Analysis of Cardinal and Ordinal Assumptions in Conjoint Analysis

R. Wes Harrison, Jeffrey Gillespie, and Deacue Fields

Of twenty-three agricultural economics conjoint analyses conducted between 1990 and 2001, seventeen used interval-rating scales, with estimation procedures varying widely. This study tests cardinality assumptions in conjoint analysis when interval-rating scales are used, and tests whether the ordered probit or two-limit tobit model is the most valid. Results indicate that cardinality assumptions are invalid, but estimates of the underlying utility scale for the two models do not differ. Thus, while the ordered probit model is theoretically more appealing, the two-limit tobit model may be more useful in practice, especially in cases with limited degrees of freedom, such as with individual-level conjoint models.

Key Words: ordered probit, two-limit probit, conjoint analysis, cardinality

Numerous applications of conjoint analysis (CA) have emerged in the agricultural economics literature in recent years. Most of these studies have analyzed consumer preferences for new food products or resource usage and willingness to pay for recreational services. Studies evaluating new food products include Gineo (1990), Prentice and Benell (1992), Halbrendt, Wirth, and Vaughn (1991), Halbrendt, Bacon, and Pesek (1992), Yoo and Ohta (1995), Hobbs (1996), Sylvia and Larkin (1995), Sy et al. (1997), Harrison, Ozayan, and Meyers (1998), Gillespie et al. (1998), and Holland and Wessells (1998). New product acceptance studies typically assume that a respondent’s total utility for a hypothetical product is a function of various product attributes. CA is used to estimate “part-worth” utilities, which measure the partial effect of a particular attribute level on the respondent’s total utility for hypothetical products. Part-worth estimates are typically used to simulate utility values for products not evaluated by respondents; thus, optimal hypothetical products can be determined.

A second category of CA studies has sought to estimate respondents’ willingness to pay for a bundle of attributes associated with a recreational site or activity. Examples include Mackenzie (1990, 1993), Gan and Luzar (1993), Lin, Payson, and Wertz (1996), Roe, Boyle, and Teisl (1996), Stevens, Barrett, and Willis (1997), Miquel, Ryan, and McIntosh (2000), and Boyle et al. (2001). As with the new product acceptance studies, this approach requires respondents to rate or rank attribute bundles as price and other attribute levels are varied (Mackenzie 1990). Willingness to pay is calculated directly from the marginal rates of substitution between price and non-price attributes estimated from conjoint data.

Two commonly used methods for coding respondent preferences are rank-order and interval-rating scales. The rank-order method requires subjects to unambiguously rank all hypothetical product choices. In these cases, the dependent variable is ordinal, and ordered regression models such as ordered probit or logit are most suitable for conjoint estimation. The interval-rating method allows subjects to express order, indifference, and intensity across product choices, a feature allowing both metric and nonmetric properties of utility to be elicited. Model selection becomes less clear if interval-rating scaling is used. As shown in Table 1, a wide range of models have been utilized when ratings have been used. Some studies have used linear regression to estimate part-worth parameters (Halbrendt, Wirth, and Vaughn 1991; Prentice and Benell 1992; Harrison, Ozayan, and Meyers 1998; Stevens, Barrett, and Willis 1997; Roe, Boyle, and Teisl 1996).
These studies assume that interval-rating scales are metric and continuous. However, even if one argues that subjects express metric information in their responses, interval-rating scales are limited by upper and lower bounds. Under these circumstances, linear regression models yield truncated residuals and asymptotically biased parameters. The censored nature of the scale can be accounted for with a two-limit tobit (TLT) model, which corrects for censoring and retains metric information between the bounds. Alternatively, some argue that the ordered probit (OP) model is more suitable since interval-rating scales are typically measured as discrete variables and ordinal preferences are more appealing theoretically (Mackenzie 1990, 1993; Sy et al. 1997; Holland and Wessells 1998).

This paper examines the cardinal versus ordinal assumptions in conjoint analysis when an interval-rating scale is used. The paper contributes in two respects. First, the theoretical underpinnings of various measurement scales and their relationships to cardinal and ordinal preferences are examined. Second, a method for analyzing the cardinality assumption in conjoint analysis is developed using two-limit tobit and ordered probit models. The analysis is applied to three separate conjoint data sets.
Literature Review

The debate among economists as to whether cardinal preferences can be assumed is not new. Van Praag (1991) discusses the history of the cardinality-ordinality debate during the late nineteenth and early twentieth centuries. Cardinal utility was central in the development of demand theory and was largely unopposed throughout the nineteenth century (e.g., Jevons 1871, Menger 1881). The notion of a cardinal utility function was appealing because it yielded a robust theory of consumer demand and allowed for interpersonal comparisons of utility. However, by the beginning of the twentieth century, some economists began to doubt whether cardinal utility actually existed, and whether it could be accurately measured. This led to Pareto’s (1909) assertion that it was not necessary to have an exact utility function to explain demand—that only indifference curves were needed. He showed that a broader class of ordinal, monotonically increasing functions is sufficient to derive all properties of demand. Pareto’s view was reinforced in the 1930s, as Robbins (1932) rejected the idea that cardinal utility was measurable at all. Other authors have given rigorous explanations of the theory of consumer demand in the absence of cardinal utility (e.g., Fisher 1918, Stigler 1950, Alchian 1953). Today, graduate-level microeconomics texts such as Varian (1947), Silberberg (1978), and Henderson and Quandt (1980) discuss the highly restrictive nature of cardinal preferences, and develop demand theory in the context of ordinal utility.

In spite of these developments, economists remain interested in cardinal utility. Three areas of economics that assume some form of cardinal utility include (i) those estimating utility functions in an expected utility framework, in the spirit of von Neumann and Morgenstern (1947), (ii) studies using conjoint and other multiattribute utility procedures to elicit strength of preferences, and (iii) studies on income equality and poverty in which interpersonal comparisons of utility are based. In each of these areas, it is assumed that, while cardinal preferences are difficult to measure with great precision, cardinal utility functions provide answers to “real world” questions.

Several studies have analyzed the effects of treating an interval-rating scale as a cardinal measure of consumer preference (Mackenzie 1993; Roe, Boyle, and Teis 1996; Stevens, Barrett, and Willis 1997). These studies compare parameter estimates and predictability of TLT with OP models, and have produced mixed results. Mackenzie (1993) found evidence that rating scales capture intensity (cardinality) in respondent preferences. The other two studies found empirical evidence suggesting the superiority of ordered probability models as frameworks for analysis and suggested that assuming ordinal preferences is theoretically more appealing. Boyle et al. (2001) examined cardinality by analyzing both rating and ranking scales for independent sub-samples of respondents. They found that TLT and OP models resulted in the same attributes being significant with the same signs. They concluded that assumptions regarding ordinal/cardinal preferences were irrelevant for their sample. They did not, however, test the cardinality assumption or examine how well the models predicted preference ordering.

More recently, Harrison, Stringer, and Prinyawiwatkul (2002) found little difference between the TLT and OP models with respect to part-worth estimates and predictive validity. The signs, relative magnitudes, and statistical significance of all part-worth estimates were consistent across the two models. They found no statistical differences between individual-level Spearman rank correlation coefficients between predicted and observed values for the two models. Swallow, Opaluch, and Weaver (2001) examined the statistical implications of utilizing strength-of-preference information in contingent valuation. They found that an ordered response model incorporating “quasi-cardinal” information provided substantial efficiency gains relative to a binary response model that assumes a purely ordinal scale. These studies examine various degrees of cardinality by conducting comparisons of alternative models. The present study differs from previous literature by developing formal procedures for determining whether interval-rating scales contain cardinal information.

\[1\] Expected utility theory relaxes the assumption of pure ordinality. Additional axioms are introduced to allow for an expected utility function that is unique up to an affine transformation.

\[2\] We should note that cardinal information does not solve all the problems associated with interpersonal comparison of utility. Even if an individual’s interval scale contains cardinal information, the meaning of the scale values may differ from one individual to another.
Theoretical Considerations in Measuring Utility

What conditions are required for cardinal utility? Torgerson (1958, pp. 15–21) describes several types of commonly used measurement scales. The first is a purely ordinal scale, which has no natural origin and whose intervals between scale values carry no meaning. This is equivalent to an individual selecting the most preferred attribute or product bundle from a set of bundles, then, once the most preferred bundle is removed, the respondent selecting the second-best bundle from the remaining bundles. The process continues until all bundles are ranked, which results in a complete ordering of the choice set. If the restriction of a natural origin is added, where zero represents the null set, the purely ordinal scale is consistent with the definition of an ordinal utility function as defined by modern economic theory (Henderson and Quandt 1980, pp. 5–8). A purely ordinal scale implies that, given a set of numbers arranged to represent rank order, an increasing monotonic transformation of the set will preserve the original ordering. Another implication is that the rank of a particular bundle in one choice set cannot be compared to bundles in other choice sets. This follows from the assumption that differences between scale values carry no meaning, which also implies that bundles are not comparable across individuals. The purely ordinal scale can be translated into a binary choice model, where the most preferred bundles from a series of ranking-tasks represent the choice variables.

An extension of the purely ordinal scale is where differences between rankings have meaning, but are still ordinal. Swallow, Opaluch, and Weaver (2001) refer to this as a “quasi-cardinal” scale, where intervals represent imprecise indicators of an individual’s strength of preferences. Another way to think about this is that differences between scale values are meaningful, but they are not evenly spaced. Ordered logit or probit models can be used with this type of scaling. Swallow, Opaluch, and Weaver (2001) found that this type of strength-of-preference information improved the efficiency of WTP estimates, provided that the analyst assumes that “quasi-cardinal” information allows for at least partial interpersonal comparisons of utility.

Another type of scale is the equal-interval scale without an origin, which is the closest scale to pure cardinality. This form of cardinality not only assumes that intervals between scale values have meaning, but that they are also equally spaced (Torgerson 1958). Temperature scales are examples. A set of numbers satisfying the property of interval scaling is sensitive to linear transformations of the form \( y = \alpha + \beta x \), where \( \alpha \) is any positive scalar. This implies a form of cardinal scaling since interval distances have metric properties, but the origin of the scale has no meaning. As previously discussed, linear regression and TLT models may be used under these more strict assumptions.

Lastly, the ratio scale is an interval scale with the additional assumption of a natural origin. The ratio scale has the property that a number \( 2x \) is twice as great as \( x \); that is, the scale is sensitive to linear transformations of the form \( y = \beta x \). The restrictive ratio scale is consistent with the concept of a purely cardinal utility function (Henderson and Quandt 1980, pp. 5–8).

Whether interval-rating scales used in CA studies meet conditions of ordinality, quasi-cardinality, equal-interval cardinality, or pure cardinality will depend upon how preferences are elicited, and on whether respondents express certain properties in their valuations. For instance, suppose respondents are asked to rate a subset of attribute bundles (selected from the total population of all possible attribute combinations) such that “0” is assigned to the least preferred and “10” is assigned to the most preferred bundle. It is possible that at least one of the untested bundles is less preferred than the one rated “0”; likewise, an untested bundle could be preferred over the one rated “10”. Under this type of rating task, at least ordinal scaling is present. However, equal-interval scaling may also result if the distributional properties of respondent valuations correspond to the equal interval properties associated with the real number system.

On the other hand, if respondents are asked to assign a “0” to the worst possible bundle selected from the total population of attribute bundles, then the argument might be made that a ratio scale could be constructed. This is usually not the case, however, since few CA studies elicit responses for all possible attribute combinations. The distinction between ratio and equal-interval scaling is important, as cardinality has been criticized on the basis that it must be synonymous
with the ratio scale. Such criticism argues that a response of “4” means that the respondent prefers the bundle twice as much as one rated a “2”. An implication here is that a ratio scale is the only scale providing enough cardinal information to allow for interpersonal comparisons of utility. However, if equal-interval scaling is present, then a degree of cardinal information is present, and this information can be used to simulate the total utilities for untested product bundles so as to provide a unique ordering for all products. Moreover, like Swallow, Opaluch, and Weaver’s (2001) quasi-cardinal scale, the equal-interval scale also assumes a degree of interpersonal comparability.

It is not our intention to argue for or against the existence of a purely cardinal utility function. It is left to the researcher to determine its usefulness given the context of a particular study. However, various degrees of cardinality are possible when interval scaling is used, and its existence may be tested empirically. This is an important empirical issue since a researcher’s choice of scaling places restrictions on the subject’s ability to reveal his or her “true” preferences in CA studies. Moreover, the subsequent choice among econometric models is dependent upon these restrictions.

Model Specification: Cardinal Versus Ordinal Assumptions

Most conjoint studies assume that a consumer’s true utility is a linear function of selected product attributes. Two-limit tobit and OP models provide a means to estimate conjoint parameters and to evaluate the cardinal and ordinal properties of interval-rating scales. The structural equation for both models is specified as

$$y_i^* = x_i \beta + e_i,$$

where $y_i^*$ is a latent variable representing $i$th individual’s total utility for a particular combination of product attributes, $\beta$ is a $(k \times 1)$ column vector with the first element being the intercept $\beta_0$ and all other elements being part-worth utility effects, $x_i$ is the $i$th $(1 \times k)$ row vector representing the product attributes with a “1” in the first column for the intercept, and $e_i$ is the error term. The latent variable, $y^*$, is assumed to be continuous and metric in nature.

A primary assumption of both models is that interval-rating scales provide only limited information about a consumer’s true preferences ($y^*$). Assume that an interval-rating scale from 0 to $J$ is used to measure respondent preferences, where 0 and $J$ are assigned to the least and most preferred bundles, respectively. The TLT model assumes the following relationship between the interval-rating scale and $y^*$:

$$y_i = \begin{cases} 
\mu_L, & \text{if } y_i^* \leq \mu_L, \\
\mu^*, & \text{if } \mu_L < y_i^* < \mu_U, \text{ and} \\
\mu_U, & \text{if } y_i^* \geq \mu_U,
\end{cases}$$

where $\mu_L$ and $\mu_U$ are known, and set equal to the lower and upper bounds of the scale (i.e., 0 and $J$, respectively), $\mu^*$ equals the observed value of the scale for the $i$th respondent, and $y^*$ is as previously defined. An important assumption is that a respondent’s true preferences are censored by upper and lower values of the scale. This implies that some respondents rating products as either 0 or $J$ would have assigned lower or higher values to untested products if allowed to do so by experimental conditions.

A key difference between TLT and OP models is the restriction each places on the measurement of $y^*$. The OP model also assumes that $y^*$ is censored, but differs as follows:

$$y_i = \begin{cases} 
0, & \text{if } y_i^* \leq \mu_0, \\
1, & \text{if } \mu_0 < y_i^* \leq \mu_1, \\
2, & \text{if } \mu_1 < y_i^* \leq \mu_2, \\
\vdots & \vdots \\
J, & \text{if } y_i^* \geq \mu_J
\end{cases}$$

where the $\mu$’s are unknown “threshold” parameters that determine the spacing between the $J$ categories of $y$. In addition to differences in the mapping of $y$ onto $y^*$, the models also differ in regard to the error structure. The TLT model assumes $e_i$ is normally distributed with zero mean and variance equal to $\sigma^2$, where $\sigma^2$ is estimated along with other model parameters (Long 1997). Ordered probit also assumes that $e_i$ is normally distributed.
distributed with zero mean, but sets $\sigma^2$ equal to one.

It is important to note that the OP model assumes only that the $\mu$'s increase as $y$ increases, so they represent only a monotonic mapping of $y$ onto $y^*$, which is consistent with the previously discussed ordinal scales. Moreover, it can be shown that the TLT model implicitly assumes equal-interval scaling. This allows us to use the spacing of the $\mu$'s in the OP model to test the equal interval assumptions of the TLT model. Readers interested in a more technical discussion of linkages between the OP and TLT models with respect to interval spacing should contact the authors.

**Data**

Three data sets were used to test for the presence of equal-interval scaling when interval-rating scales were used to elicit consumer preferences. All three were collected to examine consumer/buyer preferences for new hypothetical products using interval-rating scales, which allows respondents to indicate ties, as well as the strength of their preferences. Models are compared by formally testing the equal interval assumption, comparing standardized parameter estimates, and analyzing predicted values across models.

The first data set was collected to examine consumer preferences for new food products derived from crawfish. The attributes and levels selected were three product forms consisting of crawfish minced-based nuggets, patties, and poppers; three package sizes consisting of a 12-, 24-, and 48-pack; three reheating methods expressed as baked, fried, or microwaved; and three price levels set at 10, 20, and 50 cents per ounce. With four 3-level attributes, a full factorial experimental design would involve 81 hypothetical product combinations. Subjects would have difficulty rating all 81 product profiles, so a fractional factorial design was used to reduce the number of profiles to nine. The questionnaire containing the nine profiles was administered via personal interview, where respondents were allowed to visually inspect and handle all nine products. After examination, the respondent was asked to rate each product profile on a scale from 1 to 10, where 1 was the least and 10 the most preferred. The sample was composed of 111 consumers who had been recruited by telephone in and around the city of Baton Rouge, Louisiana. Greater detail on the procedures is found in Harrison, Ozayan, and Meyers (1998).

The second data set was collected to examine retailer preferences of ostrich meat products. Four attributes and their levels were selected: portion size, including non-portioned, four-ounce, and six-ounce portions; product type, including ground, processed, and filet; whether or not the product was branded; and purchase price from the processor in dollars per pound: $4.00, $8.00, and $12.00. A full factorial experimental design would involve 54 hypothetical product combinations. A fractional factorial design reduced the number of profiles to nine. The questionnaire was administered via mail to retailers in the south-central United States. Respondents were asked to rate each profile from 0 to 10, where 0 was the least and 10 the most preferred product. Conjoint results were collected from 133 retail outlets. See Gillespie et al. (1998) for greater detail on the study.

The third data set was collected to examine consumer preferences of crawfish sausage products. Four attributes and their respective levels were selected: price, which included $3.00 and $3.50 per pound levels; package size, which included 16- and 48-ounce sizes; cooking method, which included baked, pan fried, and deep fried; and product form, which included short, medium, and long sausage links. A full factorial experimental design would involve 36 hypothetical product combinations. A fractional factorial design reduced the number of profiles to nine. The questionnaire was administered via personal interview. Respondents were asked to rate each profile from 1 to 10, where 1 was the least preferred and 10 the most preferred product. Conjoint results were collected from 144 consumers. See Harrison, Stringer, and Prinyawiwatkul (2002) for greater detail on the study.

**Results**

A Wald statistic is used to test the following null hypothesis: $H_0$: $(\mu_j - \mu_k) - (\mu_k - \mu_l) = 0$, for all $j$ and $k$, $j \neq k$, where $(\mu_j - \mu_k)$ and $(\mu_k - \mu_l)$ measure the interval distances between threshold parameters in the OP model. Rejection of $H_0$ provides evidence that equal interval assumptions of
the TLT model do not hold. The Wald statistic \((W)\) is distributed as chi square and calculated as

\[
W = \frac{[(\mu_j - \mu_{j+1}) - (\mu_k - \mu_{k+1})]^2}{\text{Var}(\mu_j - \mu_{j+1}) + \text{Var}(\mu_k - \mu_{k+1}) - 2\text{Cov}(\mu_j - \mu_{j+1}, \mu_k - \mu_{k+1})}.
\]

where \((\mu_j - \mu_{j+1})\) and \((\mu_k - \mu_{k+1})\) are distances between threshold parameters in the OP model. The Wald statistic \((W)\) is distributed as chi square and calculated as

\[
W = \frac{[(\mu_j - \mu_{j+1}) - (\mu_k - \mu_{k+1})]^2}{\text{Var}(\mu_j - \mu_{j+1}) + \text{Var}(\mu_k - \mu_{k+1}) - 2\text{Cov}(\mu_j - \mu_{j+1}, \mu_k - \mu_{k+1})}.
\]

A Wald test is used for the following null hypothesis: \(H_0: (\mu_j - \mu_{j+1}) - (\mu_k - \mu_{k+1}) = 0\), for all \(j\) and \(k\), \(j \neq k\), where \((\mu_j - \mu_{j+1})\) and \((\mu_k - \mu_{k+1})\) measure distances between threshold parameters in the OP model. The Wald statistic \((W)\) is distributed as chi square and calculated as

\[
W = \frac{[(\mu_j - \mu_{j+1}) - (\mu_k - \mu_{k+1})]^2}{\text{Var}(\mu_j - \mu_{j+1}) + \text{Var}(\mu_k - \mu_{k+1}) - 2\text{Cov}(\mu_j - \mu_{j+1}, \mu_k - \mu_{k+1})}.
\]

*, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

### Table 2. Wald Tests of Differences in Intervals

<table>
<thead>
<tr>
<th>(\mu_2 - \mu_1)</th>
<th>(\mu_3 - \mu_2)</th>
<th>(\mu_4 - \mu_3)</th>
<th>(\mu_5 - \mu_4)</th>
<th>(\mu_6 - \mu_5)</th>
<th>(\mu_7 - \mu_6)</th>
<th>(\mu_8 - \mu_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_1 - 0)</td>
<td>0.082</td>
<td>0.103***</td>
<td>0.097*</td>
<td>0.132</td>
<td>0.064</td>
<td>0.019</td>
</tr>
<tr>
<td>(\mu_2 - \mu_1)</td>
<td>0.021</td>
<td>0.015</td>
<td>0.050</td>
<td>-0.018</td>
<td>-0.063</td>
<td>-0.158***</td>
</tr>
<tr>
<td>(\mu_3 - \mu_2)</td>
<td>-0.006</td>
<td>0.029</td>
<td>-0.039</td>
<td>-0.084*</td>
<td>-0.180***</td>
<td>n/a</td>
</tr>
<tr>
<td>(\mu_4 - \mu_3)</td>
<td>0.035</td>
<td>-0.033</td>
<td>-0.078*</td>
<td>-0.174***</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>(\mu_5 - \mu_4)</td>
<td>-0.068</td>
<td>-0.113**</td>
<td>-0.208***</td>
<td>n/a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu_6 - \mu_5)</td>
<td>-0.045</td>
<td>-0.140**</td>
<td>n/a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu_7 - \mu_6)</td>
<td>-0.095</td>
<td>n/a</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A Wald test is used for the following null hypothesis: \(H_0: (\mu_j - \mu_{j+1}) - (\mu_k - \mu_{k+1}) = 0\), for all \(j\) and \(k\), \(j \neq k\), where \((\mu_j - \mu_{j+1})\) and \((\mu_k - \mu_{k+1})\) measure distances between threshold parameters in the OP model. The Wald statistic \((W)\) is distributed as chi square and calculated as

\[
W = \frac{[(\mu_j - \mu_{j+1}) - (\mu_k - \mu_{k+1})]^2}{\text{Var}(\mu_j - \mu_{j+1}) + \text{Var}(\mu_k - \mu_{k+1}) - 2\text{Cov}(\mu_j - \mu_{j+1}, \mu_k - \mu_{k+1})}.
\]

*, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

The test results are presented in Table 2. For the crawfish nugget data set, seven of the 28 pairwise comparisons of intervals were significantly different at the 0.05 level or greater. An additional four were significantly different at the 0.10 level. The \(\mu_8 - \mu_7\) interval is the widest and, therefore, is significantly different from most of the other intervals. For the ostrich meat data set, 24 of 36 combinations of intervals were significantly different at the 0.05 level or greater. An additional two were significant at the 0.10 level. The \(\mu_8 - \mu_7\) interval was the widest interval and, therefore, was significantly different from all other intervals at the 0.01 level of significance. The \(\mu_i\)’s estimated with this data set were the most unevenly spaced of the three data sets. For the sausage data set, ten of 28 combinations were significantly different at the 0.05 level or greater.
Table 3. Two-Limit Tobit and Ordered Probit Part-Worth Estimates for the Nugget-Based Crawfish Products Analysis

<table>
<thead>
<tr>
<th>Attribute</th>
<th>TLT and OP Index Function Estimates</th>
<th>OP Threshold Estimates</th>
<th>TLT and OP Standardized Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_{\text{TLT}}$</td>
<td>$\beta_{\text{OP}}$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Constant</td>
<td>0.646 (1.047)</td>
<td>0.146 (0.240)</td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>Average rating</td>
<td>1.145 ***b (9.522)</td>
<td>0.383 *** (7.873)</td>
<td>$\mu_2$</td>
</tr>
<tr>
<td>Packaging</td>
<td>0.010 (1.556)</td>
<td>0.003 (1.492)</td>
<td>$\mu_3$</td>
</tr>
<tr>
<td>Price</td>
<td>-6.046 ***(-10.676)</td>
<td>-2.044 ***(-10.275)</td>
<td>$\mu_4$</td>
</tr>
<tr>
<td>Nugget form</td>
<td>-0.411 ***(-3.023)</td>
<td>-0.137 ***(-2.976)</td>
<td>$\mu_5$</td>
</tr>
<tr>
<td>Patty form</td>
<td>0.841 *** (6.214)</td>
<td>0.283 *** (5.799)</td>
<td>$\mu_6$</td>
</tr>
<tr>
<td>Fried reheat</td>
<td>-0.378 ***(-2.786)</td>
<td>-0.129 ***(-2.816)</td>
<td>$\mu_7$</td>
</tr>
<tr>
<td>Microwave reheat</td>
<td>0.400 *** (2.952)</td>
<td>0.138 *** (2.913)</td>
<td>$\mu_8$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.974 *** (38.126)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Log L. ratio $\chi^2$</td>
<td>229.78 ***</td>
<td>227.37 ***</td>
<td></td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>.21</td>
<td>.21</td>
<td></td>
</tr>
<tr>
<td>Hausman statistic</td>
<td>7.17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a The notation $\beta^S_{\text{TLT}}$ and $\beta^S_{\text{OP}}$ refers to the standardized estimates, which are calculated using the following formula: $\beta^S_k = \beta_k / \sigma_{y^*}$, where $\sigma_{y^*}$ is the unconditional standard deviation of $y^*$ and is estimated by $\sigma_{y^*}^2 = \sigma_x^2 \cdot Var(\beta) + Var(\varepsilon)$. 

b ***, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

c Hausman’s procedure is used to test the null hypothesis that TLT standardized estimates are inconsistent relative to OP standardized estimates. The Hausman statistic is distributed as chi square and defined as follows:

$$H = (\beta_{11}^{\text{OP}} - \beta_{11}^{\text{TLT}}) \cdot [Var(\beta_{11}^{\text{OP}}) - Var(\beta_{11}^{\text{OP}})]^{-1} (\beta_{11}^{\text{OP}} - \beta_{11}^{\text{TLT}}).$$

An additional one was significantly different at the 0.10 level. Each of these analyses showed significant differences in the intervals between the $\mu$’s, suggesting that the equal interval assumption of the TLT model does not hold for any of the three data sets.

Comparison of Part-Worth Estimates

Given that equal interval assumptions of the TLT model do not hold for any of the data sets, it is useful to determine whether part-worth estimates differ between the two models. The index function estimates for the TLT and OP models are presented in Tables 3, 4, and 5 for the crawfish nugget, ostrich meat, and crawfish sausage data sets, respectively. The index function $\beta$s represent the part-worth estimates utilized in all of the previously cited CA studies. For both models, the $\beta$s are interpreted as the change in the underlying utility scale given a unit change in $x$. Casual observation of the results suggests that estimates of the TLT and OP analyses are consistent in the sense that, in all cases, the signs are the same, and variables that are significant for the TLT model are also significant for the OP model. However, since parameters in the TLT and OP models are
Table 4. Two-Limit Tobit and Ordered Probit Part-Worth Estimates for the Ostrich Meat Products Analysis

<table>
<thead>
<tr>
<th>Attribute</th>
<th>TLT and OP Index Function Estimates</th>
<th>OP Threshold Estimates</th>
<th>TLT and OP Standardized Estimates ( ^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{\beta}_{TLT} )</td>
<td>( \hat{\beta}_{OP} )</td>
<td>( \mu )</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.636 ** ** b</td>
<td>-0.754 **</td>
<td>0.174 **</td>
</tr>
<tr>
<td>Average rating</td>
<td>1.460 **</td>
<td>0.357 **</td>
<td>0.414 **</td>
</tr>
<tr>
<td>Branded product</td>
<td>0.431 **</td>
<td>-0.102 **</td>
<td>0.663 **</td>
</tr>
<tr>
<td>Ground form</td>
<td>0.132</td>
<td>0.039</td>
<td>0.870 **</td>
</tr>
<tr>
<td>Processed form</td>
<td>-0.681 **</td>
<td>-0.168 **</td>
<td>1.238 **</td>
</tr>
<tr>
<td>4-ounce portion</td>
<td>0.012</td>
<td>0.002</td>
<td>1.482 **</td>
</tr>
<tr>
<td>6-ounce portion</td>
<td>0.424 **</td>
<td>0.101 **</td>
<td>1.619 **</td>
</tr>
<tr>
<td>Price $12</td>
<td>-1.458 **</td>
<td>-0.353 **</td>
<td>2.183 **</td>
</tr>
<tr>
<td>Price $8</td>
<td>-0.303 *</td>
<td>-0.085 *</td>
<td>2.327 **</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>4.156 **</td>
<td>1</td>
<td>4.4282 **</td>
</tr>
<tr>
<td>Log L. Ratio ( \chi^2 )</td>
<td>442.28 **</td>
<td>449.10 **</td>
<td></td>
</tr>
<tr>
<td>Pseudo R(^2)</td>
<td>.34</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>Hausman statistic ( ^c )</td>
<td>4.30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( ^a \) The notation \( \hat{\beta}_{TLT}^S \) and \( \hat{\beta}_{OP}^S \) refers to the standardized estimates, which are calculated using the following formula: \( \hat{\beta}_{TLT}^S = \beta_{TLT}/\sigma_{\hat{y}^*} \), where \( \sigma_{\hat{y}^*} \) is the unconditional standard deviation of \( \hat{y}^* \) and is estimated by \( \sigma_{\hat{y}^*} = \sqrt{\text{Var}(x)\beta + \text{Var}(e)} \).

\( ^b \) *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

\( ^c \) Hausman’s procedure is used to test the null hypothesis that TLT standardized estimates are inconsistent relative to OP standardized estimates. The Hausman statistic is distributed as a chi square and defined as follows:

\[
H = (\hat{\beta}_{OP}^S - \hat{\beta}_{TLT}^S) [\text{Var}(\hat{\beta}_{OP}) - \text{Var}(\hat{\beta}_{TLT})]^{-1} (\hat{\beta}_{OP}^S - \hat{\beta}_{TLT}^S).
\]

The standardized \( \beta \)s are calculated as

\[
\hat{\beta}_{s}^x = \hat{\beta}_{T} / \sigma_{\hat{y}^*},
\]

where \( \sigma_{\hat{y}^*} \) is the unconditional standard deviation of \( \hat{y}^* \) and is estimated as

\[
\sigma_{\hat{y}^*} = \sqrt{\text{Var}(x)\beta + \text{Var}(e)}.
\]

Estimated under different assumptions regarding the variance of \( \sigma \), direct comparisons of the magnitudes of \( \beta \)s are not meaningful. McKelvey and Zavoina (1975) introduce a formula to standardize the coefficients for cross-model comparisons. The standardized \( \beta \)s are calculated as

\[
\hat{\beta}_{s}^x = \hat{\beta}_{T}/\sigma_{\hat{y}^*},
\]
Table 5. Two-Limit Tobit and Ordered Probit Part-Worth Estimates for the Crawfish Sausage Products Analysis

<table>
<thead>
<tr>
<th>Attribute</th>
<th>TLT and OP Index Function Estimates</th>
<th>OP Threshold Estimates</th>
<th>TLT and OP Standardized Estimates a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β_{TLT}</td>
<td>β_{OP}</td>
<td>μ_1</td>
</tr>
<tr>
<td>Constant</td>
<td>2.736 **b</td>
<td>0.813 *</td>
<td></td>
</tr>
<tr>
<td>Average rating</td>
<td>1.305 ***</td>
<td>0.426 ***</td>
<td>μ_2</td>
</tr>
<tr>
<td>Price</td>
<td>-1.026 ***</td>
<td>-0.334 ***</td>
<td>μ_3</td>
</tr>
<tr>
<td>Package size</td>
<td>-0.035 ***</td>
<td>-0.011 ***</td>
<td>μ_4</td>
</tr>
<tr>
<td>Link size: long</td>
<td>0.385 ***</td>
<td>0.126 ***</td>
<td>μ_5</td>
</tr>
<tr>
<td>Link size: medium</td>
<td>0.390 ***</td>
<td>0.127 ***</td>
<td>μ_6</td>
</tr>
<tr>
<td>Baked</td>
<td>0.955 ***</td>
<td>0.312 ***</td>
<td>μ_7</td>
</tr>
<tr>
<td>Deep fried</td>
<td>1.010 ***</td>
<td>0.329 ***</td>
<td>μ_8</td>
</tr>
<tr>
<td>σ</td>
<td>3.091 ***</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Log L. Ratio χ^2</td>
<td>522.91 ***</td>
<td>522.84 ***</td>
<td></td>
</tr>
<tr>
<td>Pseudo R^2</td>
<td>.34</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>Hausman statistic b</td>
<td>6.13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a The notation β_{TLT}^S and β_{OP}^S refers to the standardized estimates, which are calculated using the following formula: 

β_{i} = β_i / σ_{y},

where σ_{y} is the unconditional standard deviation of y and is estimated by σ_{y}^2 = Var(y) + Var(ε).

b *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels.

c Hausman’s procedure is used to test the null hypothesis that TLT standardized estimates are inconsistent relative to OP standardized estimates. The Hausman statistic is distributed as a chi square and defined as follows:

H = (β_{OP}^S - β_{TLT}^S)^T [Var(β_{OP}^S) - Var(β_{TLT}^S)]^{-1} (β_{OP}^S - β_{TLT}^S).

estimates. Results show that the standardized TLT estimates are not significantly different from the OP standardized estimates at the 0.10 level of significance (Tables 3, 4, and 5). Thus, despite rejection of the equal interval assumption, the two models do not differ significantly with respect to their estimates of the underlying utility scale.

Analysis of Predicted Values

Although the estimated TLT and OP models are consistent with respect to estimation of the underlying utility index, predicted values of the two models differ. Since there is no conditional mean for y in the OP model, the predicted value is the category having the highest probability of occurrence, given x. Consequently, the spacing of the μ’s has a significant effect on predicted values of the OP model. Predicted values are calculated as

Max_{y} Pr[y = j | x] = Φ(μ_j - xβ) - Φ(μ_{j-1} - xβ),

where ^ denotes the maximum likelihood estimates (Long 1997). (For simplification, ^ is dropped in the following discussion.) Since the conditional mean of y (i.e., xβ) is common
between the first and second terms in (5), the probability of \( y^* \) falling in the \( j \)th category is uniquely determined by the spacing of \( \mu_j \) and \( \mu_{j-1} \).

Figure 1 shows how equally spaced \( \mu \)'s affect the cell probabilities for the case where \( J \) equals 9. The vertical axis shows the value of the conditional cumulative density function (CDF) for \( y^* \), i.e., the probability that \( y^* \leq j \) given \( x \). The horizontal axis shows the value of \( y^* \) with the \( J - 1 \) thresholds spaced uniformly along its scale. The probability that \( y^* = j \) depends on the difference between values of the conditional CDF evaluated at \( \mu_j \) and \( \mu_{j-1} \). For instance, the probability that \( y^* \) falls in category 6 is given by the difference between points a and b in Figure 1. Notice that equally spaced thresholds imply that the CDF for \( y^* \) is divided equally across the \( J - 1 \) categories, but does not imply that cell probabilities are equal across intervals. Unequally spaced thresholds imply that the conditional CDF is divided asymmetrically by the \( \mu \)'s, and that cell probabilities are skewed toward intervals with the largest spacing (Figure 2).

Since the TLT model assumes that the interval rating scale is continuous between its upper and lower limits, its predicted values differ from those of the OP model. The TLT predicted values are given by the conditional mean of \( y \) given \( x \). The formula is

\[
E(y \mid x) = \frac{\Phi(\delta_l / \sigma) - \Phi(\delta_u / \sigma)}{\Phi(\delta_u / \sigma) - \Phi(\delta_l / \sigma)} \left[ \mu_l - \mu_u \right] + \frac{\Phi(\delta_{u-1} / \sigma) - \Phi(\delta_l / \sigma)}{\Phi(\delta_u / \sigma) - \Phi(\delta_l / \sigma)} \left[ \mu_{u-1} - \mu_l \right] + \frac{\Phi(\delta_{j-1} / \sigma) - \Phi(\delta_l / \sigma)}{\Phi(\delta_u / \sigma) - \Phi(\delta_l / \sigma)} \left[ \mu_{j-1} - \mu_l \right]
\]

where \( \delta_l = (\mu_l - x \beta) / \sigma \) and \( \delta_u = (\mu_u - x \beta) / \sigma \), and all other variables are as previously defined (Long 1997, p. 213). Note that OP predicted values correspond to one of the \( J \) discrete values, while TLT predicted values may take non-integer values. This complicates comparison of predicted values for the two models.

Several studies have used individual Spearman rank correlation (SRC) coefficients to examine the correlation between actual and predicted values (Harrison, Stringer, and Prinyawiwatkul 2002; Roe, Boyle, and Teisl 1996). SRC coefficients and Wilcoxon signed rank (WSR) tests are used to examine the in-sample correlation between actual and predicted values for each respondent. The SRC is used because the OP model provides ordinal predicted values. The SRC coefficient, corrected for tied data, may be found in Zar (1984, p. 320). Once the SRC for each model
and individual in the sample is calculated, the WSR statistic is used to test the null hypothesis that the SRC coefficients differ between TLT and OP models. The WSR test calculates absolute differences between each respondent’s TLT and OP SRC coefficients, ranks the absolute values across the entire sample, assigns the sign of the original difference to the rank, and then sums the ranks (Roe, Boyle, and Teisl 1996).

Results of the SRC analysis suggest that, for all three data sets, the rank order of the TLT results is more highly correlated with the actual ranking than the OP results (Table 6). The SRC coefficients were greater for the TLT relative to the OP results in 64 percent, 73 percent, and 55 percent of the cases for the crawfish nugget, ostrich meat, and crawfish sausage data sets, respectively. This is not surprising given the distribution of the predicted values for both TLT and OP analyses. Ordered probit predicted values tend to cluster among a limited number of predicted values. For the crawfish nugget, ostrich meat, and crawfish sausage data sets, only five, three, and four values were predicted, respectively. These ratings generally fell within the widest intervals of the μ’s. For example, the ostrich meat OP model yielded predicted values of 0, 8, and 10, with a frequency distribution of 51.1, 25.8, and 23.3 percent of the total sample, respectively. In contrast, TLT predicted ratings calculated using equation (6) are real numbers in the interval [0, 10]. In calculating SRC, the squared difference between the actual and the predicted values is expected to be higher for the OP results than for the TLT, since there are few numbers that OP predicted values will take, resulting in a lower SRC for the OP model. The SRC coefficient penalizes OP results because of the discrete nature in which its predicted values are estimated. Note the large number of cases in which SRC was greater for the TLT model in the ostrich meat data set (Table 6). Given that there were only three predicted values in that data set using OP (0, 8, and 10), the result is not surprising.

Given the tendency for the SRC coefficient to favor TLT results, a second rank-order correlation coefficient, gamma (γ), was estimated (see Blalock 1979, pp. 416–426). This statistic tests whether pairs are concordant or discordant. Pairings are concordant if the estimated ranking increases along with the actual ranking. Discordance occurs in the opposite case. Ties are counted as neither concordant nor discordant. Contrary to results of the SRC coefficients, the γ coefficients were greater for the OP results for all three data sets, and significantly different in the crawfish nugget and sausage data sets. Ordered probit results had greater associated γ’s in 63, 56,
Table 6. Results of the Spearman and Gamma Analyses

<table>
<thead>
<tr>
<th></th>
<th>Median Value</th>
<th>Number of Cases</th>
<th>Wilcoxon Signed Rank Test $^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ordered Probit</td>
<td>Two-Limit Tobit</td>
<td>OP &gt; TLT</td>
</tr>
<tr>
<td>Crawfish nugget Spearman $r_a$</td>
<td>0.35</td>
<td>0.38</td>
<td>71</td>
</tr>
<tr>
<td>Crawfish nugget Gamma $\gamma$</td>
<td>0.31</td>
<td>0.28</td>
<td>40</td>
</tr>
<tr>
<td>Ostrich meat Spearman $r_a$</td>
<td>0.32</td>
<td>0.43</td>
<td>74</td>
</tr>
<tr>
<td>Ostrich meat Gamma $\gamma$</td>
<td>0.43</td>
<td>0.39</td>
<td>43</td>
</tr>
<tr>
<td>Sausage Spearman $r_a$</td>
<td>0.58</td>
<td>0.62</td>
<td>76</td>
</tr>
<tr>
<td>Sausage Gamma $\gamma$</td>
<td>0.67</td>
<td>0.52</td>
<td>40</td>
</tr>
</tbody>
</table>

$^a$ Spearman rank correlation and gamma coefficients are calculated between actual and predicted rankings for each respondent in the sample.

$^b$ The Wilcoxon signed rank test tests the null hypothesis that differences between Spearman rank and gamma values for TLT and OP models are equal to zero.

*, **, and ### indicate significance at the 0.10, 0.05, and 0.01 levels.

and 70 percent of the cases for the crawfish nugget, ostrich meat, and crawfish sausage data sets, respectively. This suggests greater concordance with the OP rankings than with the TLT rankings. Unfortunately, dependence on $\gamma$ for determination of rank-order correlation has problems as well. Consider a hypothetical case where the actual ranking for a set of 10 profiles is 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, compared to TLT model predicted values of 1, 2, 3, 4, 5, 6, 7.51, 7.49, 9, and 10, and OP model predicted values of 1, 1, 1, 1, 1, 1, 1, 1, 1, and 10. In this case, $\gamma = 0.96$ for TLT predicted values and 1 for OP predicted values. Though TLT values are more closely aligned with the actual results, they are not in complete concordance. While OP predicted values generally differ greatly from actual values, they are in concordance. These results suggest that neither SRC nor $\gamma$ coefficients are entirely suitable for comparison of predicted values across the TLT and OP models. Moreover, Kendall’s $\tau$ coefficient suffers from similar problems as the estimation is also based upon concordance. The validity of using such techniques to compare the two models is questionable, leading us to depend upon tests of differences in the index values for both models, as was done earlier in the paper.

Conclusions

Conjoint analysis (CA) has increased in popularity among agricultural economists in recent years. The technique has been used to estimate consumer preferences for a variety of new food products, to analyze consumer preferences for food safety attributes, and to estimate consumers’ willingness-to-pay for recreational services. An important methodological question among these studies is whether interval-rating scales capture cardinal information, which has implications for model selection. This paper develops formal procedures for testing cardinal and ordinal assumptions in conjoint analysis. Theoretical concepts of equal interval and ordinal scaling are linked to the underlying assumptions of the TLT and OP models. The paper shows how an OP model can be used to test the equal interval assumptions of the TLT model. The analysis involves using Wald procedures to test the null hypothesis of equally spaced thresholds in the OP model. The null hypothesis
was tested and rejected for three separate conjoint data sets, implying that the underlying cardinality assumption of the TLT model is invalid for these samples. Therefore, we conclude that the OP model is the theoretically correct model for estimating part-worth parameters for these samples. However, the equal-interval assumption may hold for other studies given different experimental conditions.

Since the TLT model is often used in the agricultural economics literature to estimate part-worth values, the effects of model misspecification are examined by analyzing index function parameters. Casual inspection of the standardized part-worth parameters showed little difference between the TLT and OP models. Moreover, the Hausman null hypothesis was rejected for all three data sets, indicating that TLT part-worth estimates are not statistically different from the OP estimates. Therefore, despite rejection of the equal interval assumptions of the TLT model, the models yield virtually identical estimates of the underlying utility scale. This implies that the TLT model provides a close approximation of the theoretically correct OP model, and that the TLT estimates are not particularly sensitive to the cardinality assumption. This is an important finding since estimating the underlying utility scale is the central focus of most CA studies. Additionally, in studies where degrees of freedom are constrained, the TLT model may be preferable since it generally requires fewer degrees of freedom for estimation. This is particularly relevant for studies that estimate individual-level models.

Unlike the analysis of index function parameters, comparisons of predicted values for the two models lead to inconclusive results. The two models yield conceptually different predictions for the observed interval-rating scale. The TLT model assumes equal spacing and continuity of the observed scale, thus yielding non-integer predictions. The OP model assumes ordinal spacing of the observed scale, thus yielding discrete predictions that result in numerous ties between actual and predicted values. Moreover, comparison of predicted values using standard techniques for analyzing rank-order data, such as SRC coefficients, $\gamma$, and Kendall’s $\tau$, are invalid. Spearman’s rank-order coefficient favors the TLT model because of the non-integer nature of its predictions, whereas $\gamma$ and Kendall’s $\tau$ favor the OP model because they measure the degree of concordance.

Our general conclusion is that, while modern economic theory and the empirical rejection of equal-interval cardinality suggest that the OP is theoretically superior to the TLT model, researchers relying upon part-worth estimates from conjoint analyses are unlikely to find significant differences in the part-worth estimates between the two models. Theory leads to recommendation of the OP model, while empirical evidence suggests that, in estimating part-worth utilities, either can be used. In cases where there are too few degrees of freedom to estimate an OP model, the TLT is likely to be the best option. The similarity in model estimates also implies that utility estimates are not particularly sensitive to assumptions regarding interpersonal comparisons. In fact, if one carefully plans the conjoint design, there are often enough degrees of freedom to estimate individual preference functions using the TLT model, which avoids the pooling of individual preferences altogether. Individual estimates are usually not possible with the OP model.

References


