Modeling farmer participation to a revenue insurance scheme by means of Positive Mathematical Programming

Simone Severini and Raffaele Cortignani
Università della Tuscia di Viterbo
Facoltà di Agraria
Via S. C. De Lellis, snc
I – 01100 Viterbo, Italy
Phone: IT-0761-357241
Fax: IT-0761-357295

Abstract

European farmers face increasing income uncertainty and the debate is growing on the role of insurance schemes and of public support in this field. This debate is further stimulated by the perspective of introducing instruments to cope with risk also in the Common Agricultural Policy. Therefore, there is a need for empirical analysis and tools aimed at providing empirical evidences on this subject.

This paper applies a PMP modelling approach that takes into explicit consideration risk aversion behaviour to test the possibility to use it to assess the implications of participating in a insurance scheme. This is done by introducing a revenue insurance scheme into a model developed on a small group of crop farms in Italy. In particular, a quadratic mix integer programming approach has been developed in order to model the choice of participating or not in the proposed insurance scheme. The model has been than used to conduct simulations considering changes in the level of the insurance premium.

The paper tries to assess the soundness of the proposed approach and to identify its limitations. The obtained results suggest that this could be a useful tool to investigate the impact of participating in insurance schemes on production patterns and farm profitability and the role of public support in this field.

Keywords: insurance schemes, PMP, farmers' participation, risk aversion, non-linear mix-integer programming.

JEL classification: Q12, C61, Q18.

1. INTRODUCTION

Farmers are perceived to face an increasing income uncertainty. Commodity prices have been characterised by increasing volatility in recent years. This has been experienced also in the domestic EU market given that the Common Agricultural Policy (CAP) has reduced its role in price stabilisation. Production risk is also expected to increase in the future because the current climate changes may bring about higher yield variability due to the increasing occurrence of extreme events and weather variability. For these reasons, the debate is growing on the potential role of private and of publicly funded instruments to manage farm risk including those measures financed by CAP.

Because of all these elements, it seems relevant to develop evaluation approaches able to provide insights on management strategies to cope with risk, including insurance schemes. In order to do so, models used in empirical analysis should explicitly take into consideration farmers' risk aversion behaviour (Moschini and Hennessy, 2001).

While other approaches have been developed to modelling revenue crop insurance at farm level (See, for example, Hansen and Henry de Frahan (2010)), this paper focuses on Positive Mathematical Programming (PMP). In particular, it applies a PMP approach proposed to taking into explicit consideration risk aversion behaviour (Cortignani and Severini, 2010) in order to test whether it can be used to evaluate the potential impact of insurance schemes. This is done by introducing a revenue insurance scheme into a model developed on a small group of field crop farms located in Central Italy. Unlike a recent paper (Severini and Cortignani, 2011), this model has been

developed to explicitly depict the choice of farmers to participating in the proposed scheme by means of a non-linear mix-integer approach. This allows to a better investigation of the impact of the insurance scheme.

The objective of the paper is to develop a preliminary attempt to assess the soundness and applicability of the proposed approach, to consider its strengths and weaknesses and to identify future developments needed to improve it. Indeed, the paper is presented with the aim of exchanging opinions with other researchers interested in the topic and to receive critiques and suggestions with the aim of improving the approach.

Despite the limited scope of the empirical application, some very preliminary and tentatively considerations on the usefulness and drawbacks of the analysis to explore policy relevant questions are also derived.

The following two paragraphs briefly provide some background information on the insurance schemes applied in agriculture and on the developed modelling approach. Paragraph 4 presents the empirical analysis while the last paragraph provides some conclusions.

2. INSURANCE SCHEMES AND THE ROLE OF GOVERNMENTAL POLICIES

Revenue insurance is the kind of insurance scheme considered in the empirical application of the model. It combines yield and price risk coverage in a single insurance product and it can be product-specific or whole farm (EC, 2006). This insurance could be cheaper than insuring independently price and yield, as the risk of a bad outcome is smaller: indeed, low yields may be compensated by high prices and vice-versa. Nevertheless, this kind of insurance is not very common in the EU but available in the USA (EC, 2006; Edwards, 2009).

Governments have traditionally developed public policies aimed at increasing the risk management ability of farmers including subsidies to premium (Cafiero et al., 2005). This is a very common instrument that is often justified on the grounds that the premium must be affordable, that a sufficient volume of insurance contracts must be underwritten and that insurance companies have to find the insurance product attractive enough to remain in the business.

The emphasis on this instrument has increased also within the CAP. The reform of the CMO wine (Reg. (EC) n. 479/2008) has introduced the possibility of providing public funds for harvest insurance in order to contribute to safeguarding producers' incomes where these are affected by natural disasters, adverse climatic events, diseases or pest infestations. A broader instrument has been introduced after the 2009 Health check of the CAP. Art. 68 of Reg. (EC) n. 73/2009 allows Member States to use up to 10% of their first pillar funds to grant specific support to farmers, among others, in the form of contributions for insurance premiums.

The role of CAP in supporting the insurance scheme is expected to increase in the near future. The Commission has proposed that "a risk management toolkit should be included to deal more effectively with income uncertainties and market volatility that hamper the agricultural sector's possibility to invest in staying competitive. The toolkit would be made available to Member States to address both production and income risks, ranging from a new WTO green box compatible income stabilization tool, to strengthened support to insurance instruments and mutual funds" (EC, 2010: page 11).

3. METHODOLOGY

Positive Mathematical Programming (PMP) models have been extensively used to evaluate farmers' adjustment to changes in market and policy conditions. However, these models generally consider risk aversion behaviour only implicitly by means of the estimated cost function included in their objective functions. Few Authors have gone forward proposing ways to explicitly consider risk aversion behaviour (Heckelei, 2002; Paris and Arfini, 2000).

Recently, a way to explicitly incorporate such behaviour into PMP models has been proposed and empirically tested (Cortignani and Severini, 2010; Severini and Cortignani, 2011). This approach, formally described in the appendix, is based on a simple expected utility framework under the uncertainty of activity gross margins and assuming constant absolute risk aversion coefficients (McCarl and Spreen, 1997).

The model has the following general structure:

$$\max_{x_n} Z = E(\widetilde{gm}_n)' x_n - d'_n x_n - \frac{1}{2} x'_n Q_n x_n - \frac{1}{2} \phi_n x'_n \Sigma_{gm} x_n \qquad [1]$$

s. to

$$A' x_n \leq b_n \qquad [\lambda_n] \qquad [2]$$

where $E(\widetilde{gm}_n)$ are the expected unitary gross margin values; x_n are the model variables that refer to the land allocated to each activities in the *n-th* farms; d_n and Q_n are the parameters of the quadratic cost function; ϕ_n are the farm specific coefficients of absolute risk aversion and Σ_{gm} the covariance matrix of the unitary gross margins.

The parameters d_n, Q_n, the λ_n dual values and ϕ_n are estimated by imposing the first-order conditions of the considered farm model taking into account exogenous information (i.e. supply elasticities) and all the observations over the considered period in which data is available (Heckelei, 2002). The Σ_{gm} has been calculated by taking into consideration the variability of gross activity margins observed in the same period in the farm sample. The estimation model is described in the Appendix.

This paper develops this kind of model in order to assess its potential use to evaluate the potential role of revenue insurance schemes. The model considers the possibility to participate in a revenue insurance scheme for a single crop (i.e. durum wheat in the empirical application). When participating in the program, the farmer pays an insurance premium and, if the unitary revenue of that crop falls below the expected level, he/she receives an indemnity calculated on the basis of the difference between the expected and the actual revenue level. In this case, the expected gross margin vector and covariance matrix of gross margins are recalculated and differ from the case without the insurance scheme.

In a preliminary application of this model, it has been assumed that all farmers participate in the insurance scheme whenever they grow durum wheat in a sort of "compulsory participation" (Severini and Cortignani, 2011). This paper goes further by removing such very restrictive hypothesis by explicitly modelling the participation choice: the model has been developed to allow for the discrete choice of participating or not to the proposed scheme by means of a quadratic mix integer formulation.

In order to do so, the simulation models have the following general structure:

$$\max_{xi_n,\ x_n} Z = E(\widetilde{gm}_{n,"un"})^{'}\, xi_{n,"un"} - \frac{1}{2}\phi_n\, xi^{'}_{n,"un"}\, \Sigma_{un}\, xi_{n,"un"}$$

$$+\ E(\widetilde{gm}_{n,"in"})^{'}\, xi_{n,"in"} - \frac{1}{2}\phi_n\, xi^{'}_{n,"in"}\, \Sigma_{in}\, xi_{n,"in"} \qquad [3]$$

$$-\ d^{'}_n\, x_n - \frac{1}{2}\, x^{'}_n\, Q_n x_n$$

s. to

$$A^{'}\, x_n\ \leq\ b_n \qquad\qquad [4]$$
$$x_n =\ xi_{n,"un"} + xi_{n,"in"} \qquad\qquad [5]$$
$$xi_{n,"in"}\ \leq\ b_n * \delta_n \qquad\qquad [6]$$
$$xi_{n,"un"}\ \leq\ b_n * (1 - \delta_n) \qquad\qquad [7]$$

$E(\widetilde{gm}_{n,"un"})$ and $E(\widetilde{gm}_{n,"in"})$ are expected unitary gross margin values for the crop *j* without (index *un*) and with (index *in*) the insurance. This latter vector takes into account both the insurance premium and the indemnities.

Variables x_n are splitted into two further variables: $xi_{n,"un"}$ and $xi_{n,"in"}$. These refer to the amount of land of each crop grown without and with insurance, respectively.

Σ_{un} and Σ_{in} are the variance-covariance matrixes of activity gross margins without and with insurance; δ_n is a farm specific dichotomous variable that can takes the values 1 or 0.

The portion of the objective function that accounts for the participation case is given by the second and the third lines of [3]. The second line accounts for the expected values and the covariance matrix of the gross margins taking into account the role of the insurance scheme.

Constraint [5] requires that the sum of the variable xi for each crop (with and without insurance) are equal to the variable x.

Constraints [6] and [7] allow to make the participation choice discrete. Indeed, when variable δ_n for a specific farm is equal to 1, this forces the farmer to participate into the program with all available land and vice-versa.

Therefore, for a farm participating in the scheme (variable δ =1), the first line of the objective function [3] cancels out and the objective function only refers to the case with insurance. The opposite occurs in the non participation case (variable δ =0).

4. EMPIRICAL ANALYSIS

A sample of 27 FADN farms (constant in the period 2005-2007) specialized in cereals, oilseed and protein crops – located in the province of Ancona (Marche, Italy) - has been taken into consideration[1].

---

[1] We tanks the Italian Institute of Agricultural Economics (INEA) of Rome that has supplied the FADN farm data.

Table 1: Share of each crop in terms of the total cropped area per year and three year average (%)

	2005	2006	2007	Average
Durum Wheat	63.3	49.9	64.1	59.1
Maize	3.8	3.8	6.4	4.7
Other cereals	4.8	3.7	5.1	4.5
Sunflower	14.4	18.9	13.8	15.7
Other crops	13.6	23.8	10.7	16

Source: Own calculation on FADN data

Most of the area is cultivated to durum wheat which, on average, uses around 60% of the cropped area (Table 1). Other important crops are sunflower and maize.

Before turning to the simulation results, it seems useful to briefly discuss the calibration results and, in particular, the levels of the recovered absolute risk aversion (ARA) coefficients. Two over the 27 farms show a null ARA coefficient suggesting a non-risk aversion behaviour. The remaining 25 farms show low levels of ARA coefficients: in 12 cases these coefficients are non-zero but lower than 0.0002, in 11 farms these range between 0.0002 and 0.0004, while only in two farms these coefficients are higher than 0.0004. The level of the ARA coefficients seems to be negatively but weakly correlated with the farm size (Correlation index = -0.553). No correlation is found between the level of these coefficients and the degree of production specialization of the considered farms.

While the calibrated model relies on the assumption that the analysed insurance scheme is not available to the farmers (BASELINE), all simulations refer to the case in which farmers can decide whether to participate in the insurance scheme or not. The baseline insurance simulation case (BLINS) is described first. Then, another set of simulations considers changes in the level of unitary premium (PREM) (Table 2).

Table 2: Synthesis of the simulation scenarios.

Simulation code	Short description of the simulations
BLINS	Baseline insurance simulation case. It refers to durum wheat only and considers full coverage (100% indemnity) and a premium set at 197.7 €/ha.
PREM	It considers different level of the premium paid by farmers: increases and decreases of: 25%, 50%, 75%, 100% from the BLINS case.

All simulations assume that an indemnity (*ind*) is paid to farmers whenever the level of unitary revenues from durum wheat is below its expected revenue level (*E(rev)*). This latter level is calculated on the basis of the weighted average of unitary revenues from the observations in the following way:

$$E(rev_n) = \frac{\sum_t rev_{n,t} * x_{n,t}^0}{\sum_t x_{n,t}^0}$$
[8]

where *x°(n,t)* are the amount of land devoted to durum wheat in each farm and period.

The unitary premium paid (pre) is identified on the basis of the arbitrary hypothesis that the expected total amount of indemnities (*E(TIND)*) should be equal to 80% of the expected total amount of premiums (*E(TPRE)*)[2]. These are calculated *ex-ante* on the basis of the available three year data set in the following way:

$$E(TIND) = \sum_{n,t}\big(E(rev) - rev_{n,t}\big) * x_{n,t}^0 \qquad\qquad [9]$$

$$E(TPRE) = \sum_{n,t} pre * x_{n,t}^0 \qquad\qquad\qquad\qquad [10]$$

Note that a uniform unitary premium per hectare of durum wheat (*pre*) is assumed to be applied to all farmers that decide to participate in the insurance scheme.

The unitary revenues for durum wheat in all observations (i.e. for all n and t) are then recalculated introducing the insurance scheme previously described. This generates a new set of unitary gross margins that differs from the original one only in the gross margins of durum wheat. This set is then used to recalculate the variance-covariance matrix for unitary gross margins.

5. ANALYSIS OF THE SIMULATION RESULTS

The empirical analysis has been developed mainly for testing the model and to assess how it responds to: a) the introduction of the insurance scheme; b) changes in the levels of the premium paid by farmers. Table 3 reports some basic parameters for durum wheat under the baseline and the BLINS scenario.

Table 3: Durum wheat. Basic economic parameters.

		BASELINE	BLINS
Expected gross margin	(€/ha)	646	605
Total variance of durum wheat gross margins	€[2]	130,160	99,629
Premium	(€/ha)	0	197.7
Expected indemnity	(€/ha)	0	263,83

Note: Data are calculated as weighted average on the whole farm sample.

The introduction of the revenue insurance generates that 7 over 27 farm models participate in the insurance scheme and that around 28% of the grown durum wheat is insured (Table 4). This also causes a small increase of the total area devoted to durum wheat.

---

[2] This ex-ante evaluation may not be satisfied ex-post because farmers can decide whether or not to subscribe the insurance contract.

Table 4 - Cropping patterns under the baseline (no insurance) and different scenarios. Whole sample.

	Baseline without insurance (BASELINE)	Baseline with insurance (BLINS)	Changes of the premium rate from BLINS (PREM scenario)							
			Relative increases of the premium				Relative decreases of the premium			
			100%	75%	50%	25%	-25%	-50%	-75%	-100%
Insured farms (n°)	0	7	0	0	2	4	11	18	23	27
	ha	*ha*	*Percentage change from the BLINS values (%)*							
Durum wheat	460	468	-1.7	-1.7	-1.5	-1.0	1.7	4.0	8.9	11.6
insured	0	133	-100.0	-100.0	-66.0	-55.4	58.4	199.3	246.9	293.4
uninsured	460	335	37.2	37.2	24.1	20.5	-20.8	-73.4	-85.4	-100.0
Maize	37	38	-1.1	-1.1	-1.1	-1.1	-5.2	-16.2	-35.1	-42.9
Other cereals	35	32	9.6	9.6	6.9	3.6	-5.1	-5.3	-1.0	-0.6
Sunflower	122	120	1.7	1.7	1.5	1.2	-1.0	-3.2	-8.6	-12.4
Other crops	91	88	3.9	3.9	3.9	2.9	-3.5	-7.9	-20.4	-26.5

Source: Own elaboration on FADN data.

The possibility to participate into the scheme increases slightly the overall farm expected gross margins (Table 5). This is due to the fact that, in average, the expected indemnities are greater than the premium paid by farmers and that, despite previous analysis, the model depicts the participation choice on a individual farm basis.

Table 5 - Main economic results under the baseline (no insurance) and different scenarios. Whole sample.

	Baseline without insurance (BASELINE)	Baseline with insurance (BLINS)	Changes of the premium rate from BLINS (PREM scenario)							
			Relative increases of the premium				Relative decreases of the premium			
			100%	75%	50%	25%	-25%	-50%	-75%	-100%
	€ 1,000	*€ 1,000*	*Percentage change from the BLINS values (%)*							
Total Gross Margins	662	671	-1.4	-1.4	-1.1	-0.7	1.3	3.7	6.9	10.4
Total amount of premiums	0	26	-100.0	-100.0	-49.1	-44.2	18.8	49.7	-13.3	-100.0
Total amount of indemnities	0	35	-100.0	-100.0	-57.5	-47.0	38.8	110.7	127.4	134.0
Loss ratio w/out gov. support ^ (%)	-	133	-	-	111	127	156	188	350	-
Government support^^ (1,000 €)	0	53	0	0	16	26	89	159	231	308
Loss ratio with gov. support (%)	-	80	-	-	80	80	80	80	80	80
Premium paid by farmers (€/ha)	-	198	-	-	297	247	148	99	49	0
Unitary gov. Support (€/ha)	-	132	-	-	116	144	141	133	167	196

^: Total expected amount of premiums paid by farmers divided by total expected value of indemnities. ^^: Government support to insurance companies to ensure ex-post a 80% loss ratio. Source: Own elaboration on FADN data.

The loss ratio is even higher than 100% clearly indicating that total expected indemnities exceed the total amount of premiums paid by farmers (Table 5)[3]. This implies that insurance companies cannot find this market attractive without government support. In order to reach a 80% loss ratio, government support must be relatively large: it represents around 2/3 of the total revenues of the insurance companies (Table 5).

Under the conditions set by the scenario BLINS, the increase of expected gross margin due to the introduction of the insurance scheme is only around 18% of the total amount of the government support (Table 5). This suggests that only a small share of this support translates into an increase of farmers income.

---

[3] The ex-post loss ratio is way higher than the one used ex-ante to identify the BLINS premium (80%). This seems consistent with the way the participation choice has been modelled.

The model has also been tested considering increases and decreases in the level of the premium paid by farmers (Scenario PREM).

As expected, when the premium increases, the number of farm models participating in the scheme declines (Table 4): when the premium is increased of at least 50%, no farm models participate into the scheme. Increasing the premium has a very negative impact on the amount of durum wheat enrolled in the insurance scheme and, to a way lower extent, also on the total amount of durum wheat (Table 4).

Increasing the premium rate clearly has a negative impact on the expected farm gross margins that, for increases of 50% and higher, go back to the baseline level (Table 5). The reduction of the amount of land enrolled in the scheme generates a decrease of the total expected indemnities and of the total premiums paid. This latter result suggests that the increase of the premium does not compensate for the reduction of the land enrolled in the scheme. Because the decrease of total indemnities is greater than that of the total premium paid, the loss ratio declines from the BLINS case (Table 5). However, it remains always higher than 100% generating a need for government support.

The relative importance of this support declines from the BLINS case but only slightly: with a premium 50% higher than in the BLINS case, government support still accounts for more than ½ of the overall revenues of the insurance companies (Table 5). Given that the farmers are asked to pay higher premium than in the BLINS case, lower shares of government support translate into increases of farm expected gross margins. With a premium 50% higher than in the BLINS case, the increase of expected gross margins is only around 11% of the overall government support (Table 5). This ratio is lower than that observed under the BLINS conditions even because only farm models with relatively high expected indemnities remain enrolled in the insurance scheme.

Opposite results are obtained when the premium decreases. In this cases more farm models participate in the scheme: all of them are enrolled when the premium is fully paid by means of government funds (Table 4). Decreasing the premium has a very positive impact on the amount of durum wheat enrolled in the insurance scheme. However, this also generates a not negligible increase of the total area devoted to durum wheat. For example, when the premium is decreased by 50% from BLINS, the total durum wheat area increases by around 4% (Table 4).

Decreasing the premium rate clearly has a positive impact on the expected farm gross margins: for example, when it is set at half the BLINS level, these increase by almost 4% (Table 5). The increase of the amount of land enrolled in the scheme generates an increase of total expected indemnities and, at least for decreases up to 50%, of the total amount of premiums paid by farmers. Clearly this increases the expected loss ratio and, in order to ensure a 80% loss ratio, the amount of support must be strongly increased in order to compensate for the reduction of unitary farm payments and the increase of participation (Table 5).

Because of the positive effect of the decrease of the premium on the expected gross margins, higher shares of government support translate into increases of such margins. With a premium 50% lower than in the BLINS case, the increase of expected gross margins is only around 22% of the overall government support (Table 5). This ratio is higher than under the BLINS conditions even because now also farm models with relatively low expected indemnities participate in the insurance scheme.

6. CONCLUSIONS

This paper has used a PMP modelling approach that includes exogenous information on gross margin variability. This permits to recover farm specific risk aversion coefficients and to develop a model that has been found to respond to simulation scenarios in a different way than other PMP models (Cortignani and Severini, 2010). Furthermore, this kind of model can be used to evaluate the likely impact of changes in the variability of gross margins and of introducing an insurance scheme (Severini and Cortignani, 2011).

In this paper the model has been used to evaluate the impact of introducing a revenue insurance scheme for a single activity and of changing the level of premium paid by farmers. The analysis presented here has overcome an important limitation that affected previous work on this subject. By using a non-linear mixed integer programming approach, it has been represented the choice of the farmers to participate or not in the insurance scheme. This approach has been applied to data from a small group of field crop farms located in Central Italy in order to develop a first preliminary empirical test.

The analysis has a couple of limitations which are important to mention before summarising its main results. First, the modelling approach relies on a simplified and restrictive expected utility framework that assumes constant absolute risk aversion coefficients. Second, the empirical test considers only one specific type of insurance scheme and a very limited and specific sample of farms.

Despite these limitations, the analysis has produced some interesting results. The model has been able to investigate the impact of introducing an insurance scheme and of changing the level of the premium paid by farmers. It has permitted to assess how this affects participation and production choices and the relative profitability of both farmers and insurance companies. Furthermore, it allows to assess if and under which conditions such scheme could remain in place.

The results of the empirical test suggest that the proposed model responds in a coherent way to the considered simulations. Introducing the insurance scheme provides an incentive for some farmers to participate and to increase the land used to grow the insured crop. Decreasing (increasing) the level of the premium paid by farmers increases (decreases) the participation to the insurance scheme and the acreage of insured but also of total durum wheat. Thus, under the considered case, providing government subsidies increases the production of the insured crop showing its small production-distorting nature. Finally, decreasing (increasing) the level of premium positively (negatively) affects farm economic results. However, in the considered empirical conditions, the proposed insurance scheme seems not to be profitable for insurance companies. Thus, such market could be developed only if government provides subsidies in order to cover a large share of the premium.

REFERENCES

Cafiero C., Capitanio F., Cioffi A., Coppola A. (2005). Risk and crisis management in agriculture. Final Report prepared for European Parliament. October 2005.

Cortignani R., Severini S. (2010). Taking into explicit consideration risk aversion behaviour in Positive Mathematical Programming models: an empirical application. Paper presented at: III Workshop on Valuation Methods in Agro-food and Environmental Economics. CREDA-UPS-IRTA, Castelldefels (Barcellona), Spain, 1-2 July 2010.

Edwards W. (2009).Crop Revenue Insurance. Iowa State University, University Extension. Ag Decision Maker. Available at: http://www.extension.iastate.edu/agdm/

European Commission (EC) (2006). Agricultural Insurance Schemes. European Commission – Joint Research Centre, Ispra (Italy).

European Commission (EC) (2010). The CAP towards 2020: Meeting the food, natural resources and territorial challenges of the future. COM(2010) 672 final. Brussels, 18 November 2010

Hansen K., B. Henry de Frahan (2010). Risk in Agriculture: Modeling Revenue Insurance for Crop Farms in Belgium. Paper presented at the III Workshop on Valuation Methods in Agro-food and Environmental Economics. CREDA-UPS-IRTA, Castelldefels (Barcellona), Spain, 1-2 July 2010.

Heckelei T. (2002). Calibration and estimation of programming models for agricultural supply analysis, University of Bonn.

McCarl B.A., Spreen T. H. (1997). Applied Mathematical Programming Using Algebraic Systems, available on-line to: http://agecon2.tamu.edu/people/faculty/mccarl-bruce/books.htm.

Moschini G., Hennessy D. A. (2001). Uncertainty, Risk Aversion and Risk Management for Agricultural Producers, Handbook of Agricultural Economics, Volume1.

Paris Q., Arfini, F. (2000). "Funzioni di costo di frontiera, auto-selezione, rischio di prezzo, PMP e Agenda 2000", Rivista di Economia Agraria, Anno LV, N.2, Giugno.

Severini S., Cortignani R. (2011). An attempt to modelling revenue insurance schemes at the farm level by means of Positive Mathematical Programming. Paper accepted for presentation at: the 122nd EAAE Seminar. Ancona, February 17-18, 2011.

APPENDIX. DESCRIPTION OF THE ESTIMATION MODEL.

We use the method proposed by Heckelei (2002) extending it to explicitly considering risk aversion[4]. This uses the Generalized Maximum Entropy (GME) approach covered by the restrictions needed to determine the appropriate curvature of the cost function and incorporates exogenous supply elasticities (Heckelei, 2002).

Considering that the data refer to several years (t = 1, ..., T), the GME problem is specified as follows:

---

[4] For details see Cortignani and Severini (2010)

$$max_{w_t,\mathbf{d},\boldsymbol{\phi},\boldsymbol{Q},\boldsymbol{L},\lambda_t} \; H(\boldsymbol{w}_t) = -\sum_{t=1}^{T} \mathbf{w}'_t \ln \mathbf{w}'_t - \mathbf{w}^{\varepsilon'} \ln \mathbf{w}^{\varepsilon}$$

s. to

$$E(\widetilde{gm}_t) - \lambda_t \boldsymbol{A} - \boldsymbol{d}_t - \boldsymbol{Q}(x_t^0 - \boldsymbol{V}w_t) - \boldsymbol{\phi}\,\Sigma_{gm}(x_t^0 - \boldsymbol{V}w_t) = 0$$

$$E(\widetilde{gm}_t) - \lambda_t \boldsymbol{A} - \boldsymbol{d}_t - \boldsymbol{Q}(x_t^0 - \boldsymbol{V}w_t) - \boldsymbol{\phi}\,\Sigma_{gm}(x_t^0 - \boldsymbol{V}w_t) < 0$$

$$\boldsymbol{A}'\left(x_t^0 - \boldsymbol{V}w_t\right) = b_t$$

$$\boldsymbol{Q} = \boldsymbol{L}\boldsymbol{L}' \qquad con\; L_{j,i'} = 0 \quad \forall\; i' > j$$

$$\boldsymbol{V}^{\varepsilon}\,\mathbf{w}^{\varepsilon} = \left[\left(\boldsymbol{Q}^{-1} - \boldsymbol{Q}^{-1}\boldsymbol{A}\,(\boldsymbol{A}'\boldsymbol{Q}^{-1}\boldsymbol{A})^{-1}\,\boldsymbol{A}'\,\boldsymbol{Q}^{-1}\right) \odot \left[\frac{gm^o}{x^0}\right]\right]$$

$$\sum_{s=1}^{S} w_{j,t,s} = 1$$

$$\sum_{s=1}^{S} w_{j,i,s}^{\varepsilon} = 1$$

where *H(wt)* is the level of entropy, the errors vector ($\boldsymbol{V}w_t$) is re-parameterized as the expected value of a discrete probability distribution by defining the $\boldsymbol{V}$ support matrix and the w_t probabilities vector; elasticities ($\boldsymbol{V}^{\varepsilon}\,\mathbf{w}^{\varepsilon}$) are re-parameterised in the same way as the error terms by defining the $\boldsymbol{V}^{\varepsilon}$ support matrix and the $\mathbf{w}^{\varepsilon}$ probabilities vector[5]; $\boldsymbol{gm}_t$ are the gross margins of each activity; λ_t is the shadow price of land over several years; $\boldsymbol{A}$ is the technical coefficients matrix; $\boldsymbol{d}_t$ and $\boldsymbol{Q}$ are respectively the parameters associated with the linear term and the quadratic term of the cost function; x_t^0 are the observed levels of activity in different years; $\boldsymbol{\phi}$ are the coefficients of absolute risk aversion for each farms n and Σ_{gm} the covariance matrix of the gross margins[6]; $\boldsymbol{L}$ is the lower triangular matrix of the Cholesky decomposition. The first two constraints impose the first order conditions for the observed and for the not observed activities. The following two equations ensures that the land allocated to different crops in each year is equal to the total available land, and the proper curvature of the cost function. The fifth constraint is the combination between the elasticity re-parameterization *($\boldsymbol{V}^{\varepsilon}\,\mathbf{w}^{\varepsilon}$)* with the Jacobian matrix that contains the partial derivates of the land demand functions ($\frac{\partial x_i}{\partial gm_i}$); the matrix $\left[\frac{gm^o}{x^0}\right]$ is defined as the sample mean of activity gross margin (gm^o) divided by the sample mean of observed land allocation (x^0). The last two constraints relate to the probability law (where *s* is the number of support values).

---

[5] The intuition behind the objective function is that the entropy criterion pulls towards the centre of the elasticity support range, in opposition to the error terms of the data constraints. The smaller the elasticity support range, the higher the penalty for deviating from the support centre. Consequently, the width of the support range reflects the precision of the *a priori* information (Heckelei and Wolff, 2003).

[6] Upper and lower bounds on the level of the coefficient of absolute risk aversion have been imposed. The E-V risk aversion coefficient equal the E-standard error risk aversion coefficient divided by twice the standard error. Because the E-standard error risk aversion coefficient usually ranges from 0 – 3 (McCarl and Spreen, 1997), these values have been chosen as lower and upper bounds. The Σ_{gm} has been calculated taking into consideration the variability of gross activity margins observed during the three-year period.