Bayesian estimation of non-stationary Markov models combining micro and macro data

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Objective:
(1) Develop a Bayesian estimation framework for non-stationary Markov models that allows combining micro and macro data based estimation techniques previously considered as alternatives only
(2) Evaluate the derived estimator using Monte Carlo Simulations

Background on Markov processes
- A Markov process allows to model the movement of individuals between a finite number of predefined states, \( i=1, \ldots, k \), as a stochastic process.
- A Markov process is characterized by a transition probability matrix \( P_t \).
- The vector \( \mathbf{n}_t \) denotes the number of individuals in each state and develops over time, \( t \), according to a (first order) Markov process:
  \[ \mathbf{n}_t = P_t \mathbf{n}_{t-1} \]

Relevance: Farm Structural Change

General aims:
(1) Identify and quantify factors that determine farm structural change
(2) Predict structural change in response to these factors
- Often modeled as a Markov Process (Zimmermann et. al 2009)
- But current estimation techniques do not allow using available micro and macro data in a satisfying way

Specification of \( P_t \):
- \( P_t \) is assumed to be a function of explanatory variables
- We propose two different specifications for ordered and unordered Markov states based on the multinomial logit model and the ordered logit model
- Main differences:
  (1) Multinomial logit model requires assumption of iid errors which might be inappropriate with ordered Markov states
  (2) Ordered logit model requires less parameter to be estimated

Combining...

macro data...    with micro data...

Definition:
- The number of individuals in each class, \( n_i \), is observed over time
- Individual transitions are not observed and many different transitions could result in the observed data
- \( P_t \) needs to be estimated

Data availability:
- (Usually) good
- Example: For the analysis of EU farm structural change it is available from the Farm Structure Survey at population level

Example: Macro data
<table>
<thead>
<tr>
<th>State</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farms in t=0</td>
<td>60</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>Farms in t=1</td>
<td>40</td>
<td>40</td>
<td>20</td>
</tr>
</tbody>
</table>

Mcro data based likelihood: \( n_i \) are distributed as weighted sum of independent multinomials (MacRae 1977)

\[ \mathbf{n}_t = \sum_{i=1}^{k} \sum_{j=1}^{k} \mathbf{P}_{ijt} \mathbf{n}_{ijt} \]

A large sample approximation is employed (Brown and Payne 1986)

Posterior

in Bayesian estimation of Markov models

Computation
- A sample from the posterior density is obtained via a random walk Metropolis algorithm
- The posterior mean, which is the optimal Bayesian estimator under squared error loss, is approximated by the mean of the posterior sample

Monte Carlo Simulation
- Aim: Analyse influence of prior on posterior and estimator performance
- Separate simulation for ordered/unordered Markov states
- 10 true models with 20 repetitions each

Monte Carlo Results
Bow-Whisker plots of the (summed) squared deviation from the true values in the 200 simulation runs and (summed) variance of the posterior density

Conclusions
- Inclusion of micro data as prior information reduces Mean Square Error (MSE) and posterior variance
- Improvement stronger the more Markov states are considered
- Prior information increases numerical stability of the estimation

References


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Acknowledgements
This research is funded by the German Research Foundation (DFG), grant No. 585064