_2)	–0.24	0.10	–0.25	0.04
Variance of intercept random effect (σ_u^2)	13.46	3.29	19.32	7.08
Variance of error term (σ_ε^2)	1.89	0.11	2.43	0.14
Variance of slope random effect (σ_v^2)	1.93	0.35		
Variance of quadratic term random effect (σ_s^2)	0.47	0.20		
Covariance (σ_u^2, σ_s^2)	1.62	1.51		
Covariance (σ_s^2, σ_v^2)	–0.004	0.38		
Covariance (σ_u^2, σ_v^2)	–0.03	0.06		
Optimal level of nitrogen (100 lbs/acre)	1.71	0.12	1.44	0.15
–2 Log-likelihood	2348.6		2433.6	

SE, standard error.

Table 2. Rye–Ryegrass Yield (1000 lbs/acre) Response to Nitrogen Using the Nonstochastic and Stochastic Linear Response Plateau Functional Form

	Stochastic Linear Response Plateau		Nonstochastic Linear Response Plateau	
Parameter	Estimate	SE	Estimate	SE
Intercept (β_0)	5.67	0.29	5.72	1.15
Slope (β_1)	1.62	0.31	1.38	0.17
Yield plateau (μ_p)	8.01	0.12	8.23	1.14
Intercept random effect $\left(\sigma_u^2\right)$	13.96	1.53	19.32	7.08
Variance of error term $\left(\sigma_\varepsilon^2\right)$	1.85	0.11	2.42	0.14
Plateau random effect $\left(\sigma_v^2\right)$	3.65	0.33		
Variance of slope random effect $\left(\sigma_s^2\right)$	0.89	0.16		
Covariance $\left(\sigma_u^2\sigma_s^2\right)$	–1.41	0.74		
Covariance $\left(\sigma_u^2\sigma_v^2\right)$	0.89	0.82		
Covariance $\left(\sigma_s^2\sigma_v^2\right)$	1.54	0.18		
Optimal level of N (100 lbs/acre)	1.44	0.14[a]	1.82	0.14[a]
–2 Log-likelihood	2295.10		2429.80	

[a] The standard error (SE) of the optimal nitrogen (N) application rate for the stochastic Linear Response Plateau is obtained by Monte Carlo methods, whereas the SE of the optimal nitrogen application rate for the nonstochastic Linear Response Plateau is obtained using the delta rule.

The likelihood ratio (LR) statistic for the stochastic quadratic vs. the nonstochastic quadratic model is 170; the LR for the stochastic linear response plateau vs. the nonstochastic linear response plateau is 269.4; and the LR for the stochastic Spillman-Mitscherlich vs. the nonstochastic Spillman-Mitscherlich is 262.8. All the LR statistics are greater than the critical chi-square $\left(X_5^2\right)$ value[1] at any conventional significance level. Thus, stochastic models fit our data better than the alternative nonstochastic models for each of the three crop response functions.

Based on the Likelihood Dominance Criterion (Pollak and Wales, 1991), we choose the functional form for the crop response function that fits our data best. The estimated maximum likelihood value for the stochastic linear response plateau is 2295.1. The likelihood value for the stochastic quadratic is 2348.6, and for the stochastic Spillman-Mitscherlich, it is 2300.0. All the three stochastic crop response functions have the same number of parameters ($n = 9$). Hypothesis testing on the stochastic crop response functions according to the Likelihood Dominance Criterion ranking favors the stochastic linear response plateau over the stochastic Spillman-Mitscherlich and the stochastic Spillman-Mitscherlich over the stochastic quadratic function form for crop response. From the illustration in Figure 1, a quadratic functional form may be considered a poor choice for this data set on the basis that it assumes symmetry. It indicates that yield decreases past the peak at the same rate it increases before the peak. We base our optimal N application rate recommendations on the stochastic linear response plateau, the best fitting functional form.

The profit maximizing level of N is evaluated at 2009 input and output prices. Although N 34–0–0 ammonium nitrate was used in the experiment, The Samuel Roberts Noble Foundation Agricultural Division currently recommends using 46–0–0 urea. The prices of N 34–0–0 and 46–0–0 as reported by input suppliers in south–central Oklahoma are \$0.51/lb of N and \$0.41/lb of N, respectively. We do a sensitivity analysis by determining N rate recommendations because N prices vary. The per pound price of forage is determined as the cost of beef gain per pound

[1] Note that there is a potential nuisance parameter problem with this hypothesis test because imposing that the two variances are zero also imposes that the three covariances are zero. We do not explore this issue because all null hypotheses are rejected even using the more conservative critical value.

Table 3. Rye–Ryegrass Yield (1000 lbs/acre) Response to Nitrogen Using the Nonstochastic and Stochastic Spillman-Mitscherlich (S-M) Functional Form

	Stochastic S-M		Nonstochastic S-M	
Parameter	Estimate	SE	Estimate	SE
Maximum (potential) yield (*a*)	7.91	0.12	8.47	1.15
Response due to nitrogen (*b*)	3.28	0.38	2.81	0.23
Ratio of successive increments (*c*)	1.31	0.26	0.89	0.16
Variance of error term (σ_ε^2)	1.85	0.11	2.42	0.14
Intercept random effect (σ_u^2)	19.44	1.10	19.35	7.09
Variance of slope random effect (σ_s^2)	5.89	1.45		
Plateau random effect (σ_v^2)	0.37	0.15		
Covariance (σ_u^2,σ_s^2)	8.36	1.16		
Covariance (σ_u^2,σ_v^2)	1.67	0.36		
Covariance (σ_u^2,σ_v^2)	0.80	0.19		
Optimal level of nitrogen (100 lbs/acre)	1.07	0.02[a]	1.13	0.09[a]
–2 Log-likelihood	2300.0		2431.4	

[a] The standard error (SE) of the optimal nitrogen application rate for the stochastic Spillman-Mitscherlich is obtained by Monte Carlo methods, whereas the SE of the optimal nitrogen application rate for the nonstochastic Spillman-Mitscherlich is obtained using the delta rule.

divided by the pounds of forage required by a stocker animal to produce a 1-pound gain (Belasco et al., 2009; Coulibaly, Bernardo, and Horn, 1996; Kaitibie et al., 2003). Based on the National Research Council (1984) net energy equations used to estimate livestock requirements, Ishrat, Epplin, and Krenzer (2003) and Krenzer et al. (1996) show that 1 pound of beef gain requires 10 pounds (dry matter) of standing forage. Within the south–central Great Plains, the cost per pound of gain has ranged from \$0.32/lb since 2005 to \$0.55/lb in 2009. Kaitibie et al. (2003) used an average daily weight gain equation and determined the cost of beef gain at \$0.54/lb. As a result of decreased prices of corn and fertilizer, this cost declined to \$0.45/lb (which is approximately the mean across the period). Therefore, at the cost of beef gain per pound of \$0.45, the price per pound of forage is \$0.45/10 = \$0.045. Our optimal N application rate recommendations are based on N prices of \$0.41/lb and forage sale prices of \$0.045/lb.

The estimated optimal N rates and their standard errors for the models are included in the respective tables of results. At the assumed prices, the profit maximizing level of N obtained with the nonstochastic linear response plateau function is 182.3 lbs/acre, the level of N required to reach the plateau. Applied N increases yield at a rate of 13.8 lbs/acre until the plateau yield level of 8235.7 lbs/acre. At \$0.045 sale price of forage, the marginal value product of N is \$ 0.62/lb, which is greater than the \$0.41/lb price of N. The 95% confidence interval of the optimal level of N obtained with the nonstochastic linear response plateau is 209.4 lbs/acre to 154.6 lbs/acre.

Maximum profits for the stochastic linear response plateau are achieved with a N fertilization rate of 143.6 lbs/acre. The 95% confidence interval for this estimate is to apply 115.5 lbs/acre to 171.8 lbs/acre of N. The nonstochastic linear response plateau gives an optimal level of N that is 38.7 lbs/acre higher than the stochastic linear response plateau. Based on the average expected plateau yield and optimal N obtained with the stochastic linear response plateau, the marginal productivity of N is higher with the stochastic model. On average, N increases forage yield at a rate of 16.3 lbs/acre compared with 13.8 lbs/acre for the nonstochastic model. The stochastic linear response plateau crop response function leads to diminishing marginal productivity of N that is supported by data from agronomic experiments (Paris, 1992).

The expected profit function of the nonstochastic linear response plateau crop response function is higher than that of the stochastic linear response plateau (Figure 2A). Figure 2A shows that the expected profit curve predicted by the

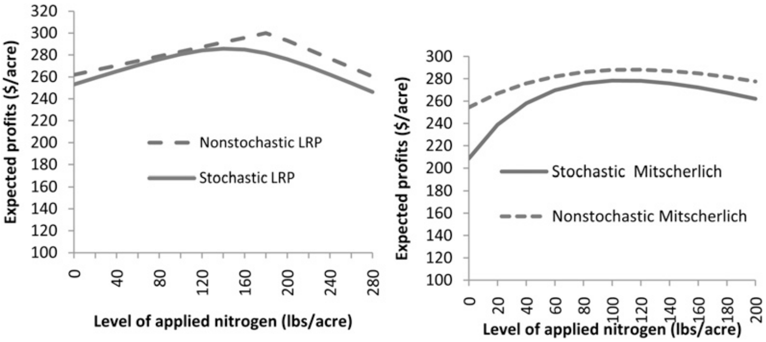

Figure 2. A–B, Expected Profit Functions for the Linear Response Plateau (LRP) and the Spillman-Mitscherlich (S-M) Functional Form
Note: Price of ryegrass = $0.0450/lb, price of nitrogen = $0.41/lb

nonstochastic linear response plateau increases linearly as a function of N and decreases sharply when N exceeds the optimal N level. Because of the initial linear section, the profit-maximizing N rate is insensitive to N prices. The nonstochastic linear response function overestimates yield potential in years when growing conditions are not good. This explains the large difference between N recommendations calculated using the stochastic and nonstochastic models. The loss (to the producer) from using the nonstochastic linear response plateau to predict optimal N levels when the stochastic linear response plateau is the true model is approximately $9.0/acre. This loss is small because the expected profit function of the stochastic linear response plateau is relatively flat. The cost of using a nonstochastic linear response plateau to determine N recommendations when the stochastic linear response plateau is the true model increases if the price of N increased relative to the price of forage.

The profit maximizing level of N obtained with the nonstochastic Spillman-Mitscherlich model is 113.5 lbs/acre. The 95% confidence interval for this estimate is 95.4 lbs/acre to 130.4 lbs/acre of N. The optimal level of N obtained with a stochastic Spillman-Mitscherlich model is 107.4 lbs/acre. The 95% confidence interval for the optimal level of N obtained with the stochastic Spillman-Mitscherlich is 103 lbs/acre to 110.6 lbs/acre. The expected profit function of the nonstochastic Spillman-Mitscherlich is higher than that with the stochastic Spillman-Mitscherlich (Figure 2B). The loss from using the nonstochastic Spillman-Mitscherlich to predict the optimal level of N when the stochastic Spillman-Mitscherlich is the true model is approximately $1.0/acre. The economic benefits of using fully stochastic models are small because optimal N rates do not differ greatly between stochastic and nonstochastic models and the expected profit functions are relatively flat.

The analysis presented here does not account for the environmental/social costs of overfertilization as a result of using a nonstochastic crop response function to determine N rates. Although not quantified, there are additional costs to overestimating crop N needs. For instance, Tumusiime et al. (2011) has shown that applying N above the consumptive potential of the growing plant can increase lime costs. There is a potential social cost resulting from potential groundwater contamination from N fertilizer overapplication. Because the stochastic models recommend lower N levels, accounting for these additional costs would increase the advantage of the stochastic crop response functions.

The profit-maximizing level of N obtained with a nonstochastic quadratic crop response model is 144.3 lbs/acre, and the optimal level of N obtained with a stochastic quadratic model is 171.4 lbs/acre. We notice from Figure 3 that fertilizer application recommendations for the stochastic linear response plateau and the stochastic Spillman-Mitscherlich models can be less or more than the fertilization rates recommended with the alternative nonstochastic model, depending on price ratios for the input and the output. The use of the stochastic linear response plateau or Spillman-Mitscherlich function to

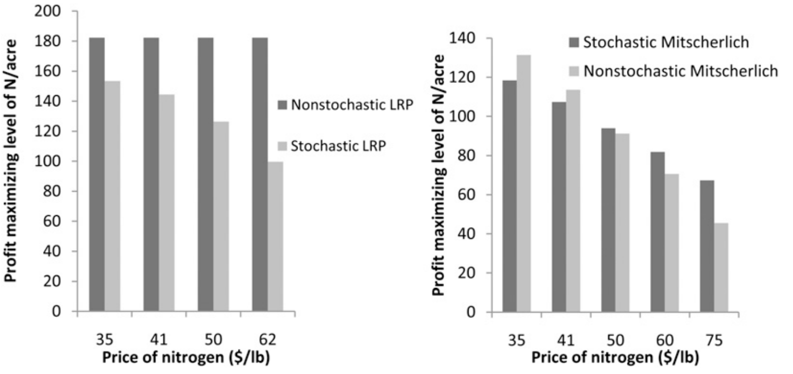

Figure 3. Optimal Level of Nitrogen Application Rate at Varying Prices for the Linear Response Plateau (LRP) and the Spillman-Mitscherlich (S-M) Functional Form
Note: Price of ryegrass is constant at $0.045

determine N application recommendations provides insight as to why some farmers may apply more or less N than would appear optimal. Also, the expected profit function is relatively flat so the optimal level is likely difficult for farmers to determine. The stochastic quadratic model consistently estimates higher optimal levels of N application rates than the alternative nonstochastic model.

Summary and Conclusions

Models predicting crop yield response to N fertilizer applications are often used to recommend optimal fertilizer application rates. Past studies usually assume the parameters of the crop yield function are nonstochastic or "limited" stochastic and that all model errors are independent. Given that research suggests that the parameters of the crop yield functions vary by year, estimating a random parameter yield function could give a more realistic model of producers' profit expectations. In this study, we consider yield functions in which all parameters are random. The approach was applied to cereal rye–ryegrass forage data collected from a long-term N fertilization experiment in south–central Oklahoma to determine and compare the profitability of N estimated from stochastic crop yield functions and the alternative nonstochastic models. The functional forms considered are the linear response plateau, the quadratic, and the Spillman-Mitscherlich.

Constant parameter models are rejected in favor of random parameter models. The quadratic functional form fits the data poorly. The stochastic linear response plateau model provided the best fit to the data among the yield functions studied. Our results support the findings of and Kaitibie et al. (2003) and Tembo et al. (2008) that the linear response plateau yield function with stochastic plateau provides a better fit than a nonstochastic plateau. The value of using a stochastic linear response plateau instead of a nonstochastic alternative functional form was estimated to be $9/acre, so the economic benefit is not huge. The finding by Makowski and Wallach (2002) that it pays to use a random parameter model to calculate N application rates is supported, but the loss from not using random parameter models to determine the optimal level of N application is small, because optimal N rates do not differ greatly between stochastic and nonstochastic models and the expected profit function is relatively flat.

Another implication of this study regarding the flatness of the profit function is that it brings into question the economic feasibility of variable rate application technologies that are being developed to improve N use efficiency. If forage producers have a wide margin of error when deciding how much N to apply, the cost of obtaining a more accurate estimate of N may not exceed the benefit, because the cost of "being roughly right" in the N application rate is not large.

The observation by Cerrato and Blackmer (1990) and other researchers that the quadratic functional form estimates a higher optimal N application rate than a linear response plateau functional form is supported for stochastic models, but not nonstochastic models. The quadratic functional form implies a yield decline beyond the maximum yield as a result of excess N fertilization, which is rarely observed in field studies. Nevertheless, our data do show an unsustained yield decline at a high N application rate. Other studies do find a quadratic functional form providing a better statistical fit (Belanger et al., 2000), which means that crop yield functions with plateau may not dominate in every situation. In a practical farm extension context, stochastic production functions provide a way of incorporating production uncertainty into input decisions. The methodology developed to determine N application recommendations is applicable to other crops as well as other areas. The methodology is of benefit to producers because it improves the precision of optimal N recommendations under production uncertainty as well as N use efficiency and farm profitability.

Current recommendations of fertilizing annual cool season cereal rye–ryegrass pastures from the Samuel Roberts Noble Foundation are to apply 100–200 lbs/acre. Our estimated optimal rates are within this range. Based on the estimates from the stochastic linear response plateau, the 95% confidence interval level is to apply between 115.5 lbs/acre to 171.8 lbs/acre annually.

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