Irreversible Investment, Uncertainty and Ambiguity: The Case of the Bioenergy Sector

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Summary : We analyse the decision of an agent to invest and engage in industrial activities that are characterized by two forms of uncertainty: market size uncertainty and competitive effect uncertainty. We apply our model on the bioenergy industries. We compare the case of an ambiguity neutral agent with that of an ambiguity adverse agent. We show that the investment decision of an agent depends on the effects of both the capital investment and the level of production on the cost and the uncertainty the agent is confronted with. Moreover, we find that ambiguity aversion tends to decrease the agent's optimal levels of production and investment. Our numerical analysis of the French case illustrates the different effects associated with market size uncertainty and competitive effect uncertainty.

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1 Introduction

Investments into renewable technologies will have to develop in order to reach the renewable energy target of 20% fixed by the European Union (EU) for 2020.[1] To reach the future targets set out by the EU, significant amounts of biomass and investments into biomass based technologies will be necessary.[2] Biomass is key to the development of renewable energies, but it must undergo a pretreatment and densification process before it can be transported and stored. Indeed, biomass is a resource that is heterogeneous in quality and is not homogeneously distributed across space. Therefore, the wide variety of biomass types does not correspond with the specifications of feeding systems and the conversion processes considered. Investment in new pre-treatment facilities is a necessary step in the total biomass supply chain in order to save transport, material, handling costs for users and to reduce investments in transformation facilities.

These pre-treatment processes are still in progress and the biomass market is emerging. A potential investor has information about the demand and the competition effect on the supply market. He might be confronted with uncertainties about the market size and competition,*i.e.*, uncertainty on the number of buyers, and an uncertainty about the supply for energy markets. Both types of uncertainty affect prices in different ways.

Uncertainty about the number of buyers is related to the variability of the number of potential buyers. The agent will either have to supply a few potential buyers such as heat and electricity producers, needing to replace coal, or a larger number of potential buyers including producers of second generation biofuel and heat and electricity producers. Market size uncertainty affects the agent's perception of the average price.

Uncertainty about competition is related to the variability of the competition from other fuel suppliers based on the selling price of pre-treated biomass. The biomass may be sold either to heating or power units as a substitute for coal (the selling price could then be indexed to coal prices) or to Biomass to Liquid (BtL) units as a substitute for fossil fuel and prices could then be indexed to oil prices, which fluctuate even more sharply than coal prices (Fuss & Szolgayova, 2009). Uncertainty about competition affects the variance price.

So, considering these two kinds of uncertainty, a biomass agent has to decide how much capital investment he will make in biomass activities taking into account selling price uncertainty. Furthermore, the cost of entry in bioenergy production represents a quasi-sunk cost due to the fact that biomass torrefaction is a specific, and relatively expensive, process. This naturally raises the issue of the effect of both types of uncertainty and of the irreversibility on the investment level and production.

Furthermore, in the energy market, the instability of the economy may lead the agent to doubt his evaluation of the variance of the output price. We use the term "ambiguity" to indicate situations in which the odds of an uncertain event are not precisely known. In other words, a situation in which there is an "uncertainty about uncertainty".[3] An agent who has doubts about the odds is considered as an ambiguity-averse agent. So a question arises: how will an ambiguity-averse agent behave when he makes his decisions concerning investment and production?

[1] In 2007, the European Commission has fixed the renewable energy target in the EU's overall mix to 20% in the final energy consumption by 2020 regarding 1990. To reach this goal, the member states have adopted the pack energy-climate and renewable energy (European Commission, 2009) in particular which defines the operational measures to develop 20% of renewable energies by 2020.

[2] Currently biomass delivers around 4% of the EU's primary energy (EEA, 2008).

[3] For more details on ambiguity approach, see Camerer (1999); Etner et al. (2010).

To understand the impact of uncertainty on investment and production in biomass activities, we propose a two-period model in which there is incomplete information about the competitive effect and the market size. Under these uncertainties, the agent has to decide how much he wants to invest for the production of his pre-treated biomass units at the following period. We study the cases of an ambiguity-neutral agent and of an ambiguity-averse agent. Following Klibanoff et al. (2005), we extend our work by presenting ambiguity as a second order prior probability distribution over the set of plausible distributions of the competitive effect. This approach allows us to analyse the impact of ambiguity on the investment choice.

The standard theory of irreversible investments or quasi sunk cost (Henry, 1974; Sutton, 1991) and options values suggests a negative relation between investment and uncertainty (Dixit & Pindyck, 1994). Empirical studies also confirm this negative relation (Carruth et al., 2000; Bond et al., 2005; Fan & Zhu, 2010). However, (Kulatilaka & Perotti, 1998) and (Sarkar, 2000) point out that an increase in uncertainty could increase the probability of investing, and thereby has a positive impact on investment. Mohn & Misund (2009) argue that any positive impact on investment arising from the fact that greater uncertainty, under certain circumstances, increases the marginal profitability of capital. While the effect of price uncertainty has been analysed in many papers (Elder & Serletis, 2009, 2010), no work has been done on the two types of uncertainty that affect prices in different ways: the perception of the average price (market size uncertainty) and the price variance (competitive effect uncertainty). Our contribution to this literature is to examine the impact of both types of uncertainty on irreversible investment.

Concerning ambiguity, we refer to the basic literature on ambiguity with (Ellsberg, 1961) and Fellner (1961, 1965), the empirical investigations by Slovic & Tversky (1974) and the recent literature with (Klibanoff et al., 2005) and Gollier (2006) to indicate situations for which the odds of an uncertain event are not precisely known. Determining how an ambiguity-averse agent decides to invest and engage in emerging technologies is an important line of research in entrepreneurial decision-making in BtL.

Using an analytical approach and numerical analysis, we first note that whatever the certainty or uncertainty context, the agent never invests or produces when he thinks that an increase of the capital increases the cost of one more unit. Moreover, the agent's capital investment decision depends on the effects of the amount of capital invested, of the level of production on the cost and on the uncertainty to which the agent is confronted. Then, we observe asymmetric effects of uncertainty on the optimal amount of investment and optimal production. We show that the effect of market size related uncertainty is stronger than that of the competition related uncertainty as the investment and production levels are higher. This is true when, in the case of certainty, the effect of competition is weak and the number of buyers is low or high if and only if the agent's prior belief in the weak competition is lower than a certain threshold. Furthermore, if in the case of certainty the agent knows the competition is weak and the market size is high, the combination of both types of uncertainty leads him to invest less. He behaves similarly if the market size is low in the certainty case if and only if his prior belief concerning market size is lower than a certain threshold. Secondly, the effect of the competition related uncertainty is stronger than the uncertainty concerning the market size uncertainty since the investment and production levels are higher. This is true when in the case of certainty the competition is strong and the number of buyers is high, or low if and only if the agent's prior belief concerning the weak competition is higher than a certain threshold. Besides, if in the certainty case the agent knows the competition is strong and the market size is low, the combination of both types of uncertainty leads him to invest more. If the market size is high in the certainty case, he has the same behaviour if and only if his prior belief about the market size is higher than a certain threshold. Finally, ambiguity aversion tends to decrease both the agent's investment in capital and in production.

The French biomass pre-treatment industry (torrefaction) is taken as an example, and the empirical results show that the model developed here can provide useful advice for pre-treatment biomass investment programs.

The remainder of the paper is organized as follows. Section 2 consists of a description of the model. Section 3 analyses and compares the optimal investment and production decisions of both an ambiguity neutral agent and an ambiguity averse agent. Section 4 and 5 present the specification of the model and a numerical analysis, respectively. Section 6 concludes.

2 Model description

We consider a two period model with a risk-neutral agent. The agent faces two types of uncertainty: a market size uncertainty and a competition-related uncertainty. Indeed, whereas the agent knows that he is competing on the market of biomass providers,[4] *i.e.* he is a price taker, he only has a subjective perception of his potential customers and of the severity of the market competition . Both types of uncertainty affect prices in different ways: market size uncertainty pertains to the perception of the price average while the competition effect uncertainty pertains to the price variance.

We define four possible states of the world: a Low market size and a Weak competition effect (LW), a High market size and a Weak competition effect (HW), a Low market size and a Strong competition effect (LS) and High market size and a Strong competition effect (HS) . We propose to divide the agent prior beliefs on these states in two kinds of beliefs: first, the agent prior beliefs are ψ on the low number of buyers, and $(1-\psi)$ on the high number of buyers; second, the agent prior beliefs are θ on the strong competition, and $(1-\theta)$ on the weak competition. In addition, we consider that the "right" value of the probability associated to the competitive effect uncertainty θ may be unknown. In this case, θ is a random variable, and it is called $\tilde{\theta}$. The agent associates a probability distribution $F(\theta)$ on $[\underline{\theta}, \overline{\theta}]$ which measures the subjective relevance of a particular θ probability. The competitive effect is then ambiguous in the sense that his beliefs depend on a probability distribution. Instability in the energy market can cause the agent to become uncertain about the true value of probability θ, which pertains to the variance of the output price. So there may be a great deal of ambiguity associated with the competition based on the output selling price. Following Klibanoff et al. (2005), we describe the agent's behaviour towards ambiguity by a function ϕ. An increasing and concave ϕ means that the agent is ambiguity averse. Similarly, ambiguity neutrality is characterized by the linear function ϕ.

We associate a selling price P_i to each state $i \in \{LW, HW, LS, HS\}$. A larger number of buyers is likely to be able to support a higher price, so we get that $P_{HW} > P_{LW}$ and $P_{HS} > P_{LS}$. Moreover, competition between fuel suppliers leads to a lower price, $P_{HW} > P_{HS}$ and $P_{LW} > P_{LS}$.

At period 0, the agent has the opportunity to invest in plant in order to produce pre-treated biomass. Let be $K \geq 0$, the stock of capital and the investment costs $I(K)$. I is an increasing and convex function such that $I(0) = 0$. As in Cairns (2009), we assume a sunk capital, *i.e.*, a capital amount that is specific to the firm.

At period 1, if the agent has invested $I(K)$ at period 0, he has to choose his production q which repre-

[4] Indeed, the agent knows that there already exists substitute to pre-treatment process which could provide the biomass consumers.

sents the units of torrefied biomass. This yields a pay-off equals to $P_i q$ in state $i \in \{LW, HW, LS, HS\}$. From this pay-off must be subtracted the cost of production $c(q, K)$ which is an increasing and convex function with q and a decreasing and convex function with K. Moreover, we assume that if $q > 0$ then $c(q, 0) = \infty$, $c(0, K) = 0$.

So, with a discount factor $\beta < 1$, the agent's expected pay-off $V(K, q; \psi, \theta)$ is expressed as follows:

$$\begin{aligned} V(K,q;\psi,\theta) = \quad & -I(K) + \beta\psi\left[\theta\left(P_{LS}q - c(q,K)\right) + (1-\theta)\left(P_{LW}q - c(q,K)\right)\right] \\ & +\beta(1-\psi)\left[\theta\left(P_{HS}q - c(q,K)\right)) + (1-\theta)\left(P_{HW}q - c(q,K)\right)\right]. \end{aligned} \tag{1}$$

Likewise, considering the ambiguity approach, the agent's expected pay-offs is given by:

$$W(K,q;\psi,\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \phi\left(V(K,q;\psi,\theta)\right) dF(\theta) \tag{2}$$

3 Optimal decision making

3.1 Neutrality to ambiguity

In this section, we consider that the agent is aware of the true value of θ . In other words, there is uncertainty about the price variance and he believes that the probability associated with this uncertainty is relevant. In this case, we consider $W(K, q; \psi, \theta) = V(K, q; \psi, \theta)$.[5] At period 0, the agent has to determine his optimal stock of capital K^* for producing pre-treated biomass. Then, at period 1, he could decide which quantity q^* to produce. By consequence, we propose to solve this model through backward induction.

We define the expected price under market size uncertainty, the expected price under competitive effect uncertainty, and the expected price under both market size and competitive effect uncertainties, respectively, as follows:

$$E_\psi P_m = \psi P_{Lm} + (1-\psi)P_{Hm}, E_\theta P_j = \theta P_{jS} + (1-\theta)P_{jW}, \text{ and } E_{\psi\theta}P = \psi E_\theta P_L + (1-\psi)E_\theta P_H.$$

with $j \in \{L, H\}$ and $m \in \{W, S\}$.

So the first order condition on quantity is

$$\frac{\partial c(q,K)}{\partial q} = c_q = E_{\psi\theta}P. \tag{3}$$

If for all $q > 0$ we get $V(K, q; \psi, \theta) \leq 0$, *i.e.*, if the project is never profitable, then the agent does not produce. On the other hand, if there exists $q > 0$ such that $V(K, q; \psi, \theta) > 0$, the project is profitable for a certain level of production. This relation implies that q is an implicit function of K, $q \equiv q(K)$

Now, we study the optimal stock of capital K which maximizes the expected pay-off $V(K, q(K); \psi, \theta)$. The condition is given by the solution of the following program:

$$-\frac{\partial I(K)}{\partial K} + \beta\frac{\partial q(K)}{\partial K}[E_{\psi\theta}P] - \beta(\frac{\partial c(q,K)}{\partial q}\frac{\partial q(K)}{\partial K} + \frac{c(q,K)}{\partial K}) = 0. \tag{4}$$

Using equation (3), we obtain,

$$\beta c_K = -\frac{\partial I(K)}{\partial K} = -I'(K). \tag{5}$$

[5] More precisely, a neutral agent maximizes the expected pay-off $\phi(V(K, q; \psi, \theta))$ where ϕ is linear function. For notation convenience, we assume ϕ is a scalar equal to 1 when agent is neutral to ambiguity.

If for all $K > 0$ we get $V(K, q(K); \psi, \theta) \leq 0$, *i.e.*, if the project is never profitable, then the agent does not invest in it. On the other hand, if there exists $K > 0$ such that $V(K, q(K); \psi, \theta) > 0$, the project is profitable for certain level of investment.

As already mentioned in McDonald & Siegel (1986) as in Gollier (2007), it is optimal for an agent to invest only if investment value exceeds his cost. Finally, the optimal decisions (q^*, K^*) are defined by equation (3) and (5). We can summarize some static comparative results in the following lemma

Lemma 1 *(i) A higher price, P_{LW}, P_{HW}, P_{LS}, and/or P_{HS}, increases the level of production, q^* and investment K^*, if $\frac{\partial^2 c(q^*,K^*)}{\partial q \partial K} < 0$.*
(ii) A higher prior belief on the realization of a low number of buyers, ψ, and/or a higher prior belief on the realization of a strong competition effect, θ, decreases the level of production, q^* and investment K^*, if $\frac{\partial^2 c(q^*,K^*)}{\partial q \partial K} < 0$.

Proof.
Part *(i)*
Increasing any price induces an increase of $E_{\psi\theta}P$. Conditions (3) and (5) imply $q^* \equiv q^*(E_{\psi\theta}P)$ and $K^* \equiv K^*(E_{\psi\theta}P)$. Denoting $\bar{P}$, the price expected value and differentiate (3) and (5), we obtain respectively, if $\frac{\partial^2 c(q^*(\bar{P}),K^*(\bar{P}))}{\partial q \partial K} \neq 0$,

$$\frac{\partial q^*(\bar{P})}{\partial \bar{P}} = \frac{1 - c_{qK}\frac{\partial K^*(\bar{P})}{\partial \bar{P}}}{c_{qq}}$$

and

$$\frac{\partial K^*(P)}{\partial P} = -\frac{c_{qK}}{c_{qq}c_{KK} - [c_{qK}]^2 + c_{qq}I_{KK}}$$

With cost convexity assumption and the convexity of $I(.)$, we have $c_{qq}c_{KK} - [c_{qK}]^2 + c_{qq}I_{KK} > 0$. Then, if $c_{qK} < 0$, $\frac{\partial K^*(\bar{P})}{\partial \bar{P}} > 0$ and $\frac{\partial q^*(\bar{P})}{\partial \bar{P}} > 0$.
Part *(ii)*
Conditions (3) and (5) imply $q^* \equiv q^*(\psi, \theta)$ and $K^* \equiv K^*(\psi, \theta)$. We differentiate (3) and (5) with respect to ψ and θ, we obtain respectively, if $\frac{\partial^2 c(q^*(\theta,\psi),K^*(\theta,\psi))}{\partial q \partial K} \neq 0$,

$$\frac{\partial q^*(\theta, \psi)}{\partial \theta} = \frac{\psi \left(P_{LS} - P_{LW}\right) + (1-\psi)\left(P_{HS} - P_{HW}\right) - c_{qK}\frac{\partial K^*(\theta,\psi)}{\partial \theta}}{c_{qq}}$$

$$\frac{\partial q^*(\theta, \psi)}{\partial \psi} = \frac{\theta \left(P_{LS} - P_{HS}\right) + (1-\theta)\left(P_{LW} - P_{HW}\right) - c_{qK}\frac{\partial K^*(\theta,\psi)}{\partial \psi}}{c_{qq}}$$

and

$$\frac{\partial K^*(\theta, \psi)}{\partial \theta} = -\frac{c_{qK}[\psi \left(P_{LS} - P_{LW}\right) + (1-\psi)\left(P_{HS} - P_{HW}\right)]}{c_{qq}c_{KK} - [c_{qK}]^2 + c_{qq}I_{KK}}$$

$$\frac{\partial K^*(\theta, \psi)}{\partial \psi} = -\frac{c_{qK}[\theta \left(P_{LS} - P_{HS}\right) + (1-\theta)\left(P_{LW} - P_{HW}\right)]}{c_{qq}c_{KK} - [c_{qK}]^2 + c_{qq}I_{KK}}$$

where $\psi \left(P_{LS} - P_{LW}\right) + (1-\psi)\left(P_{HS} - P_{HW}\right) < 0$ and $\theta \left(P_{LS} - P_{HS}\right) + (1-\theta)\left(P_{LW} - P_{HW}\right) < 0$. Then, $\frac{\partial q^*(\theta,\psi)}{\partial \theta} < 0$, $\frac{\partial K^*(\theta,\psi)}{\partial \theta} < 0$, $\frac{\partial q^*(\theta,\psi)}{\partial \psi} < 0$ and $\frac{\partial K^*(\theta,\psi)}{\partial \psi} < 0$ if $\frac{\partial^2 c(q^*(\theta,\psi),K^*(\theta,\psi))}{\partial q \partial K} < 0$.
■

So the opportunity to sell each unit at a higher price, and then getting a higher pay-off, prompts the agent to produce more. This opportunity may come from an increase in the possible selling prices, a lower belief in the realization of a low number of buyers, and/or a lower belief in the realization of a strong competition effect.

Moreover, if the marginal production cost decreases with the capital investment, a higher price, a lower belief in the realization of a low number of buyers, and/or a lower belief in the realization of a strong competition effect increases the optimal level of investment in capital. Besides, if the capital investment has no impact on the marginal production costs then the prices and the two beliefs do not affect the agent's decision concerning the level of investment in capital. Furthermore, if $\frac{\partial^2 c(q,K)}{\partial K \partial q} > 0$ we get that $K^* = 0$ and $q^* = 0$.

In addition, if the marginal production cost decreases with the capital investment, a higher price, a lower belief in the realization of a low number of buyers, and/or a lower belief on the realization of a strong competition effect prompts the agent to invest and produce.

So the effect of the capital investment on the marginal production cost plays a major role in the agent's decision concerning his investment in capital and production.

It is natural now to compare the situation of certainty with the situations in which there is one type of uncertainty (either market size uncertainty, or competitive effect uncertainty), and with the situation in which there are both types of uncertainty. To do so, we define the marginal rate of substitution associated to the cost as follows:

$$TMSC(q,K) = -\frac{\frac{\partial c(q,K)}{\partial q}}{\frac{\partial c(q,K)}{\partial K}},$$

which represents the increase of K for which the cost is maintained when the agent produces one more unit. Using relations (3) and (5), we have

$$TMSC(q,K) = -\frac{c_q}{c_K} = \frac{E_{\psi\theta}P}{I'(K)} \tag{6}$$

The situation of certainty, *i.e.* that in which the agent knows the number of buyers on the market and the severity of the market competition effect, corresponds to $E_{\psi\theta}P = P$. In this case, we denote by q_C^* and K_C^* the optimal quantity and investment .

The cases in which there is only one type of uncertainty: first, market size uncertainty, *i.e.*, the agent has perfect knowledge of the level of the effect of market competition , correspond to $E_{\psi\theta}P = E_{\psi}P_m$ with $\theta = 0$ or $\theta = 1$. In the case of Market size Uncertainty , we denote by q_{MU}^* and K_{MU}^* the optimal quantity and investment; secondly, the competition effect uncertainty (*i.e.*, the agent does not initially know the effect of competition but he knows the market size of the future market) corresponds to $E_{\psi\theta}P = E_{\theta}P_j$ with $\psi = 0$ or $\psi = 1$. In the case of Competitive effect Uncertainty , we denote by q_{CU}^* and K_{CU}^* the optimal quantity and investment .

We first note that regardless of the certainty or uncertainty level, the agent never invests nor produces when he thinks that an increase of the capital increases the cost of one more unit.

The lack of information on the true level of the price implies that the agent tends to undervalue the price when the true price is high and overvalue it when it is low. This directly impacts on the level of production, which decreases when the agent undervalues the price, and increases when he overvalues it. Even though the level of capital investment is affected by this erroneous evaluation, the agent's decision also takes into account the effect on the cost of both the level of production and the level of capital invested. Actually, producing more leads the agent to choose a level of investment in capital that reduces his unit production cost. Then under uncertainty, the agent makes a lower (higher) investment when he undervalues (overvalues) the price and the increase of the capital decreases the cost of one more unit. Elder & Serletis (2009, 2010) find empirical evidence that uncertainty about oil prices has tended to depress investment in Canada and United States. Our model could explain their result with considering that the investors expect at once a price that is lower than the true one and that an increase of the capital reduces the marginal cost of production.

Moreover, under both uncertainties, the agent produces less when he does not know that the price is the highest, *i.e.*, the number of buyers is high and there is little market competition. In this context, he invests less capital when he thinks that an increase of the capital increases the cost of one more unit. On the contrary, the agent produces more when he does not know that the price is the lowest, *i.e.*, the number of buyers is low and there is a strong market competition effect. Then, he invests more in capital when he thinks that an increase of the capital decreases the cost of one more unit.

3.2 Aversion to ambiguity

In this section, we seek to understand how choices concerning capital and capacity investment are affected by ambiguity aversion. We then propose to compare the optimal production and capital investment decisions of an agent who is averse to ambiguity with those of an ambiguity neutral agent. To formalize the aversion to ambiguity, we consider that the "right" value of the probability associated to the competition severity uncertainty θ may be unknown. The agent's belief, θ, is then represented not as a single probability measure on the set of states but as a set of probability measures. Such a framework is relevant to the decision concerning investment and production; indeed, as quoted in Heath & Tversky (1991): *the ambiguity aversion is particularly strong in cases in which people feel that their competence in assessing the relevant probabilities is low.*

We then extend the model by considering that θ is a random variable. The agent now associates a probability distribution $F(\theta)$ on $[\underline{\theta}, \overline{\theta}]$ which measures the subjective relevance of a particular θ probability. Following Klibanoff et al. (2005), we assume that the preferences of the agent indicate smooth ambiguity aversion. So, the agent considers that its expected pay-off is defined by equation (2):

$$W(K, q; \psi, \theta) = \int_{\underline{\theta}}^{\overline{\theta}} \phi\left(V(K, q; \psi, \theta)\right) dF(\theta)$$

with $\phi(.)$ defined by an increasing and concave function when the agent is ambiguity averse.

As mentioned previously, we consider the problem in two steps. First, we focus on the impact of ambiguity aversion on the optimal production, $\hat{q}^*$ and second on the optimal capital investment, $\hat{K}^*$.

For a given stock of investment, the first order condition for production is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi'(V(K, q; \psi, \theta)) \frac{\partial V(K, q; \psi, \theta)}{\partial q} dF(\theta) = 0 \tag{7}$$

where

$$\frac{\partial V(K, q; \psi, \theta)}{\partial q} = (E_{\psi\theta} P - c_q). \tag{8}$$

Proposition 1 *For a given initial stock of capital investment, ambiguity aversion tends to decrease the agent's optimal level of production, $\hat{q}^* < q^*$.*

Proof. We use the following notations:

$$\Delta(q, \theta) = \phi'(V(K, q; \psi, \theta))$$
$$\Lambda(q, \theta) = E_{\psi\theta} P - c_q.$$

By definition the covariance is:

$$\text{cov}(\Delta(q,\theta),\Lambda(q,\theta)) = E(\Delta(q,\theta)\Lambda(q,\theta)) - E(\Delta(q,\theta))E(\Lambda(q,\theta)).$$

Then, with condition (7), we have

$$E(\Delta(\hat{q}^*,\theta)\Lambda(\hat{q}^*,\theta)) = J(\hat{q}^*) = 0$$

Comparison to neutrality ambiguity case.

From condition (3), we know that $E(\Delta(q^*,\theta)) = 0$ and then $\text{cov}(\Delta(q^*,\theta),\Lambda(q^*,\theta)) = E(\Delta(q^*,\theta)\Lambda(q^*,\theta))$.

So if $\text{cov}(\Delta(q^*,\theta),\Lambda(q^*,\theta)) < 0$ that implies $E(\Delta(q^*,\theta)\Lambda(q^*,\theta)) < 0$. This is equivalent to $J(q^*) < 0 = J(\hat{q}^*)$. Since ϕ is increasing and concave, $J(.)$ is decreasing function and $q^* > \hat{q}^*$. The sign of covariance is given by differentiate $\Delta(\theta)$ and $\Lambda(\theta)$ with respect to θ where

$$\frac{\partial\Lambda(\theta)}{\partial\theta} = \frac{\partial E_{\psi\theta}}{\partial\theta} < 0$$

and

$$\frac{\partial\Lambda(\theta)}{\partial\theta} = \phi''(V(K,q;\psi,\theta))\frac{\partial V(K,q;\psi,\theta)}{\partial\theta} > 0$$

with $\phi''(V(K,q;\psi,\theta)) < 0$ and $\frac{\partial V(K,q;\psi,\theta)}{\partial\theta} < 0$. Therefore, $\text{cov}(\Delta(q^*,\theta),\Lambda(q^*,\theta)) < 0$ and $q^* > \hat{q}^*$.

■

Let us now turn to the analyze of the agent's optimal investment in capital. Equation (7) implies that $\hat{q}^* \equiv \hat{q}^*(K)$ and the first order condition is:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi'(V(K,\hat{q}^*(K);\psi,\theta))\frac{\partial V(K,\hat{q}^*(K);\psi,\theta)}{\partial K}dF(\theta) = 0 \tag{9}$$

where

$$\frac{\partial V(K,\hat{q}^*(K);\psi,\theta)}{\partial K} = -I^{'}(K) + \beta\frac{\partial\hat{q}^*(K)}{\partial K}[E_{\psi\theta}P] - \beta(c_q\frac{\partial\hat{q}^*(K)}{\partial K} + c_K). \tag{10}$$

Proposition 2 *Ambiguity aversion tends to decrease the agent's optimal investment level,* $\hat{K}^* < K^*$.

Proof. Similar to the proof of Proposition 1 with conditions (5) and (9), thus omitted.

■

Aversion to ambiguity concerning the competition effect leads the agent to reduce his investment in capital and his production. Actually, the agent has doubts about its own subjective beliefs on the competition effect. This adds a new uncertainty dimension for him and discourages him from investing and producing. Ambiguity aversion then restrains investment and production in the new process. This may have drastic consequences on the development of emerging processes.

4 Specification

The empirical analysis is based on the French biomass pre-treatment industry. The case of France is a particularly interesting subject of study, because active research studies are being conducted on second generation biofuel technologies (Ademe, 2009). One of the pilot programmes in which five French partners and one German partner participate, has launched BioTfueL, a million Euro project that uses the Fischer-Tropsch process to convert torrefied wood biomass into drop-in renewable fuel. This group will launch pilot projects in France that will commence in 2012. The domestic biomass resources available are also large. [6] Prospects for the diffusion of torrefaction technology in such a dynamic and expanding market are also of particular interest if the economical profitability is to be enhanced.

To determine the profit flow the firm receives when the project is implemented, we suppose, as is frequently done, that sunk investment costs are linear: $I(K) = p_K K, p_K > 0$ and $I'(K) = p_K$ with $p_K \in [0, 1]$, the investment coefficient (Cairns, 2009). The quantity of pre-treated biomass is a function of the amount of capital, K, that have to be paid for the installation of a production facility. Using (6), we can easily define different probability thresholds, θ and ψ by comparing different cases. In the appendix we expose in tables (1), (2), (3), (4), (5) and (6) the comparisons between both the agent's optimal levels of investment in capital and the agent's optimal levels of production according to the uncertainty to which he is confronted.

4.1 Some static comparative results

Comparisons between the certainty situation and the uncertainty situation.

For $P_i = P_{LW}$ and $P_i = P_{HS}$, we compute $TMSC(q^*, K^*) = TMSC(q_C^*, K_C^*)$, and we get, respectively:

$$\theta = \frac{E_\psi P_W - P_{LW}}{E_\psi P_W - E_\psi P_S} \equiv h_1(\psi) \text{ and } \theta = \frac{E_\psi P_W - P_{HS}}{E_\psi P_W - E_\psi P_S} \equiv h_2(\psi).$$

According to table (3), when the agent does not know that the number of buyers is low and the competition is weak, but he thinks that an increase of capital increases (decreases) the marginal production cost, he may invest less and produce less (invest more and produce more) if his prior belief on the strong competition is higher than $h_1(\psi)$, which depends on his prior belief on the low number of buyers. Otherwise, if his prior belief on the strong competition is lower than $h_1(\psi)$, he may invest more and produce more (invest less and produce less).

Similarly, according to table (4), when the agent does not know that the number of buyers is high and the competition is strong, but he thinks that an increase of capital increases (decreases) the marginal production cost, he may invest less and produce less (invest more and produce more) if his prior belief on the strong competition is higher than $h_2(\psi)$, which depends on his prior belief on the low number of buyers. Otherwise, if his prior belief on the strong competition is lower than $h_2(\psi)$, he may invest more and produce more (invest less and produce less).

So the agent's behaviour in terms of capital investment and production is affected by the interaction between the two beliefs.

Comparisons between the situation where there are both types of uncertainty and the situation in which there is only one type of uncertainty.

The lack of information on both the number of buyers and the severity of the competition effect leads the agent to produce less (more) than an agent who knows that the number of buyers is low (high).

[6] The French potential of forest residues was estimated at over 30 Mt per year available for energetic use in 2015 (MEEDDAT, 2010).

Moreover, under both types of uncertainty, the agent chooses to produce less (more) than an agent who knows that the competition effect is weak (strong).

The agent's decision concerning capital investment depends on both the effects of the capital investment and of the level of production on the cost, and the uncertainty to which the agent is exposed. Hence, in the situation in which the agents think that an increase in capital decreases the marginal production cost, an agent who does not get any information chooses a larger (lower) investment in capital than an agent who knows that the number of buyers is low (high) and an agent who knows that the competition is strong (weak). On the other hand, in the situation where the agents think that an increase of the capital increases the cost of one more unit then whatever the uncertainty they are facing, the agents never invest nor produce.

Comparison between the situations in which there is only one type of uncertainty.

For $P_i = P_{HW}$ and $P_i = P_{LS}$, we compute $TMSC(q^*, K^*) = TMSC(q^*_{CU}, K^*_{CU})$, and we get, respectively:

$$\theta = \frac{P_{HW} - E_\psi P_W}{P_{HW} - P_{HS}} \equiv h_3(\psi) \text{ and } \theta = \frac{P_{LW} - E_\psi P_S}{P_{LW} - P_{LS}} \equiv h_4(\psi).$$

According to table (6), an agent, called here *agent 1 (agent 1')*, who does not have information on the severity of the competition effect but knows that the number of buyers is low (high) produces more (less) than an agent, called *agent 2 (agent 2')*, who does not have information on the number of buyers but knows that the competition is weak (strong). Besides, when the agents think that an increase of capital decreases the marginal production cost, *agent 1 (agent 1')* invests more (less) in capital than *agent 2 (agent 2')*.

In addition, when the agents think that an increase of capital decreases the marginal production cost, *agent 1' (agent 1)*'s prior belief on the strong competition is lower than $h_3(\psi)$ ($h_4(\psi)$), he may invest more in capital and produce more (a lower investment in capital and produce less) than *agent 2 (agent 2')*.

On the other hand, when the agents think that an increase of capital increases the marginal production cost, then whatever the uncertainty that they face, the agents never invest nor produce.

4.2 Determination of the cost function

Like Cairns (2009) and Tsatsaronis & Park (2002) , we consider the avoidable cost of production $c(q, K)$ as a function of the amount of capital, K and the output production, q. The avoidable costs are commonly calculated by subtracting the unavoidable cost from the respective total cost excluding the sunk cost $I(K)$ such that:

$$c(q, K) = c^T(q, K) - c^{UN}(q, K)$$

where $c^T(q, K)$ is the total cost and $c^{UN}(q, K)$, the unavoidable costs.

The total cost function is a convex function composed of the capital costs and the production costs. We use a limited development at the order one of the translog function to represent the cost minimizing behaviour of the agent who uses the amount K of capital to produce a quantity q of output. For the torrefaction technology, the cost function is:

$$\ln(c^T(q, K)) = a_1 + a_2 \ln(q) + a_3 \ln(K) + a_4 \ln(q) \ln(K)$$

where $a_1 > 0$, is a fixed cost, a_2 and a_3 are the cost elasticity of the production and the capital respectively, a_4, is the cross elasticity between production and capital. We assume that the unit costs of production are increasing in accumulated production so $a_2 > 0$ and the investment costs of invested capital K are decreasing in accumulated capital so $a_3 < 0$. The data on operating costs of a torrefaction plant were taken from the existing literature and consists of engineering estimates. The technology exists today but it is tested at pilot scale, so we estimate our coefficients on the basis of economic data for different possible capacities of units (c.f. table (7) in appendix). We assume that unit runs at full capacity. The estimates are presented in table 8 in appendix.

We then determine the unavoidable cost rate related to the production and the investment as follows (Tsatsaronis & Park, 2002). Due to technical limitations imposed by the availability and/or costs of materials and manufacturing methods, a maximum value of the mass efficiency of the torrefaction process cannot be exceeded regardless of the amount invested. This efficiency is achieved at the point where the investment cost becomes infinite. This point determines the unavoidable destruction of raw biomass per unit of torrefied biomass . Thus, we could determine the cost rate associated to the unavoidable raw biomass destruction Z_q^{UN}. Similarly, the unavoidable investment costs per unit of torrefied biomass, Z_K^{UN}, are obtained by considering an extremely inefficient version of the technology, that is a version that would never be feasible in practice because of the very high biomass costs associated with it.[7] We assume that the percentage of the total costs that cannot theoretically be avoided, in view of today's technology and economic environment of the torrefaction, technology is between 20% and 50% (Tsatsaronis & Park, 2002). We take an average unavoidable cost Z^{UN} such as $Z^{UN} = Z_q^{UN} = Z_K^{UN} = 35\%$. Then the avoidable costs are calculated by subtracting the unavoidable cost rates from the respective cost rates:

$$c(q, K) = (1 - Z^{UN})e^{a_1 + a_4 \ln(K) \ln(q)} K^{a_3} q^{a_2}.$$

Then we have to select an appropriate discount rate. It is an important topic in investment decision (Kumbaroglu et al., 2008). Various ways of calculating discount rates adjusted for risk, have been proposed by Trigeorgis (1996). Schmit et al. (2009) assume a discount rate of 8% to reflect a relatively high credit risk for the investment in ethanol plant, whereas Uslu et al. (2008) chooses a discount rate of 12.5% for an investment in torrefaction. The discount rate of the refinery unit is between 8 and 10% (Dangl, 1999; Felfli et al., 2005). In our analysis, we assume it is equal to 10% because torrefaction units will supply BtL and refinery units. We will vary this rate in the sensitive analysis.

We illustrate the results determined in section 3 for an ambiguity neutral agent and an ambiguity averse agent.

5 Numerical analysis

5.1 Optimal decisions

5.1.1 Ambiguity-neutral agent

According to operating data for torrefaction plant, the marginal cost of production decreases with K and increases with q (table (7)). Furthermore, the cost function does not vary with the number of uncertainty such that $c(q_C^*, K_C^*) = c(q^*, K^*)$ and $c(q_{MU}^*, K_{MU}^*) = c(q_{CU}^*, K_{CU}^*)$. To illustrate our results, we consider

[7] In practical applications, this term is determined by arbitrarily selecting a set of parameters for this technology that lead to a very inefficient solution and by estimating the investment costs for this solution.

the following scenario: $p_K = 0.5$,[8] $\theta = 0.3$, $\psi = 0.5$. We take the prices of torrefied biomass collected during a survey conducted among potential buyers of torrefied biomass in France such as $P_{LW} = 100$ Euros/t, $P_{HW} = 200$ Euros/t, $P_{LS} = 80$ Euros/t and $P_{HS} = 148$ Euros/t. For these values, we have $h_1(\psi) \geq \theta$, $h_2(\psi) \leq \theta$, $h_3(\psi) \geq \theta$ and $h_4(\psi) \leq \theta$. From (3) and (5), we determine the optimal level of production and investment for the situation in which there are both types of uncertainty, the situation in which there is certainty and the situation in which there is only one uncertainty. The results are summed up in table (9) for the different cases $P_i = P_{jm}$ for $j \in \{L, H\}$ and $m \in \{W, S\}$.

Taking into account of the operating costs, the optimal investment in capital, K^* and the optimal levels of production, q^* are ranked to the uncertainty which the agent faces, and summed up in tables (1), (2), (3), (4), (5) and (6).

We observe the asymmetric effects of uncertainty on the optimal amount of investment and optimal production. We show that the effect of market size related uncertainty is stronger than that of the competition related uncertainty as the investment and the production levels are higher. This is true when, in the case of certainty, the effect of competition is weak and the number of buyers is low, or high if and only if the agent's prior belief on the weak competition is lower than a certain threshold h_3. Furthermore, if in the situation of certainty the agent knows the competition effect is weak and the market size is high, the combination of both types of uncertainty leads him to invest less. He behaves similarly if the market size is low in the certainty case if and only if his prior belief concerning market size is lower than a certain threshold h_1 (cases $P_{LW} = 100$ Euros/t and $P_{HW} = 200$ Euros/t of table (9)).

Secondly, the effect of the competition related uncertainty is stronger than the uncertainty concerning the market size uncertainty as the investment and production levels are higher. This is true when in the case of certainty the competition is strong and the number of buyers is high, or low if and only if the agent's prior belief concerning the weak competition is higher than a certain threshold h_4. Besides, if in the certainty case the agent knows the competition is strong and the market size is low, the combination of both types of uncertainty leads him to invest more. If the market size is high in the certainty case, he has the same behaviour if and only if his prior belief on market size is higher than a certain threshold h_2 (cases $P_{HS} = 148$ Euros/t and $P_{LS} = 80$ Euros/t of table (9)).

5.1.2 Ambiguity-averse agent

We now illustrate propositions 1 and 2 to examine the difference ambiguity causes in the results. We use a Gaussian quadrature to produce the Legendre-Gauss weights and nodes for computing the integral of the continuous function W on interval $[\underline{\theta}, \overline{\theta}]$ (Judd, 1999; Miranda & Fackler, 2004). We use a beta distribution $B(\theta; \eta, \mu)$ with the parameter $\eta = 0.5$ and $\mu = 0.5$ [9] to specify the probability distribution over the set of plausible distribution of the competitive effect (Miranda & Fackler, 2004). Following Klibanoff et al. (2005), we consider a constant absolute ambiguity aversion (CAAA) utility function (Engle-Warnick et al., 2008):

$$\phi(V(K, q; \psi, \theta)) = \begin{cases} \frac{1-e^{-\tau V_0}}{1-e^{-\tau}} & \text{if } \tau > 0 \\ V_0 & \text{if } \tau = 0 \end{cases}$$

[8] We study the effect of different values of this parameter in the sensitivity analysis.

[9] The beta distribution is often used to describe the distribution of an unknown probability value, typically, as the prior distribution over a probability parameter. It is defined on the interval $[0, 1]$. η and μ give the shape of the probability density function. If $\eta < 1$ and $\mu < 1$, the beta density function is U-shaped and symmetric about $1/2$ if $\eta = \mu$.

For the sake of clarity, we take $\tau = 15$ to compare the case of an agent who is highly averse to ambiguity with that of an ambiguity neutral agent. Figures (1) and (2) below show the marginal payoffs, $\partial W(q, K; 0.5, 0.3)/\partial q$ in function to q to determine q^* and $\partial W(q(k), K; 0.5, 0.3)/\partial K$ in function to K to determine K^* for both an ambiguity neutral and an ambiguity averse agents.

[Insert figures (1)and (2)]

The marginal values are decreasing with q and K. Thus we find that an ambiguity averse agent produces less than an ambiguity-neutral agent and invests less as defined in the proposition 1 and 2. The results are the same for different value of τ. Due to ambiguity, an agent who invest in biomass torrefaction facilities chooses a lower capacity for his units than he would if he were ambiguity neutral. Ambiguity aversion leads the investor to evaluate probabilities distribution according to the least-favourable state, in this case the lowest pay-off. This behaviour could have consequences on the development of emerging BtL process. Indeed, as mentioned before, the pre-treatment could enhance the deployment of BtL process because it can improve the economics of the overall production chain. If the producer invests less, the buyer takes the risk of not being supplied the right quantity. The buyer of the torrefied biomass perceives uncertainty about the availability of their inputs. They would be reluctant to invest in the new renewable energy process.

5.2 Sensitivity analysis

5.2.1 Sensitivity of the optimum strategy as regards the unavoidable cost rate Z^{UN}

In the reference example, the percentage of total costs that could be not avoided in view of today's technology and economic environment, is an average of maximum and minimum percentages that are possible today. The reference unit produced torrefied biomass relatively expensively compared with units for which the unavoidable cost rate is higher but the mass efficiency of the torrefaction is also better. Increasing the unavoidable cost rates should increase the competitive advantage of a high-output capacity due to the fact the avoidable cost is lower, but regardless the amount invested, the mass efficiency is lower. This can be verified in the table (11) in which tests are made for minimum and maximum values in the range of possible unavoidable cost rates determined for this torrefaction process. A decrease in the proportion of the unavoidable cost reduces the profitability by increasing the avoidable cost price and therefore deters the entrepreneur for investing in high capacity. An increase of the unavoidable cost rate increases the optimal capital investment. However, whatever the unavoidable cost rate Z^{UN}, the ranking of optimal investment in capital is unchanged.

5.2.2 Variation in the discount rate and the investment coefficient, p_K

In our reference simulation, we considered a discount rate of 10% per year. In this section, we examine the effect of increasing and decreasing this rate respectively, to 12% and 10%. A change in the discount rate modifies the optimum investment strategy. A higher discount rate penalizes the waiting time and therefore encourages the entrepreneur to invest earlier. Indeed, the sensitive analysis (c.f. table (12)) shows the increase (decrease) in the discount rate leads to a high (low) investment in capital.

Then , we study the effect of increasing and decreasing the investment coefficient. We sum up the results in table (10) in appendix for different value of $p_K \in [0, 1]$. We notice that K^* decreases when p_K increases whether there is one or two types of uncertainty. As proved in the Lemma 1, a higher investment coefficient increases the cost of investment so the agent decreases his capital investment.

Nevertheless, whatever the discount rate or the value of p_K, the ranking of optimal investments in capital is unchanged.

6 Conclusion

In this paper, we assess the impact of two types of uncertainty and of the ambiguity aversion of the agent on his investment and production strategy. We develop a formal model for decision making in which agents are neutral to risk and averse to ambiguity about the true distribution of the competitive effect. We analyse the optimal capacity and production choices in this model. We show analytically that the model has the following implications, which are consistent with the theoretical findings: (i) whatever the certainty or the uncertainty context, the agent never invests nor produces when he thinks that an increase of the capital increases the cost of one more unit; (ii) the agent's decision in terms of capital investment depends on both the effects of the capital investment and the level of production on the cost and the uncertainty which the agent faces ; (iii) as we know, an increase of capital decreases the marginal production; therefore, the effect of the market size uncertainty is stronger than that of the competitive effect uncertainty since the levels of investment and of production are higher. This is true when in the case of certainty the competition is weak and the number of buyers is low, or high if and only if the agent's prior belief about the weak competition is lower than a certain threshold. Secondly, the impact of the competition effect uncertainty is stronger than that of the market size uncertainty since the levels of investment and of production are higher. This is true when, in the certainty case, the competition is strong and the number of buyers is high, or low if and only if the agent's prior belief about the weak competition is higher than a certain threshold; (iv) in the presence of ambiguity about the competition effect, agents will invest less in their units and their level of production is lower. The main feature of this model is that it helps to understand the behaviour of an agent who faces uncertainty about the market size and market competition if he is averse to ambiguity. From a theoretical point of view, this paper emphasizes the need to reduce the effects of ambiguity in the European policy framework that encourages the development of renewable energy production. The introduction of long-term contracts could contribute to reducing them. An attractive feature of the model is to determine how the risk and ambiguity aversions of the buyer will affect the investment strategy of torrefied biomass producers. Finally, it will be important to check empirically, with potential agents (private forest owners, cooperatives...) the theoretical results obtained in our model and to evaluate the degree of their ambiguity aversion.

References

Ademe (2009). *Fonds Chaleur biomasse.* Technical report.

Bond, S., Moessner, R., Mumtaz, H., & M.Syed (2005). *Microeconometric Evidence on Uncertainty and Investment.* Mimeo, Institute for financial studies.

Cairns, R. D. (2009). *Sunk Cost and Cost Functions.* mimeo, Departement of Economics, McGill University.

Camerer, C. (1999). *Ambiguity-Aversion and Non-Additive Probability: Experimental Evidence, Models and Applications.*, chapter Uncertain Decisions: Bridging Theory and Experiments, (pp. 53–80). Kluwer Academic Publishers.

Carruth, A., Dickerson, A., & A.Henley (2000). What do we know about investment under uncertainty? *Journal of Economic Surveys*, 14, 119–153.

Dangl, T. (1999). Investment and capacity choice under uncertain demand. *European Journal of Operational Research*, 117, 415–428.

Dixit & Pindyck (1994). *Investment under uncertainty.* Princeton University Press.

Elder, J. & Serletis, A. (2009). Oil price uncertainty in canada. *Energy Economics*, 31(6), 852–856. Energy Sector Pricing and Macroeconomic Dynamics.

Elder, J. & Serletis, A. (2010). Oil price uncertainty. *Journal of Money, Credit, and Banking*, 42(6), 1137–1159.

Ellsberg, D. (1961). Risk, ambiguity, and the savage axioms. *Quaterly Journal of Economics*, 110(3), 585–603.

Engle-Warnick, J., Escobal, J., & Laszlo, S. (2008). Ambiguity aversion and portfolio choice in small-scale peruvian farming. mimeo.

Etner, J., Jeleva, M., & Tallon, J.-M. (2010). Decision theory under uncertainty. *Journal of Economic Surveys.*

Fan, Y. & Zhu, L. (2010). A real options based model and its application to china's overseas oil investment decisions. *Energy Economics*, 32(3), 627–637.

Felfli, F. F., Luengo, C. A., & Rocha, J. D. (2005). Torrefied briquettes: technical and economic feasibility and perspectives in the brazilian market. *Energy for Sustainable Development*, 9, 23–29.

Fellner, W. (1961). Two propositions in the theory of induced innovations. *The Economic Journal*, 71(282), 305–308.

Fellner, W. (1965). *Probability and Profit.* Yale University.

Fuss, S. & Szolgayova, J. (2009). Fuel price and technological uncertainty in a real options model for electricity planning. *Applied Energy*, In Press.

Gollier, C. (2006). Does ambiguity aversion reinforce risk aversion? applications to portfolio choices and asset prices. In *Séminaire d'Economie Théorique*: Université de Toulouse 1 Sciences Sociales.

Gollier, C. (2007). Comment intégrer le risque dans le calcul économique ? *Revue d'économie politique*, 117(2/2007), 209–223.

Heath, C. & Tversky, A. (1991). Preference and belief: Ambiguity and competence in choice under uncertainty. *Journal of Risk and Uncertainty*, 4, 5–28.

Henry, C. (1974). Investment decisions under uncertainty: The "irreversibility effect". *The American Economic Review*, 64(6), 1006–1012.

Judd, K. L. (1999). *Numerical methods in economics.* MIT Press.

Klibanoff, P., Marinacci, M., & Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6), 1849–1892.

Kulatilaka, N. & Perotti, E. (1998). Strategic growth options. *Management Science*, 44, 1021–1031.

Kumbaroglu, G., Madlener, R., & Demirel, M. (2008). A real options evaluation model for the diffusion prospects of new renewable power generation technologies. *Energy Economics*, 30(4), 1882–1908.

McDonald, R. & Siegel, D. (1986). The value of waiting to invest. *The Quarterly Journal of Economics*, 101(4), 707–728.

MEEDDAT (2010). *La fiscalité des biocarburants en France.* Technical report, Ministère de l'Ecologie, de l'Energie, du Développement Durable et de l'Amménagement du territoire, http://www.developpement-durable.gouv.fr/La-fiscalite-des-biocarburants-en.html.

Miranda, M. J. & Fackler, P. L. (2004). *Applied computational economics and finance.* The MIT Press.

Mohn, K. & Misund, B. (2009). Investment and uncertainty in the international oil and gas industry. *Energy Economics*, 31(2), 240–248.

Sarkar, S. (2000). On the investment-uncertainty relationship in a real options model. *Journal of Economic Dynamics and Control*, 24(2), 219–225.

Schmit, T. M., Luo, J., & Tauer, L. W. (2009). Ethanol plant investment using net present value and real options analyses. *Biomass & Energy*, 33, 1442–1451.

Slovic, P. & Tversky, A. (1974). Who accepts savage's axiom? *Behavioral Science*, 19, 368–373.

Sutton, J. (1991). *Sunk costs and Market structure.* MIT Press.

Trigeorgis, L. (1996). *Real Options.* MIT Press.

Tsatsaronis, G. & Park, M.-H. (2002). On avoidable and unavoidable exergy destructions and investment costs in thermal systems. *Energy Conversion and Management*, 43(9-12), 1259–1270.

Uslu, A., Faaij, A. P., & Bergman, P. (2008). Pre-treatment technologies, and their effect on international bioenergy supply chain logistics. techno-economic evaluation of torrefaction, fast pyrolysis and pelletisation. *Energy*, 33(8), 1206–1223.

7 Appendix

Figures

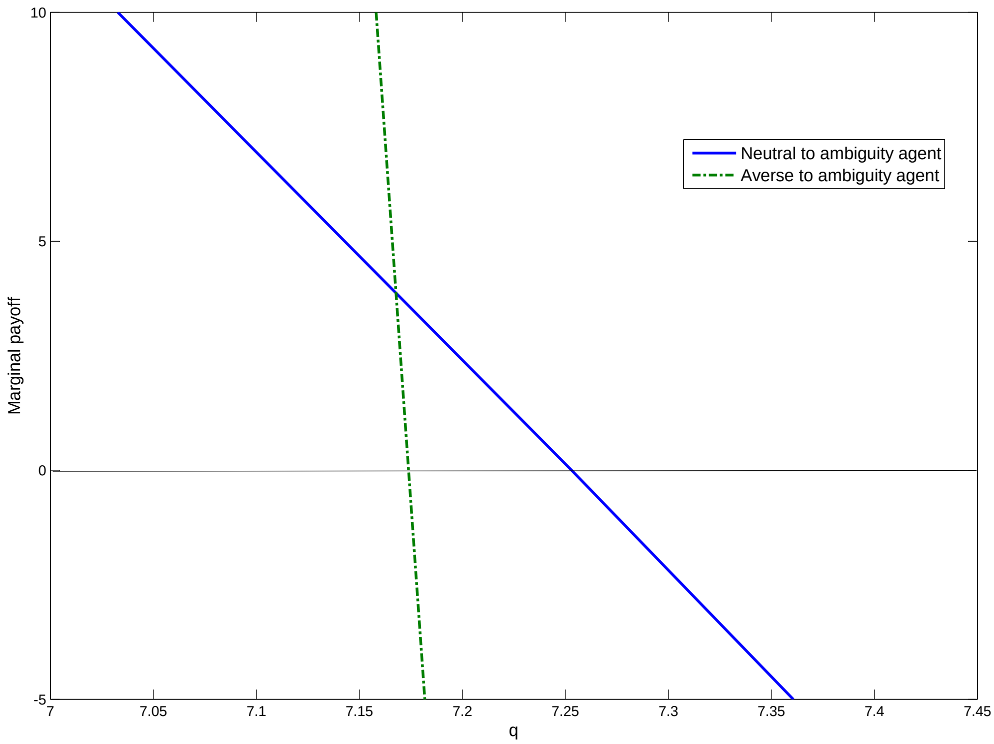

Figure 1: The marginal payoff $\partial W(6.34, q; 0.5, 0.3)/\partial q$ in function to q. Calculated with β=0.1, τ=15 and p_K=0.5 .

Tables

Table 1: Ranking of the agent's optimal level of investment in capital and the agent's optimal level of production according to the uncertainty to which he is confronted when in the certainty case $P_i = P_{HW}$.

$P_i = P_{HW}$: **in the certainty case, the agent knows that he will be in state** HW		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K_C^* = K_{MU}^* = K_{CU}^* = K^* = 0$	$q_C^* = q_{MU}^* = q_{CU}^* = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K_C^* > K_{MU}^*$ $K_C^* > K_{CU}^*$ $K_C^* > K^*$	$q_C^* > q_{MU}^*$ $q_C^* > q_{CU}^*$ $q_C^* > q^*$

■

Table 2: Ranking of the agent's optimal level of investment in capital and the agent's optimal level of production according to the uncertainty to which he is confronted when in the certainty case $P_i = P_{LS}$.

$P_i = P_{LS}$: **in the certainty case, the agent knows that he will be in state** LS		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K_C^* = K_{MU}^* = K_{CU}^* = K^* = 0$	$q_C^* = q_{MU}^* = q_{CU}^* = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K_C^* < K_{MU}^*$ $K_C^* < K_{CU}^*$ $K_C^* < K^*$	$q_C^* < q_{MU}^*$ $q_C^* < q_{CU}^*$ $q_C^* < q^*$

Table 3: Ranking of the agent's optimal level of investment in capital and the agent's optimal level of production according to the uncertainty to which he is confronted when in the certainty case $P_i = P_{LW}$.

$P_i = P_{LW}$: **in the certainty case, the agent knows that he will be in state** LW		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K_C^* = K_{MU}^* = K_{CU}^* = K^* = 0$	$q_C^* = q_{MU}^* = q_{CU}^* = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K_C^* \leq K_{MU}^*$ $K_C^* \geq K_{CU}^*$	$q_C^* \leq q_{MU}^*$ $q_C^* \geq q_{CU}^*$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q_C^*, K_C^*) = c(q^*, K^*)$:		
* if $h_1(\psi) \leq \theta$	$K^* \geq K_C^*$	$q^* \geq q_C^*$
* if $h_1(\psi) \geq \theta$	$K^* \leq K_C^*$	$q^* \leq q_C^*$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q_C^*, K_C^*) > c(q^*, K^*)$:		
* if $h_1(\psi) \leq \theta$	$K^* \geq K_C^*$	$q^* ? q_C^*$
* if $h_1(\psi) \geq \theta$	$K^* ? K_C^*$	$q^* \geq q_C^*$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q_C^*, K_C^*) < c(q^*, K^*)$:		
* if $h_1(\psi) \leq \theta$	$K^* \leq K_C^*$	$q^* ? q_C^*$
* if $h_1(\psi) \geq \theta$	$K^* ? K_C^*$	$q^* \leq q_C^*$

Table 4: Ranking of the agent's optimal level of investment in capital and the agent's optimal level of production according to the uncertainty to which he is confronted when in the certainty case $P_i = P_{HS}$.

$P_i = P_{HS}$: **in the certainty case, the agent knows that he will be in state** HS		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K_C^* = K_{MU}^* = K_{CU}^* = K^* = 0$	$q_C^* = q_{MU}^* = q_{CU}^* = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K_C^* \leq K_{MU}^*$ $K_C^* \geq K_{CU}^*$	$q_C^* \geq q_{MU}^*$ $q_C^* \leq q_{CU}^*$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q_C^*, K_C^*) = c(q^*, K^*)$:		
* if $h_2(\psi) \leq \theta$	$K^* \geq K_C^*$	$q^* \geq q_C^*$
* if $h_2(\psi) \geq \theta$	$K^* \leq K_C^*$	$q^* \leq q_C^*$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q_C^*, K_C^*) > c(q^*, K^*)$:		
* if $h_2(\psi) \leq \theta$	$K^* \geq K_C^*$	$q^* ? q_C^*$
* if $h_2(\psi) \geq \theta$	$K^* ? K_C^*$	$q^* \geq q_C^*$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q_C^*, K_C^*) < c(q^*, K^*)$:		
* if $h_2(\psi) \leq \theta$	$K^* \leq K_C^*$	$q^* ? q_C^*$
* if $h_2(\psi) \geq \theta$	$K^* ? K_C^*$	$q^* \leq q_C^*$

Table 5: Ranking of the agent's optimal level of investment in capital and the agent's optimal level of production according to the uncertainty to which he is confronted when in the certainty case the price is equal to P_i.

$P_i = P_{HW}$: **in the certainty case, the agent knows that he will be in state** HS		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = K^* = 0$	$q^*_{MU} = q^*_{CU} = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K^* < K^*_{MU}$ $K^* < K^*_{CU}$	$q^* < q^*_{MU}$ $q^* < q^*_{CU}$
$P_i = P_{LS}$: **in the certainty case, the agent knows that he will be in state** LS .		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = K^* = 0$	$q^*_{MU} = q^*_{CU} = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K^* > K^*_{MU}$ $K^* > K^*_{CU}$	$q^* > q^*_{MU}$ $q^* > q^*_{CU}$
$P_i = P_{LW}$: **in the certainty case, the agent knows that he will be in state** LW .		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = K^* = 0$	$q^*_{MU} = q^*_{CU} = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K^* \leq K^*_{MU}$ $K^* > K^*_{CU}$	$q^* < q^*_{MU}$ $q^* > q^*_{CU}$
$P_i = P_{HS}$: **in the certainty case, the agent knows that he will be in state** HS .		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = K^* = 0$	$q^*_{MU} = q^*_{CU} = q^* = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K^* > K^*_{MU}$ $K^* < K^*_{CU}$	$q^* > q^*_{MU}$ $q^* < q^*_{CU}$

Table 6: Ranking of the agent's optimal level of investment in capital and the agent's optimal level of production according to the uncertainty to which he is confronted when in the certainty case the price is equal to P_i.

Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
$P_i = P_{HW}$: **in the certainty case, the agent knows that he will be in state** HS		
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = 0$	$q^*_{MU} = q^*_{CU} = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q^*_{MU}, K^*_{MU}) = c(q^*_{CU}, K^*_{CU})$:		
* if $h_3(\psi) \leq \theta$	$K^*_{CU} \geq K^*_{MU}$	$q^*_{CU} \geq q^*_{MU}$
* if $h_3(\psi) \geq \theta$	$K^*_{CU} \leq K^*_{MU}$	$q^*_{CU} \leq q^*_{MU}$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q^*_{MU}, K^*_{MU}) > c(q^*_{CU}, K^*_{CU})$:		
* if $h_3(\psi) \leq \theta$	$K^*_{CU} \geq K^*_{MU}$	$q^*_{CU} ? q^*_{MU}$
* if $h_3(\psi) \geq \theta$	$K^*_{CU} ? K^*_{MU}$	$q^*_{CU} \geq q^*_{MU}$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q^*_{MU}, K^*_{MU}) < c(q^*_{CU}, K^*_{CU})$:		
* if $h_3(\psi) \leq \theta$	$K^*_{CU} \leq K^*_{MU}$	$q^*_{CU} ? q^*_{MU}$
* if $h_3(\psi) \geq \theta$	$K^*_{CU} ? K^*_{MU}$	$q^*_{CU} \leq q^*_{MU}$

$P_i = P_{LS}$: **in the certainty case, the agent knows that he will be in state** LS .		
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = 0$	$q^*_{MU} = q^*_{CU} = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q^*_{MU}, K^*_{MU}) = c(q^*_{CU}, K^*_{CU})$:		
* if $h_4(\psi) \leq \theta$	$K^*_{CU} \geq K^*_{MU}$	$q^*_{CU} \geq q^*_{MU}$
* if $h_4(\psi) \geq \theta$	$K^*_{CU} \leq K^*_{MU}$	$q^*_{CU} \leq q^*_{MU}$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q^*_{MU}, K^*_{MU}) > c(q^*_{CU}, K^*_{CU})$:		
* if $h_4(\psi) \leq \theta$	$K^*_{CU} \geq K^*_{MU}$	$q^*_{CU} ? q^*_{MU}$
* if $h_4(\psi) \geq \theta$	$K^*_{CU} ? K^*_{MU}$	$q^*_{CU} \geq q^*_{MU}$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$ and $c(q^*_{MU}, K^*_{MU}) < c(q^*_{CU}, K^*_{CU})$:		
* if $h_4(\psi) \leq \theta$	$K^*_{CU} \leq K^*_{MU}$	$q^*_{CU} ? q^*_{MU}$
* if $h_4(\psi) \geq \theta$	$K^*_{CU} ? K^*_{MU}$	$q^*_{CU} \leq q^*_{MU}$

$P_i = P_{LW}$: **in the certainty case, the agent knows that he will be in state** LW .		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{MU} = K^*_{CU} = 0$	$q^*_{MU} = q^*_{CU} = 0$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K^*_{CU} < K^*_{MU}$	$q^*_{CU} < q^*_{MU}$

$P_i = P_{HS}$: **in the certainty case, the agent knows that he will be in state** HS .		
Conditions fulfilled	Ranking of optimal investment in capital	Ranking of optimal level of production
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} \geq 0$	$K^*_{CU} \leq K^*_{MU}$	$q^*_{CU} \geq q^*_{MU}$
If $\frac{\partial^2 c(q,K)}{\partial q \partial K} < 0$	$K^*_{CU} > K^*_{MU}$	$q^*_{CU} > q^*_{MU}$

Table 7: Operating expenses for the different scale of unit

Parameters	Units	Capacity		
	t/an	80000	200000	400000
K	MEuros	7.5	15.6	27.18
$\frac{\partial c(q,K)}{\partial K}$ (in absolute value)	Euros/t	35.6	29.6	25.8
$\frac{\partial c(q,K)}{\partial q}$	Euros/t	26	20.9	20.9
Biomass cost (1)	Euros/t	137	137	137
Total marginal cost	Euros/t	198.6	187.6	183.6

(1) We assume that biomass is sold at the same price regardless of unit capacity.

Table 8: Estimation results for the cost function parameters

Parameters	Values
a_1	1
a_2	2.33
a_3	-2.33
a_4	0.12

Table 9: Optimal level of production and investment in function of the uncertainties for $P_{LW} = 100$ Euros/t, $P_{HW} = 200$ Euros/t, $P_{LS} = 80$ Euros/t and $P_{HS} = 148$ Euros/t

Case	Number of uncertainties	ψ	θ	q^1	K^2	Ranking of optimal levels of q and K
	Two Uncertainty	0.5	0.3	11.91	6.01	
$P_i = P_{LW}$	Certainty(C)	1	1	13.91	6.84	$q^*_{MU} > q^*_C > q^* > q^*_{CU}$ $K^*_{MU} > K^*_C > K^* > K^*_{CU}$
	Competitive effect (CU)	1	0.3	10.93	5.61	
	Market size (MU)	0.5	1	15.39	7.44	
$P_i = P_{HW}$	Certainty(C)	0	1	16.79	8.00	$q^*_C > q^*_{MU} > q^*_{CU} > q^*$ $K^*_C > K^*_{MU} > K^*_{CU} > K^*$
	Competitive effect(CU)	0	0.3	12.84	6.34	
	Market size (MU)	0.5	1	15.39	7.44	
$P_i = P_{LS}$	Certainty (C)	1	0	9.49	5.00	$q^* > q^*_{CU} > q^*_{MU} > q^*_C$ $K^* > K^*_{CU} > K^*_{MU} > K^*_C$
	Competitive effect(CU)	1	0.3	10.93	5.61	
	Market size (MU)	0.5	0	10.21	5.31	
$P_i = P_{HS}$	Certainty (C)	0	0	10.90	5.59	$q^*_{CU} > q^* > q^*_C > q^*_{MU}$ $K^*_{CU} > K^* > K^*_C > K^*_{MU}$
	Competitive effect (CU)	0	0.3	12.84	6.34	
	Market size (MU)	0.5	0	10.21	5.31	

(1) In ton per hour; (2) In MEuros.

Table 10: Sensitive analysis as regards the investment coefficient, p_K for $\theta = 0.3$, $\psi = 0.5$ in case $P_i = P_{HS} = 148$ Euros/t.

Number of uncertainties	K^1	Values of p_K			
		0.2	0.5	0.8	1
Certainty (C)	K^*_C	7.56	5.59	4.79	4.46
Two Uncertainty	K^*	8.14	6.01	5.15	4.79
Competitive effect (CU)	K^*_{CU}	8.66	6.39	5.48	5.09
Market size(MU)	K^*_{MU}	7.16	5.31	4.55	4.23

(1) In MEuros.

Table 11: Sensitive analysis as regards the unavoidable cost rate for $\theta = 0.3$, $\psi = 0.5$ in $P_i = P_{HS} = 148$ Euros/t.

Z^{UN}	Number of uncertainties	q^1	K^2	Ranking of optimal levels of q and K
20 %	Two Uncertainty	10.66	5.79	$q^*_{CU} > q^* > q^*_C > q^*_{MU}$
	Certainty (C)	9.58	5.39	
	Competitive effect (CU)	11.28	6.16	$K^*_{CU} > K^* > K^*_C > K^*_{MU}$
	Market size (MU)	8.97	5.12	
50 %	Two Uncertainty	14.02	6.31	$q^*_{CU} > q^* > q^*_C > q^*_{MU}$
	Certainty (C)	12.84	5.87	
	Competitive effect (CU)	15.12	6.72	$K^*_{CU} > K^* > K^*_C > K^*_{MU}$
	Market size (MU)	12.03	5.56	

(1) In ton per hour; (2) In MEuros.

Table 12: Sensitive analysis as regards the discount rate for $\theta = 0.3$, $\psi = 0.5$ in $P_i = P_{HS} = 148$ Euros/t.

Discount rate	Number of uncertainties	q^1	K^2	Ranking of optimal levels of q and K
8 %	Two Uncertainty	11.91	5.59	$q^*_{CU} > q^* > q^*_C > q^*_{MU}$
	Certainty (C)	10.90	5.20	
	Competitive effect (CU)	12.84	5.94	$K^*_{CU} > K^* > K^*_C > K^*_{MU}$
	Market size (MU)	10.21	4.93	
12 %	Two Uncertainty	11.91	6.39	$q^*_{CU} > q^* > q^*_C > q^*_{MU}$
	Certainty (C)	11.90	5.94	
	Competitive effect (CU)	12.84	6.79	$K^*_{CU} > K^* > K^*_C > K^*_{MU}$
	Market size (MU)	10.21	5.63	

(1) In ton per hour; (2) In MEuros.

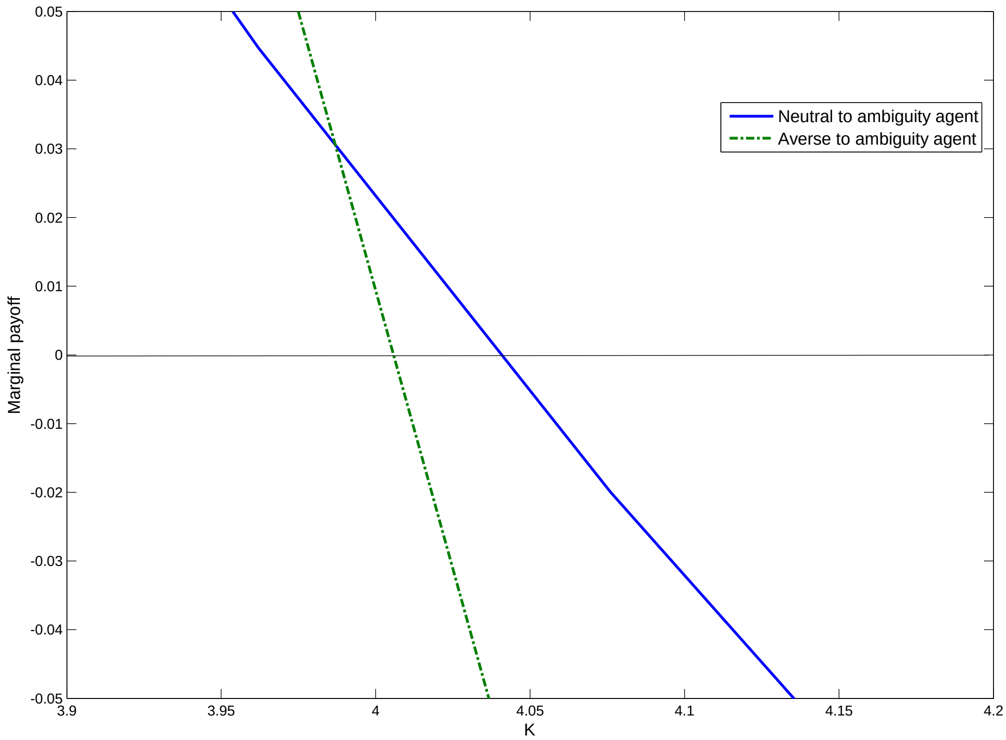

Figure 2: The marginal payoff $\partial W(K, q; 0.5, 0.3)/\partial K$ in function to K for $\hat{q}^*(K) = 7.18$ and $q^*(K) = 7.26$. Calculated with β=0.1, τ=15 and p_K=0.5.

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